2aAA8. The increase of the sound absorption through rectangular slot perforations

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This paper describes several methods to predict the sound absorption of a surface with rectangular perforations. Theoretical and experimental works about rectangular perforations are reviewed and compared. In particular, the transfer matrix method is considered together with possible expressions for the surface, hole and cavity impedances. In this model, the surface impedance may be calculated summing resistance and mass terms to the characteristic impedance. Besides, fundamentals of models for numerical cavity modes, for microperforations and for double-porosity are described. A comparison of all previous models is also reported. Then, the paper shows the results of a few measurements of sound absorption of walls where rectangular slot perforations have been left between stone elements. Comparisons with theoretical models are carried out to explain different mechanisms of absorption. Finally, this paper aims to establish practical methods to develop broadband sound insulation devices through rectangular perforations.
1. INTRODUCTION

By using the phenomenon of resonance it is possible to gain absorption at frequencies where it would be difficult to achieve it with traditional porous materials. In fact, the low absorption of porous materials at low frequencies and their scarce performance wherever the particle velocity is low (which is often the case at environmental boundaries where they are commonly placed) require the introduction of other mechanisms for sound absorption [1,2]. The most common solution in these cases is offered by resonant devices. Two typical systems of this kind exist: the Helmholtz resonators and the resonant panels. This paper focuses on the early.

Helmholtz resonators appear as panels with holes. They were deeply studied by the physician Hermann von Helmholtz, and nowadays they still gain a significant attention worldwide. Their mechanism of absorption is based on the shortcoming of the sound waves out and in the holes: the air in the neck behaves as the vibrating mass against the spring constituted by the air in the cavity beyond the panel [3]. By changing the vibrating mass, and the stiffness and resistance of the rear air spring, the resonance frequency and absorption values may be tuned. Helmholtz resonators are often characterized by a high absorption in a small frequency range (this characteristics is measured by the ratio between the frequency range which corresponds to a reduction of 3 dB and the resonance frequency). To obtain absorption in a larger bandwidth, losses in the air cavity are often proposed by introducing porous material [4]. Another way for increasing losses is offered by microperforations, where viscous losses are created within the neck itself [5]. These different strategies are discussed below.

This paper focuses on rectangular perforations. These have several advantages as they may not need complex and expensive manufacture processes of bridling, but they can be realized by splitting elements which constitute the surface panel. In fact, an investigation of this system was also realized through tufa walls, whose blocks where mounted living air gaps between them.

This paper is structured in the following way: section 2 describes the different models for predicting the sound absorption of panels with slot perforations; section 3 reports the sound absorption measurements of a slotted wall and compare these with theoretical predictions; section 4 reports concluding remarks.

2. THEORETICAL MODEL OF ABSORPTION

This section describes different models for predicting the sound absorption of a slotted perforated panel. The following models are reported from the simplest to the most complex.

Single-layer method

The surface impedance of a simple absorber obtained by a perforated panel in front of a cavity is given by the summation of a resistance term, a mass term, and a cavity term:

\[ z_i = r_i + j \left( \omega m - \rho c \cdot \cot(kd) \right) \]  

where \(\omega\) is the angular frequency, \(m\) the acoustic mass per unit area of the panel, \(\rho\) the air density, \(c\) the sound velocity, \(k\) the wavenumber in air, and \(d\) is the distance of the surface from the rear wall. This simple formula ignores the presence of absorbent material in the cavity, which is hence assumed empty, and it is only valid for a thin sheet [6]. Known the expression of the surface impedance, it is possible to calculate the resonance frequency equaling to zero the imaginary part of the expression (1). For example, in the case of cavity much smaller than the acoustic wavelength, \(\cot(kd) \rightarrow 1/kd\), the resonance frequency \(f_r\) is:

\[ f_r = \frac{c}{2\pi} \sqrt{\frac{\rho}{md}} \]  

