4pAO1. Acoustic scattering from a water-filled cylindrical shell: Mode identification and interpretation via finite element and analytical models

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Understanding the physics governing the interaction of sound with targets in an underwater environment is essential to improving upon existing target detection and classification algorithms. Simple models are viable tools for meaningful interpretation of scattering results. To illustrate this, two modeling techniques are employed to study the acoustic scattering from a water-filled cylindrical shell. The first model is a hybrid 2-D/3-D finite element (FE) model, whereby the scattering in close proximity to the target is handled via a 2-D axisymmetric FE model, and the subsequent 3-D propagation to the farfield is determined via a Helmholtz integral. This model is characterized by the decomposition of the fluid pressure and its derivative in a series of azimuthal Fourier modes, a technique that has previously facilitated mode identification [A. L. Espana et al., J. Acoust. Soc. Am. 130, 2332 (2011)]. The second is an analytical solution for an infinitely long cylindrical shell, coupled with a simple approximation that converts the results to an analogous finite length form function. These two model results, when examined together on a mode-by-mode basis, offer visualization of the mode dynamics and the ability to distinguish the different physics driving the target response (i.e. structural modes versus water-waveguide modes).

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1. INTRODUCTION

The ability to locate and classify objects in contact with the ocean sediment is a problem receiving increasing attention. The acoustic response of targets near the sea floor not only depends on the target physics (such as material and shape), but also on the surrounding environment and its location within this environment. This paper aims to tackle one piece of this problem, namely identifying how various aspects of the target physics contribute to the acoustic scattering. It does so in the context of a water-filled, aluminum, cylindrical shell (no end-caps). While a simple geometrical shape characterizes this target, its acoustic response is rich with resonances driven by either the elastic/structural properties of the shell, the liquid inside, or a combination of the two.

To aid in the interpretation of the acoustic response of this target, two different models will be used, each employing different levels of approximation. This leads to the secondary goal of this paper, which is to evaluate how these approximations impact the fidelity as compared to experimental data, in addition to highlighting the usefulness of these models as a means to interpreting the physics driving the target response. The first model examined is designed to model the scattering problem as closely as possible using a hybrid finite element (FE) method. The second model is the analytical solution for the forward scattering from an infinitely long shell, of identical radius, wall thickness, and material properties.

The paper is organized as follows. Section 2 shows the results from an experiment that measures the acoustic scattering from a water-filled cylindrical shell in the free field, the aim being to familiarize the reader with the complexity of this particular target's acoustic response. This measured data will also provide a means to validating the model results. The next two sections each focus on a particular modeling technique. Section 3 presents FE calculations for the elastic shell in the free field environment, while Section 4 presents the full analytical solution for the forward scattering from an infinitely long cylindrical shell of identical inner and outer-radii and material properties. Comparison of these model results offers incite into the physics driving the shell's response, which forms the discussion in Section 5. Identifying the key physics that dominate the target response is the first step towards being able to make predictions about how a target's signature evolves from environment to environment, and under different deployment conditions.

2. EXPERIMENT

The target investigated here is an aluminum cylindrical shell having an outer radius \( a = 0.0135 \) m, an inner radius \( b = 0.0127 \) m, and measuring 0.0508 m in length. The acoustic response of this target was measured in a tank facility located at Washington State University. The facility consists of a large redwood tank, 12 feet in diameter, filled with water to a depth of approximately 7.5 ft. The source is a 4-panel composite, piezoelectric transducer, mounted on a pole roughly 4 ft below the surface of the water. A relay system was used to allow the transducer to be used as a source and receiver to measure the backscattered pressure. The source outputs a linear-phase FIR signal with bandwidth of 50-400 kHz, resulting in a \( k a \) range of 2.7 - 21.8. The shell was suspended in the water from a rotation stage located above the tank. The backscattering was recorded as a function of the target's azimuthal rotation angle, running from 0 deg. (broadside) to 90 deg. (end-on). The measured pressure amplitudes are converted to absolute target strength and plotted as a function of \( k a \) and azimuthal rotation angle in Figure 1 (a). Further details of the experimental system, including the calibration method, are given in Ref. 1.
3. HYBRID 2-D/3-D FINITE ELEMENT MODEL

