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1aAAa1. An objective measure for the sensitivity of the room impulse response
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This study is relevant for a number of important acoustic measurements in reverberation rooms such as measurement of sound transmission and measurement of sound power levels of noise sources. From a pair of impulse responses measured in a room differing only in the position of the sound source, it might be possible to quantify the sensitivity of the room due to changes in initial conditions. Such changes are linked to mixing. The proposed measure is the maximum of the absolute value of the cross-correlation between the time windowed sections of the two impulse responses. By integrating this quantity normalized by the energy of the impulse response of the room, a single number rating is obtained. The proposed measure is examined experimentally and the results are discussed. The results indicate that the number of absorbers and diffusers in the room influences the proposed measures systematically.

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INTRODUCTION

A vast majority of room acoustic parameters are calculated on the basis of measured room impulse responses (RIRs) or their frequency domain counterpart, i.e. frequency responses. Supplementary, it is possible to introduce room acoustic parameters which are calculated on the basis of the variation of the RIR and can be understood as second order room parameters. It is reasonable to expect that additional information about the acoustic properties of the room can be obtained from such parameters.

According to the authors’ knowledge there have not been many attempts to define and investigate second order parameters as they are less intuitive to understand and more complex to interpret compared to first order parameters, which have been, in contrast, more intensively studied and standardised [1]. In this perspective this study can be understood as an attempt of finding a consistent description of a simple example of a second order room acoustic parameter, to define it and test its behaviour.

This is done by first presenting the analogy between a sound source in a room and a billiard game. The latter is extensively studied in the theory of dynamical systems [2] and concepts like mixing, Lyapunov exponents and sensitivity are presented. On this basis a sensitivity function ($\Omega$) is defined that measures the sensitivity of the RIR on the change in the sound source location. $\Omega$ is the basis to introduce the purposed objective measure named sensitivity rating ($\Gamma$). Both, $\Gamma$’s and $\Omega$’s, behaviour is systematically tested in different experimental conditions and the results are analysed and discussed. The main hypothesis being tested is that the complexity of the room shape increases the sensitivity of the RIR to the change of the sound source location. For a more detailed and complete study the reader is directed to [3].

METHOD AND THEORY

A sound impulse being radiated from a point source in a room can be described as a dynamical system. To establish properly the analogy, the theory of dynamical systems must first be introduced and only next the measure is derived.

The sound field in a room as a dynamical system

The most representative example in the theory of dynamical systems systems is the billiard game [4] where a bouncing ball is shoot from the initial position into a given direction and specularly reflected on the boundaries. By assuming that the kinetic energy of the ball is preserved in wall reflections the evolution of the dynamical system is analogue to the propagation of a portion of the sound front of the sound impulse radiated from a point source. The analogy is justified in the high frequency regime (geometrical acoustics [5]) where the propagation of sound as rays is justified. For the description of the sound field as a dynamical system the reader is directed to [6].

It follows that by shooting many balls evenly distributed in space angle with speed equal to the speed of sound propagation is analogous of the sound source radiation of a pulse. As there is no energy loss assumed in the billiard case, there should be also no energy loss in the acoustic case. This is of course never true, and a sound impulse can only be observed for a limited amount of time.

Systems that are highly sensitive to a change in the initial conditions are of special interest in theory of dynamical systems [2]. The measure for the sensitivity on the initial condition is the Lyapunov exponent which is a quantity characterizing the degree of separation of infinitesimally close trajectories. Systems having positive Lyapunov exponents are referred to as chaotic [2] and have the property of mixing. Mixing systems behave in a way that when $t \rightarrow \infty$ the state of the system cannot be linked to its initial condition [6]. This at the same time means that the information about the initial condition of the system is being lost during the evolution of the system. The dynamics with which the information is being lost is characterized by the Lyapunov exponent.