One of the most important aspects in a Helmholtz resonator is represented by its vibrating mass. The acoustic mass is obtained from the thickness of the perforated sheet plus the end corrections. In particular, being \(\varepsilon\) the fraction of open area and \(t'\) the thickness of the sheet plus the end corrections, the acoustic mass may be expressed as:

\[ m = \rho t' / \varepsilon \]  

This allows expressing the formula (2) with the simpler and more known formula:

\[ f_r = \frac{c}{2\pi} \sqrt{\frac{\varepsilon}{td}} \]
The vibrating air within the perforations provides the acoustic mass of the device. For this, it is important to study the sound field in the neck and in the cavity. In general, a perforated panel may be thought as a series of Helmholtz resonators. Consequently, it is not always possible to assume that the cells are independent each other, especially at low frequency. In fact, when the wavelength becomes larger and the absorber faces random incidence, lateral propagation in the cavity occurs. This problem is generally avoided by the introduction of absorbent materials in the cavity. The introduction of these absorbent layers makes important the normal sound field only.

A full expression of the acoustic mass in (2), if the holes have a rectangular slot of width $w$, is:

$$m = \frac{\rho}{\varepsilon} \left( t + 2\delta w + \sqrt{\frac{8\nu}{\omega}} \left( 1 + \frac{t}{2w} \right) \right)$$  \hspace{1cm} (5)

where the last term is due to the boundary layer effect and it depends from the kinematic viscosity of air $\nu$. This last term may be generally avoided, except in case of microperforations as it will be described below. What it is more challenge is the term $\delta$, which is related to the radiation impedance of the resonator. For example, in case of a square aperture, and assuming a perforation rate $\varepsilon$ lower than 0.16, the following relationship is assumed:

$$\delta = 0.845(1 - 1.25\varepsilon)$$  \hspace{1cm} (6)

A more precise expression for the evaluation of the end correction has been formulated by Smits and Kosten [7]

$$\delta = -\frac{1}{\pi} \ln \left( \sin \left( \frac{1}{2} \pi \varepsilon \right) \right).$$  \hspace{1cm} (7)

This leads to express the acoustic mass of a slotted resonator simply as:

$$m = \frac{\rho}{\varepsilon} (t + 2\delta w).$$  \hspace{1cm} (8)

The above formulas only allow calculating the resonance frequency. In order to predict the absorption law, it is necessary to express the resistance term $r_m$ in (1) to be able to know the surface impedance. The resistance term accounts for the losses within the cavity, and it is particularly influenced by the presence of a porous material in it.

For a Helmholtz device with no porous layer in the cavity, and assuming that the perforations are not microperforations, the resistance term may be expressed as:

$$r_m = \frac{\rho}{\varepsilon} \sqrt{8\nu \omega} \left( 1 + \frac{t}{2w} \right).$$  \hspace{1cm} (9)

Ingard reported another expression for this term [3], which has largely been tested in literature [8-10]:

$$r_m = \frac{\sqrt{8\rho \eta \omega}}{2\varepsilon}$$  \hspace{1cm} (10)

where $\eta$ is a constant corresponding to the viscosity of the air. Ingard tested the previous expression with impedance tube measurements and concluded that the double of the value from (10) could be assumed too. However, both formulas (9) and (10) result in small resistance, poor effect over the impedance and, finally, small contribution to the absorption of the resonator.

In order to increase the resistance term and hence the cavity losses, it is possible to introduce an absorbent material in the cavity. In this case, the position of the material should be considered carefully, as a higher or lower absorption is obtained when the porous material is nearer or farther from the back of the perforated panel, respectively (this coincides with a higher or lower particle velocity in the cavity). In particular, as the passage of the sound in the holes alters the particle motions, it should be considered that if the porous layer is behind the perforation, or within a slot width, then it is possible to assume that the flux is not in a free field and it is influenced by the amount of perforation. Consequently, the resistance term may be expressed as:

$$r_m = \frac{\sigma t}{\varepsilon},$$  \hspace{1cm} (11)

where $\sigma$ is the flow resistivity of the porous layer. Reversely, if the porous layer in the cavity is further than one slot width from the perforated panel, then the sound waves behave as in a free field and the resistance term is:
\[ r_\rho = \sigma t_\rho \] (12)