3.1. Model Formulation

The word 'hybrid' is used to indicate that two different models are employed, namely an FE model to handle the target scattering in the near field, coupled to a 3-D propagation model to calculate the pressure scattered to the far field. The model takes advantage of the cylindrical symmetry of the problem by decomposing the full 3-D problem into a series of independent, 2-D Fourier modal sub-problems. The commercial FE program COMSOL is used to build up a 2-D axisymmetric model of the shell surrounded by water, with a perfectly matched layer (PML) surrounding the entire domain. The shell is subjected to plane waves and the resulting pressure and its derivative are calculated on a discrete set of points on a surface closely surrounding the target (much less than one wavelength at the highest frequency). These pressure and derivatives, coupled with the free field Green's function, are subsequently used in the discrete form of the Helmholtz integral to calculate the far field pressure scattered back to the source location. Further details of this process can be found in Refs. 2, 3 and 4.

3.2. Results

Figure 1 (b) shows the absolute target strength as a function of ka and the target's azimuthal rotation angle (equal to the incident angle in this free field case). Comparison to the free field data in Figure 1 (a) shows that the model does a good job of capturing the target's response. To better gauge how well the two agree, slices are taken at six different angles in the acoustic templates and are shown in Figure 2. There is especially good agreement at broadside (0 deg.) and end-on (90 deg.), and even in the fine structure at angles in between. The largest differences occur in the sharp resonances observed in the model results. These will not be excited in experiments, due to the finite pulse width of the incoming field not having the required duration to excite the resonance.
3.3. Modal Decomposition

Now that the FE calculations have been validated by experimental results, the task of interpreting the physical mechanisms driving the target response remains. Herein lies the advantage to running models, specifically models of this nature that use a coupled target scattering/propagation method. It’s possible to separate the different pieces of physics that drive the target’s response by examining them on a mode-by-mode basis, where the mode number $m$ is the modal order in the Fourier decomposition. Figure 3 shows the free field FE results for modes 0 - 5.

There are a number of interesting observations that can be made when looking at these modal plots. The first is that there are distinct arcs in every one of these templates. Furthermore, the onset of these arcs changes as one transitions up in mode number. This appears in the templates as a progression of the dark space in the plots, less and less of the template shows any acoustic response from the shell.

The second important observation is that the strongest response from the shell, other than broadside of course, is the region from 10 - 30 degrees, a fact that is consistent over all the mode numbers. There are sharp resonances at higher angles, but these are difficult to excite when conducting experiments in a real ocean environment. Due to this, focus turns to understanding the manifestation of these arcs, and more importantly, the areas of strongest response located below 30 deg. In what follows, discussion is restricted to only looking at mode 1 results (Fig. 3 (b)) with the understanding that an analogous interpretation and analysis exists for all other modes.
4. INFINITE SHELL MODEL

4.1. Scaling infinite results to an equivalent finite form function

To aid in the interpretation of the shell results, it’s useful to look at the scattering from an infinitely long shell. The scattering from an infinite cylindrical shell has the advantage of being a problem that is solved exactly by a partial-wave series, the details of which can be found in Ref. 5. Although this model lacks a major geometrical feature that contributes to the scattering, namely the truncated ends, this type of calculation does contain all of the elasticity of the shell, and can presumably aid in our physical interpretation. The details for this calculation are well established in literature,\(^5\) and therefore are not discussed here, however it is important to establish the relevance of this calculation to the actual measured data and FE calculation.

In this section we outline the concept of scaling the infinite form function results of Ref. 5 to an equivalent finite form function. We start at the expression for the far field scattered pressure from a finite cylinder widely used in literature\(^6,7\)

\[
|p_s| = \frac{a}{2\pi} |f_{cyt}| |p_{inc}|
\]  

The finite form function \(f_{cyt}\) can be written as a product between an infinite form function, called \(f_{2D}\), and a second piece that encompasses all of the end effects, called \(f_{3D}\), as follows

\[
f_{cyt} = f_{2D} f_{3D}
\]  

The advantage of writing the form function in this manner is that \(f_{2D}\) encompasses all of the elasticity of the target, while \(f_{3D}\) contains effects solely arising from the finite length of the target, and thus is given by the equivalent form function for a finite, rigid cylinder\(^8,9\)

\[
f_{3D} = \frac{2L}{a} \left( \frac{ka}{4\pi} \right)^{1/2}
\]  