There have been attempts to introduce mixing and Lyapunov exponent into acoustics[7], [8] but it is impossible to go around the fact that that an isolated trajectory in the acoustic analogy does not exist. As a consequence the degree of separation of trajectories cannot be observed nor measured. Therefore the sensitivity of the system should be rated differently.
The Sensitivity Function

The RIR represents the time evolution of a sound impulse being radiated from a sound source and observed at a microphone position. By observing the evolution of such an impulse and comparing it to the evolution of an impulse radiated from the sound source being moved by a small extent there is a time progressing divergence between them. For a graphical representation of the above-mentioned two time sections of two RIRs measured with sound sources displaced for 3.8 cm are plotted on Fig 1. The two RIRs almost coincide at the direct contribution (the peak at 2 ms) while they are becoming less alike with time.

![FIGURE 1. Two time sections of two measured RIRs differing only in the location of the sound source which was moved for 3.8 cm in between the measurements.](image)

To measure the degree of the divergence a sensitivity function is constructed as follows. The two RIRs measured at a slightly different sound source locations are labelled $s_1$ and $s_2$. First octave bands filtration on both RIRs is performed so as to be able to observe frequency related phenomenon. This is done according to the ANSI standardised octave bands[9]. Next time windowed sections of $s_1, T, \Delta T(t)$ and $s_2, T, \Delta T(t)$ are considered with $T$ and $\Delta T$ as parameters of windowing as shown in Fig. 2.

![FIGURE 2. Time windowing of a RIR. The windowed part of the impulse response is the part between the vertical dashed lines where $T$ and $\Delta T$ are parameters of windowing.](image)

In the following step the cross-correlation [10] of both widowed sections of RIRs is calculated as:

$$X_{T,\Delta T}(t) = \int s_{1, T, \Delta T}(\tau) \cdot s_{2, T, \Delta T}(\tau + t) d\tau.$$  

The amount of information both signals share increases with the increasing value of the absolute maximum of the cross-correlation function, which is a broadly used concept in statistics. Labelling it $\Omega$, it reads mathematically:

$$\Omega_{\Delta T}(T) = \max(|X_{T,\Delta T}(t)|).$$  

To be able to compare results is $\Omega$ normalized to the energy of both RIRs

$$E_1(T) = \int s_{1, T, \Delta T}^2(\tau) \, d\tau, \quad E_2(T) = \int s_{2, T, \Delta T}^2(\tau) \, d\tau.$$  

So the sensitivity function $\Omega$ is defined

$$\Omega_{\Delta T}(T) = \frac{\max(|X_{\Delta T}(t)|)}{\sqrt{E_1E_2}},$$ \hspace{1cm} (4)

which is a measure of how much of the same information both RIRs contain as a function of time, with $\Delta T$ as a parameter. The value of $\Omega$ can be between 1, if both time cropped sections of the RIRs are equal, and 0 if both time cropped sections of RIRs are completely uncorrelated. The latter is only true if $\Delta T$ limits toward infinity, but as $\Delta T$ is always limited, $\Omega = 0$ is never achieved. Additionally as $\Omega$ is calculated from experimentally measured RIR, some noise is introduced and the limiting cases of $\Omega = 1$ is never achieved as well. It is assumed, however, that the noise is stochastic and uncorrelated and contributes insignificantly to the cross-correlation value.

Considering the mentioned measuring and signal processing limitations, the following behaviour is expected. If time windowing includes only the direct contribution of the RIR, which is not influenced by the room, then it is expected to observe $\Omega \approx 1$. On the contrary, the latest part of the RIR is composed only of measurement noise, and if the noise in between measurements is uncorrelated, then $\Omega \approx 0$. It is important to emphasize that due to the maximum function in eq. 2 are high values of $\Omega$ achieved if both RIRs are time delayed versions of each other.