2.2 The Transfer Matrix Method

A more general way to describe the impedance of a multi-layer system is through the use of the transfer matrix method [2,11]. This method has the advantage to be flexible as it allows estimating the surface impedance of different designs. In this way, each layer of the absorber is represented by a matrix which allows relating the sound pressure and the particle velocity at the sides of the layer:

\[
\begin{bmatrix}
  p_{n+1} \\
  v_{n+1}
\end{bmatrix} =
\begin{bmatrix}
  a_{11} & a_{12} \\
  a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
  p_n \\
  v_n
\end{bmatrix}
\] (14)

For a layered system, the matrix elements are obtained by multiplying the matrices representing each individual layer. The surface impedance of the total system is then obtained by:

\[
z_1 = \frac{p_0}{v_0} = \frac{A_{11}Z_1 + A_{12}}{A_{21}Z_1 + A_{22}}.
\] (15)

The simplest case is when concentrated layers are considered, as in this case, \(a_{11}=a_{22}=1\), \(a_{21}=0\) and \(a_{12}=Z\), and the matrix reduces to simple equations. In cases of porous layers, the following expressions are valid:

\[
a_{11}=a_{22} = \cosh - j k_d \cdot \cos (\theta_n)
\]

\[
a_{21} = \sinh \left( j k_d \cdot \cos (\theta_n) / \left( z_a / \cos (\theta_n) \right) \right)
\]

\[
a_{12} = \left( z_a / \cos (\theta_n) \right) / \sinh \left( j k_d \cdot \cos (\theta_n) \right)
\] (16)

where \(\theta_n\) is the angle of propagation through the layer and \(z_a\) is the complex function of the wave impedance in the porous layer. The previous formulas in (16) allow the evaluation of the surface impedance (and hence of the sound absorption) for any incident angle. However, the simpler and more common case of normal wave is still valid. For example, considering a Helmholtz resonator panel whose back cavity is filled with a porous layer, and assuming only sound waves normal to the panel, then the impedance below the perforations is:

\[
z_1 = - j z_a \cdot \cot (k_l d_l)
\] (17)

where \(z_a\) is the characteristic impedance and \(k_l\) is the wavenumber of the porous absorbent. From this, it is possible to assess the impedance of the panel by adding the effect of the perforated panel:

\[
z_2 = r_m + \frac{j \omega \rho}{\varepsilon} (2 \delta a + t) + z_1
\] (18)

where the term \(\delta\) can be calculated from (6) or better from (7), and the resistance term \(r_m\) from (9) or (10).

The resistance term may generally not be avoided in case of an air gap between the porous layer and the perforated panel (so three layers are presented). In this case, the impedances at the end of the porous layer, at the end of the air gap and of the whole Helmholtz resonator are respectively:

\[
z_1 = - j z_a \cdot \cot (k_l d_l)
\] (19)

\[
z_2 = \frac{-z_a j \rho c \cot (kd_1) + \rho' c'^2}{z_a j \rho c \cot (kd_1)}
\] (20)

\[
z_3 = r_m + j (\omega m - \rho c \cdot \cot (kd_1)) = \frac{\rho}{\varepsilon} \sqrt{8 \delta a \alpha} \left(1 + \frac{t}{2a}\right) + \frac{j \omega \rho}{\varepsilon} (2 \delta a + t) + z_2
\] (21)

where \(d_1\) is the thickness of the porous layer and \(d_2\) the thickness of the air gap.
2.3 The Modes Method

A different approach to study the absorption of a slot panel is given by a numerical analysis of the sound field behind it. This method corresponds to a numerical estimation of the cavity modes. A solution of this problem has been developed by Takahashi by using the Rayleigh’s studies about diffraction [12]. He focused on the importance of the effect of diffraction phenomenon caused by impedance discontinuities of the boundary surface. This approach aims to write the incident wave and the scattered waves after the panel. Both waves depend on the incidental angle and material properties. The equation of the waves may be time consuming and they are generally solved by Fourier expansion and truncation of a finite number of equation. This method is not further discussed in this paper.