Plugging the above into the expression for \(f_{cyt}\), and subsequently into \(|p_s|\), yields the following

\[
|p_s| = \frac{L}{r} \left( \frac{ka}{4\pi} \right)^{1/2} |f_{2D}| |p_{inc}|
\]
In previous sections, the data and FE calculations were presented in the form of target strength, or

\[ TS = 20 \log_{10} \left( \frac{r_{\text{p}}}{r_{\text{incl}}} \right) \]  \hspace{1cm} (5)

Based on Eqn. 4, the quantity \( L \left( \frac{ka}{4\pi} \right)^{1/2} |f_{2D}| \) is equivalent to target strength, and thus reveals that simply multiplying the infinite shell results by the factor \( L \left( \frac{ka}{4\pi} \right)^{1/2} \) will convert it to an equivalent finite form function that can be compared to the measured data and FE results. It should be noted that the above result is equivalent to that obtained by Stanton\(^8\) via an alternate derivation.

Figure 4 shows this comparison, for the backscattering at broadside orientation, as a function of \( ka \). The blue-dashed curve is the infinite shell calculation with the aforementioned scaling applied to convert it to an equivalent finite shell result. The data is represented by the solid black curve and the hybrid 2-D/3-D FE calculation is the red-dashed curve. The scaled infinite results agree remarkably well with the data and FE model, thereby establishing the validity of the approach.

**FIGURE 4.** Comparison of the absolute target strength as a function of \( ka \) for the backscattering of sound by a water-filled cylindrical shell in the free field, calculated using the hybrid 2-D/3-D FE model (red-dashed), an analytical model for an infinite shell (blue-dashed), and acquired experimentally in a large tank facility (black).

4.2. Air-filled, water-filled and the like

Ultimately the goal is to understand the backscattering from the shell over a broad range of angles, not just broadside. Looking at the exact solution for the scattering in the specular direction from an infinite shell is a viable tool for this. It has been shown that strong contributions to the scattering in the specular direction arise from various elastic responses, including various types of guided waves down the shell.\(^{10}\) When dealing with a finite length shell, these same mechanisms exist, only now the energy reflects from the end of the shell and strongly radiates sound in the backward direction.\(^{10,11}\)

Enhancements due to guided waves have been studied extensively in literature (Refs. 10-12 just to name a few). Many times a rigid cylinder form function is used as a background which is subtracted from the elastic response, the result being to highlight only the elastic contributions to the backscattering. Instead, a subtraction technique is offered here that serves to identify the specific drivers of the resonances, namely the structural/elastic piece of the target, the liquid inside, or a combination of the two. The exact PWS solution is calculated for an infinitely long, air-filled, cylindrical shell, as well as a water-filled shell. Figure 5 shows the absolute target strength in the specular direction for the (a) air-filled and (b) water-filled shells, as a function of azimuthal rotation angle and \( ka \), for mode 1 only. The appropriate scaling has been applied to convert these results to an equivalent finite shell form function as discussed in the previous section. The complex amplitude in the air-filled case is subtracted from the complex amplitude in the water-filled case, the result of which is shown in Fig. 5 (c), again for mode 1 only. This plot shows the areas of enhancements that are due to motion of the water inside the shell, which appear as the bright arcs extending from end-on down to broadside. These coincide with the arcs of Fig. 3 (b) for the FE calculation, with the exception that the FE arcs have an added modulation to them due to end effects arising from the finite size of the shell used in the calculation. Another observation is the arcs appearing as deep nulls that asymptote to...
approximately 15 deg. and 28 deg. Since these appear as deep nulls, it indicates that they correspond to elastic modes of the shell itself and are not driven by the liquid inside. These particular modes here are known to correspond to leaky Lamb waves launched on the shell, travelling in a helical path down the shell, reflecting from the end and ultimately radiate sound in the backward direction. These waves can be compressional, shear or flexural in nature. The compressional wave has a cutoff angle defined by $\gamma_c = \sin^{-1}(c/c_p)$, where $c_p$ is the plate speed. The shear wave cutoff is defined by $\gamma_T = \sin^{-1}(c/c_s)$, where $c_s$ is the shear speed of the shell material. For the aluminum shell used here, the cutoff angles are $\gamma_c = 15.8^\circ$ and $\gamma_T = 28^\circ$. A simple relationship can be derived that defines the frequency-angle combinations where coupling to these surface guided waves is possible. Starting with the helical ray coupling condition