Between the direct component and background noise at the end of the measured RIR a transition from $\Omega \approx 1$ to $\Omega \approx 0$ is expected. There are two underlying processes causing this transition:

1. The SNR is decreasing, meaning that there is more stochastic content (background noise) in the signal (in a relative sense). This causes $\Omega$ to decrease.
2. RIRs are very similar in the beginning part, which includes the direct contribution and first reflection, but are becoming more and more uncorrelated with time. The dynamics of this process is defined by the sensitivity of the room as a dynamical system.

The intention is to rate the process number 2) while the process number 1) is limiting the observation.

The sensitivity rating

The main idea is that $\Omega$ drops more rapidly if the sensitivity of the room is higher. In any case $\Omega$ cannot be used to order rooms according to their sensitivity as $\Omega$ is not a single number quantity. Therefore an alternative measure, $\Gamma$, is defined. $\Omega$ as defined above is a factor between 0 and 1 describing the amount of correlation between signals. In an analogue way, $1 - \Omega$ is a factor that describes the amount of “uncorrelation” between the two signals and is again taking values between 0 and 1.

So it is possible to define the correlated and the uncorrelated share of energy in the RIR, by a weighted integration of RIR’s energy with the mentioned factor:

$$\Omega_{\Delta T}(T) = \int_T^\infty |H_{\Delta T}(t)| dt,$$

$$\int_0^\infty \Omega(t) \sqrt{E_1E_2}(t) dt.$$

(5)

The integration limit $T$ is the time when the RIR hits the background noise level of the measurement. The ratio of uncorrelated ($E_{uncorr}$) and total ($E_{corr} + E_{uncorr}$) is defined as the sensitivity rating,

$$\Gamma = \frac{E_{uncorr}}{E_{corr} + E_{uncorr}}.$$ \hspace{1cm} (6)

The evaluation of $\Gamma$ is performed with the expression

$$\Gamma = \frac{\int_T^\infty (1 - \Omega(t)) \sqrt{E_1(t)} dt}{\int_0^\infty \sqrt{E_1(t)} dt}.$$ \hspace{1cm} (7)

$\Gamma$ is a scalar quantity without units with the following important properties:

- If RIRs are measured in the free field, only the direct contribution exists and $E_{uncorr} = 0$ leading to $\Gamma = 0$.
- $\Gamma$ is increases if the sensitivity of the RIR to the change in the sound source location is higher.
- $\Gamma$ approaches 1, if the sensitivity to the change of the sound source location is very high and both RIRs are completely uncorrelated immediately after the direct contribution.
It should be clear that due to signal processing limitations and measurement noise sharp values of $\Gamma = 0$ and $\Gamma = 1$ are never achieved.

**EXPERIMENTS**

RIR pairs were measured, where each of them with a slightly different sound source location. The change in the sound source location is referred to as displacement and was in the range between 1 cm and 10 cm. It has to be emphasized that except for the sound source location nothing else was changed inside the room in between RIR pair measurements.

The measurements were performed with a high quality loudspeaker, a measurement free field microphone, an external soundcard and an amplifier. The used excitation signal was an exponential sweep.

The measurements of RIR pairs were performed in different conditions so that the amount of diffusing and absorbing elements was changed systematically. During the measurements there were no additional objects inside the room except the measuring equipment. The only consideration about the microphone and sound source positions was to avoid placing them at the same height, while the influence of their position on $\Gamma$ and $\Omega$ was not analysed systematically. Three experimental setups were performed.

Experimental setup 1. The intention of this experimental setup was to evaluate the influence of diffusing elements and absorption on $\Omega$ and $\Gamma$. The experiment was conducted in a rectangular room of dimensions $w \times l \times h = 328 \, \text{cm} \times 438 \, \text{cm} \times 295 \, \text{cm} \, (V \approx 42 \, \text{m}^3)$ with concrete walls.