2.4 Sub-Millimeter Slots (and Slit)

In section 2.2, it was assumed that the losses due to viscous boundary layer effects in the perforation may be neglected. However, in case of microporations (with sub-millimeters holes) this is not more the case, and these losses must be considered. In this case, the Helmholtz panel generates losses in its holes, without the use of porous layers in the back cavity. The surface impedance in the case of cylindrical holes results:

\[
 z_i = -j\omega\rho \left(1 - \frac{2J_1(k'\sqrt{-j})}{k'\sqrt{-j}J_0}\right)^{-1}
\]

where \(J_0\) and \(J_1\) are the Bessel functions of the first kind of zero and first order respectively. Also in this case, the sound absorption can then be calculated using the transfer matrix method, summing the surface impedance of three specific impedances: the acoustics impedance of the holes, the mass resistance of the end correction and any other impedance of the cavity.

A more interesting case for the present paper is the evaluation of the impedance with micro-slot holes. In particular, the theory developed by Allard for the propagation in porous media [13] has recently been adapted to slits [14]. This theoretical evaluation assumes that the pressure only changes in the direction normal to the panel and the velocity only depends on the parallel direction within the slot. For an infinitely long slit of width \(w\) in a plate of thickness \(t\), the specific impedance may be expressed as:

\[
 z_1 = j\rho \omega m = j\frac{\rho \omega m}{\tan \left(\frac{k'w}{2}\right)}
\]

where \(\rho\), the effective air density in the slit due to the viscosity effects in it, and \(k'\) are given by

\[
 \rho = \frac{\rho_0}{\tan \left(\frac{k'w}{2}\right)}
\]

\[
 k' = \sqrt{\frac{\omega\rho_0}{j\mu}}
\]

where \(\mu\) is the viscosity coefficient of air (~1.8·10^{-5} kg/ms).

The formula (23) may be solved using series expansion in the domain of the angular frequency as:

\[
 z_1 = \frac{12\mu}{b^3} + \frac{1}{700} \frac{b^3\rho^2\omega^6}{\mu} + j\frac{6}{5} \rho \omega t + ...
\]

Finally, to calculate the impedance of the micro-slotted panel, it is necessary to consider the end corrections of thickness and to add the specific impedance for the air space behind the panel:

\[
 z_2 = \frac{1}{c} \left(z_i + j\rho \omega (2\Delta t) - j\rho \omega c \cdot \cot \left(\frac{kd}{c}\right)\right)
\]
2.5 Double Porosity Model: Meso- and Micro-perforation

Models to study “double porosity materials” have received a large interest in the last years [6]. These models promise to be effective for studying the sound absorption of materials which have two interconnected networks of pores of different characteristic size. According to the model proposed by Sgard et al. [6], a meso-perforation scale is assumed in a material with a micro-porosity structure. To be effective, the characteristic size of the meso-pores has to be much larger than the material micro-pores size and much smaller than the wavelength of the sound. These materials have gained attention because the interactions between the fluid and the solid phase of the material are responsible for strong dissipation effects.

Materials with a double porosity structure either exist at a natural state (fractured material with a porous frame) or result from a building or manufacturing process. In fact, a perforated porous material may be thought as a double porosity material. Section 3 focuses on walls composed by porous stones mounted leaving rectangular slot perforations. A solution for the double porosity model is based on the homogenization technique for periodic separated scales media. The procedure consists of evaluating the characteristic impedance together with the propagation constant inside an equivalent homogeneous material. The characteristic impedance of this equivalent material is

$$z_{char} = \sqrt{\rho_{dp} K_{dp}},$$  \hspace{1cm} (28)

where the effective density $\rho_{dp}$ can be expressed as a function of the dynamic viscosity $\eta$ and the dynamic permeability $\Pi_{dp}$:

$$\rho_{dp} = \frac{\eta}{j\omega \Pi_{dp}},$$  \hspace{1cm} (30)

and the dynamic macroscopic bulk density $K_{dp}$ depends on the dynamic bulk moduli of the fictive medium (consisting of the network of pores where the micro-porous part has been replaced by an impervious material) and the single porosity medium.