$$\sin(\gamma_c)\cos(\psi) = \sin(\gamma)$$

and combining it with the circumferential reinforcement condition

$$\sin(\psi) = (\omega_n/\omega)$$

yields the following for the coupling condition

$$\sin^2(\gamma_c)[1 - (\omega_n/\omega)^2] = \sin^2(\gamma)$$

Here, $\gamma_c$ corresponds to the cutoff angle for the specific helical wave, which was calculated above for the compressional and shear waves for the aluminum shell of this paper. $\omega_n$ is the threshold frequency, defined as the value of $\omega$ at broadside orientation $\gamma = 0^\circ$. The significance of these coupling curves is discussed in the section that follows.

![Figure 5](image)

**FIGURE 5.** Free field, backscattered, target strength as a function of $ka$ and angle for mode 1 only, calculated using an analytical model for an infinite (a) air-filled shell and (b) water-filled shell. (c) shows the subtraction of the two, revealing areas of the scattering that are driven primarily by the water inside the shell.

## 5. DISCUSSION

While the previous section illustrated a technique that allows for the identification of resonance conditions that are driven by either the water inside of the shell or the elastic/structural properties of the shell, these were all done using the exact PWS solution for an infinite shell. It is important to tie this back into the backscattering from a finite length shell. Figure 6 (a) re-plots the result of Fig. 5 (c) for mode 1, along with the coupling curves for the shell defined by Eqn. 8. The compressional wave coupling is given by the red curve and the shear wave coupling the green curve. These same coupling curves appear in Fig. 6 (b), on top of the FE model results for the backscattering from the finite length shell (mode 1), the solid curve for compressional waves and the dashed-dot curve for shear waves. Both of these curves lie directly on top of the strong resonances characterizing this particular mode. It is important to note that these coupling curves give the frequency-angle locations where it is possible for sound to couple into helical waves on the shell, but they do not dictate the exact location where a strong resonance will be observed. To understand how the resonances arise it’s helpful to turn to an analysis by Rumerman, who shows that a resonance arises when the frequency and angle of the incident wave vector are such that the axial projection of the
incident wave is equal to an integer number of half-wavelengths. This is expressed mathematically by the following expression

\[ \text{ka} \sin(\gamma) = n\pi a/L \]  

These curves are plotted on Fig. 6 (b) as the black long-dashed lines, for the first six modes. The intersection of these axial curves with the elastic coupling curves coincide with bright resonances and represent the locations where the incident wave can not only couple onto helical modes of the shell, but is also trace-matched for axial resonances.

![FIGURE 6](image)

To summarize, the backscattering from a water-filled cylindrical shell was measured in a free field tank experiment, revealing the complexity of the acoustic response from this simple geometrical target. A hybrid 2-D/3-D FE model was presented and was shown to agree well with the experimental results. The hybrid model technique, characterized by the decomposition of the 3-D problem into a series of axisymmetric 2-D Fourier sub-problems, enabled the analysis of the target response on a mode-by-mode basis. While this highlighted the dominant characteristics making up the target response, it was not until the analysis of the scattering from an infinitely long shell was presented that a meaningful interpretation was possible. The scattering from an infinite air-filled and water-filled shell was calculated and a simple method of scaling the results to an equivalent finite form function was highlighted. By subtracting the two complex amplitudes, it was possible to identify the regions of the target response that are driven by the motion of the water inside the shell, as well as locations that are instead driven by the elasticity of the shell. A simple coupling condition for elastic modes was derived. Ultimately, the dominant resonances characterizing the finite shell’s backscattering response are found to lie on these elastic coupling curves, resonance locations defined by the frequency-angle pairs for which there is both trace matching with the incident field and an integral number of half wavelengths along the length of the shell. In the end, the combination of the hybrid FE model and the exact solution for an infinitely long shell revealed the different pieces of physics driving the target response.

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7. REFERENCES