Four series of measurements, designated a, b, c and d, were performed as summarised in table 1 together with their influence on the reverberation time ($T_{20}$). The addition of diffusing elements and absorbing elements are shown in Fig. 3. The diffusing elements were 3 concrete bell shaped objects and a wooden wedge, all of them placed on the floor. The absorbing elements were mineral wool and polyurethane foam plates evenly distributed on walls, corners and floor.

<table>
<thead>
<tr>
<th>designation</th>
<th>diffusers</th>
<th>absorbers</th>
<th>$T_{20}$ [s] @ 500 Hz</th>
<th>$T_{20}$ [s] @ 1 kHz</th>
<th>$T_{20}$ [s] @ 2 kHz</th>
<th>$T_{20}$ [s] @ 4 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>no</td>
<td>no</td>
<td>2.95 ± 0.03</td>
<td>2.85 ± 0.01</td>
<td>2.14 ± 0.01</td>
<td>1.61 ± 0.01</td>
</tr>
<tr>
<td>b</td>
<td>yes</td>
<td>no</td>
<td>2.63 ± 0.01</td>
<td>2.63 ± 0.01</td>
<td>2.02 ± 0.02</td>
<td>1.54 ± 0.02</td>
</tr>
<tr>
<td>c</td>
<td>no</td>
<td>yes</td>
<td>1.27 ± 0.02</td>
<td>1.28 ± 0.01</td>
<td>1.12 ± 0.01</td>
<td>1.01 ± 0.01</td>
</tr>
<tr>
<td>d</td>
<td>yes</td>
<td>yes</td>
<td>1.23 ± 0.09</td>
<td>1.19 ± 0.09</td>
<td>1.10 ± 0.06</td>
<td>1.02 ± 0.04</td>
</tr>
</tbody>
</table>

FIGURE 3. Experimental setup 1: the addition of diffusing elements (left) and absorbing elements (right).

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1 The author is aware that a omnidirectional source would be needed for proper RIR measurements, but the directionality of the used loudspeaker is believed not be of central importance for the conducted study.

2 The following equipment was used: conditioning preamplifier (Brüel&Kjær, NEXUS), sound source (Dynaudio, BM6, passive nearfield monitor), sound card (M-AUDIO, Mobilepre Usb), pressure field microphone (Brüel&Kjær, BK-4192), personal computer (HP, EliteBook 8540p), amplifier (built at DTU, single channel).
Experimental setup 2. The intention of this experimental setup was to evaluate the influence of different types of diffusing elements on $\Omega$ and $\Gamma$. The experiment was conducted in the same room as Experimental setup 1, but with a different sound source and microphone location.

Three series of measurements, designated A, B and C, were performed as summarised in table 2 together with the corresponding reverberation time. The used diffusing elements were 2 gypsum spheres on a stand and wooden wall diffusers (Fig. 4).

TABLE 2. Experimental setup 2 summary.

<table>
<thead>
<tr>
<th>designation</th>
<th>diffusers</th>
<th>$T_{20}$ [s] @ 500 Hz</th>
<th>$T_{20}$ [s] @ 1 kHz</th>
<th>$T_{20}$ [s] @ 2 kHz</th>
<th>$T_{20}$ [s] @ 4 kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>no</td>
<td>3.02 ± 0.06</td>
<td>2.76 ± 0.02</td>
<td>2.18 ± 0.01</td>
<td>1.70 ± 0.01</td>
</tr>
<tr>
<td>B</td>
<td>2 spheres</td>
<td>2.88 ± 0.04</td>
<td>2.67 ± 0.02</td>
<td>2.07 ± 0.01</td>
<td>1.65 ± 0.02</td>
</tr>
<tr>
<td>C</td>
<td>wall diffusers</td>
<td>2.76 ± 0.05</td>
<td>2.52 ± 0.02</td>
<td>1.89 ± 0.02</td>
<td>1.52 ± 0.02</td>
</tr>
</tbody>
</table>

FIGURE 4. Experimental setup 2: the addition of 2 spheres as diffusing elements (left) and wall diffusers as diffusing elements (right).

Experimental setup 3. The intention of this experimental setup was to check whether the room size influences $\Gamma$ and $\Omega$. For this purpose the experiment was performed in a larger room $w \times l \times h = 836\,\text{cm} \times 627\,\text{cm} \times 510\,\text{cm}$, ($V \approx 267\,\text{m}^3$).

Three series of measurements, designated $\alpha$, $\beta$ and $\gamma$, were performed as summarised in table 3 together with the corresponding reverberation time. The used diffusing elements were the same spheres as in the experimental setup 2 and 20 plastic panels hanging evenly distributed in the room as shown in Fig. 5.

FIGURE 5. Experimental setup 3: the addition of 20 panels (left) and 2 spheres (right) as diffusing elements.

TABLE 3. Experimental setup 3 summary.

<table>
<thead>
<tr>
<th>designation</th>
<th>diffusers</th>
<th>$T_{20}$ [s] @ 500 Hz</th>
<th>$T_{20}$ [s] @ 1000 Hz</th>
<th>$T_{20}$ [s] @ 2000 Hz</th>
<th>$T_{20}$ [s] @ 4000 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>no</td>
<td>10.77 ± 0.13</td>
<td>9.14 ± 0.05</td>
<td>6.44 ± 0.04</td>
<td>3.58 ± 0.03</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2 spheres</td>
<td>10.69 ± 0.01</td>
<td>9.15 ± 0.08</td>
<td>6.33 ± 0.05</td>
<td>3.55 ± 0.03</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>20 panels</td>
<td>8.26 ± 0.07</td>
<td>6.83 ± 0.04</td>
<td>5.25 ± 0.02</td>
<td>3.14 ± 0.03</td>
</tr>
</tbody>
</table>
RESULTS AND ANALYSIS

From the measured RIR pairs $\Omega$ and $\Gamma$ were calculated using $\Delta T = 100$ ms and in standardised octave bands [9]. The result analysis is performed for each experimental setup separately.

Experimental setup 1. The frequency and displacement dependence of $\Gamma$ is presented in Fig. 6. Both of them show an increasing trend, which can be explained. First, the increase of displacement directly leads to a larger change in the initial condition. This leads to a more divergent RIR pair which is reflected in the value of $\Gamma$. This is in agreement with the introduced theory. Second, at higher frequencies are the observed wavelengths shorter. This means that a constant displacement becomes more drastic relative to the wavelength when the frequency is increasing. To check that other phenomenon linked to frequency filtration are not the reason for the observed behaviour the same analysis in third octave bands was performed and similar behaviour was observed.

From Fig. 6 it can be also concluded that the addition of diffusing elements increase $\Gamma$ systematically while the addition of absorbing elements cause a drop of $\Gamma$ at frequencies below 2 kHz.

![Figure 6](image_url)

**FIGURE 6.** Experimental setup 1: left pane: frequency dependence of $\Gamma$ for a 3.8 cm displacement. Right pane: displacement dependence of $\Gamma$ in the 2000 Hz octave band. Designations a, b, c and d correspond to table 1.

In Fig. 7 the behaviour of $\Omega$ is presented for a 3.8 cm displacement used as a sample displacement as similar behaviour is seen for other displacements. The result with the designation 0 corresponds to a consecutive measurement of the RIR with no displacement and is meant to show the limiting values of $\Gamma$ and $\Omega$ as only noise is differing both RIR pairs. For the comparison of the results it can be assumed that $\Gamma_0$ represents the range of the measurement error.

![Figure 7](image_url)

**FIGURE 7.** Experimental setup 1: octave band behaviour of $\Gamma$ and $\Omega$ for a 3.8 cm displacement. Designations a, b, c and d correspond to table 1 while the 0 designation is the reference measurements.
From Fig. 7 the following conclusions can be made:

- The addition of diffusing elements causes a systematic rise of $\Gamma$, which is increasing with frequency.
- The addition of absorbing elements cause a drop of $\Gamma$ at lower frequencies (the effect can also be observed in Fig. 6).
- The addition of absorbing elements strongly affect $\Omega$ as it drops significantly earlier when absorption is added.

It must be also mentioned that the addition of diffusing elements slightly reduces the reverberation time in the room. Anyway it is clear that $\Gamma$ is not a direct function of the reverberation time, as $\Gamma$ in not changed substantially when high quantity of absorption is added (case $c$ and $d$).

**Experimental setup 2.** Analogue results as before are shown in Fig. 8 but for a displacement of only 3 cm. It is interesting to observe that the values of $\Gamma$ are not smaller in all octave bands although the displacement was smaller. This is due to a different measuring position as before (different microphone and sound source location).

From Fig. 8 it can be concluded that diffusing elements increase $\Gamma$ and the effect is increasing with frequency. It can also be concluded that spheres in this case cause a larger increase of $\Gamma$ compared to wall diffusers.

**FIGURE 8.** Experimental setup 2: octave band behaviour of $\Gamma$ and $\Omega$ for a 3 cm displacement. Designations A, B and C correspond to table 2 while the 0 designation is the reference measurements.

**Experimental setup 3.** Analogue results as in experimental sets 1 and 2 are shown again in Fig. 9 for a displacement of 3 cm. It can be concluded as before that the addition of diffusing elements cause $\Gamma$ to increase. It is interesting to observe that all the values of $\Gamma$ are smaller in the larger room compared to experimental setups 1 and 2 which were performed in a smaller room.

**DISCUSSION**

The most clear conclusion is that results show a systematic increase of the sensitivity rating $\Gamma$ with the addition of diffusing elements in the room. The effect increases with frequency and with the displacement.

The definition of a diffusing object and its effectiveness is a matter of discussion and in practice they are always objects that increase the complexity of the room boundaries making them uneven. The authors’ hypothesis is that increasing the complexity of the room shape leads to a higher sensitivity of the room. In this perspective are highly diffusing objects those that strongly increase the complexity of the room and $\Gamma$ could eventually be used as a basis to quantify the effectiveness of diffusing objects. It is important to be aware of the fact that the sound field interacts with the room geometry differently at different wavelengths. In this aspect the presented experiments do not give the basis for any conclusion to be made as the shape and size of the diffusing elements was not systematically studied.

On the other hand it was shown that the addition of absorption does only influence the sensitivity at lower frequencies where $\Gamma$ drops. In contrast to $\Gamma$, absorption strongly affects $\Omega$ in all octave bands by making it drop much earlier.
FIGURE 9. Experimental setup 3: octave band behaviour of $\Gamma$ and $\Omega$ for a 3 cm displacement. Designations $\alpha$, $\beta$ and $\gamma$ correspond to table 3 while the 0 designation is the reference measurement.

The experiment carried in a larger room showed the sensitivity to drop substantially but this result cannot lead to conclusive findings. First, because the sample of different rooms was only two and, additionally, because the variation of $\Gamma$ due to the microphone and sound source position in the room was not studied systematically. Nevertheless, it is clear that $\Gamma$ and $\Omega$ can be used in rooms of various sizes.

CONCLUSIONS

In this study an objective measure named sensitivity rating was introduced. It is a second order room acoustic parameter that can be used to quantify the sensitivity of the RIR on the sound source displacement. The measure was theoretically introduced and links to mixing and Lyapunov exponents were shown.

Experimental results have shown some good agreements with the hypothesis that more complexity of the room causes higher sensitivity. The absorption on the room boundaries was tested as well but seems it does not influence the sensitivity in the whole frequency range.

Although the results can be described as promising, some future work is needed to investigate the importance of the measurement position, diffusing elements shape, elements size and the size of the room.

REFERENCES