For the expression of the dynamic permeability, Sgard et al. report an expression in which it is obtained as function of the dynamic permeability in the normal direction to the fictive medium consisting of the network of pores and THE dynamic permeability of the single porosity micro-porous medium.

Besides the classical assumptions used in the modeling of porous media, this model also assumes that: (i) the double porosity medium is periodic; (ii) the skeleton is motionless; (iii) the size of the micropores is much smaller than the one of the mesopores and the size of the mesopores is much smaller than the wavelength in the material [6]. Although the many assumptions which may even be not satisfied, this model is able to capture most of the physics occurring in the material and will be implemented in the following section.

3. RECTANGULAR SLOT PERFORATIONS IN A CASE STUDY WALL

3.1 The design and construction of a slotted wall

The two main limits of the resonant Helmholtz panels are the risk of specular reflection given by their flat surface and the absence of a broadband absorption. In particular, the sharp peaks of Helmholtz resonators may only partially be reduced by the introduction of porous layers in the cavity (which indeed reduce the absorption at the resonant frequency). In fact, one of the main requests in room acoustics is to increase broadband absorption, with particular emphasis for low frequencies, where rooms may exhibit modes and strong modal response. Two important parameters to consider in Helmholtz resonators are the cost of the sound absorption treatments and their esthetic value. In this section a possible device which solves all these problems is presented [15].

A slot wall composed by natural tufa stone was created. Tufa is porous sandstone typically encountered in the Mediterranean area. It is generally fabricated in blocks, whose dimensions are 8 cm (width), 12 cm (height) and 24.5 cm (length). The stone has a density of 1700 kg/m$^3$, and it is generally used in masonry wall. When used in the external side of a wall it is generally furnished, meanwhile on the internal side it can be unprotected, increasing its rugosity, its high frequency absorption and preventing against the risk of specular reflections. The idea was hence to use tufa blocks assembled to obtain slot holes. To study the effect of multi layer systems with internal losses, polyester fiber sheets were added in the cavity.

3.2 Experimental measurements of sound absorption

Different tests were performed in the reverberant chamber of the Politecnico of Bari to investigate the behaviour of the slotted wall [15]. A surface of 10 m$^2$ was built with the tufa block against a wall of the reverberant chamber. The
Surface was firstly realized against the wall directly, and then at a distance of 10 or 35 cm from the wall (L10 or L35); slots were obtained by distancing blocks of the same row 1 or 2 cm apart (S1 or S2). Finally, to increase the sound absorption of the system a fiberboard was added (P) where the particle velocity was large, i.e. near the neck of the Helmholtz resonator. The sound absorption of the wall in each configuration was calculated taking into account corrections for temperature and humidity grade, according to [16]. Results for the reference vertical wall and the different configurations studied are reported in figure 1 [15]. This shows that a large increase of the sound absorption was obtained at low frequencies, especially when the fiber material was added in the cavity.

FIGURE 1. Random incidence sound absorption for different slot walls (in the legend, for example, S1 corresponds to slot width 1 cm, L10 corresponds to back cavity 10 cm depth, and P corresponds to the presence of a porous layer in the cavity).

3.3 Comparison between the numerical model and experiment
Theoretical values of the absorption are hence compared with measured results. The absorbent system shows two resonances, the first and greater at 150 Hz for a distance from the rear wall of 10 cm and at 125 Hz band for a distance of 35 cm, the second one in the frequency band of 1.6 kHz. Measured absorption for 2 cm slot configuration (S2) is higher than for the configuration with 1 cm slots (S1). The difference among $\alpha_{SAB}$ of the slotted walls and the $\alpha_{SAB}$ values for the un-slotted wall permits to focus on the effect of the slot configuration only.

Figure 2 reports a comparison among experimental data for the configuration S1-L10 and theory evaluation calculated with the transfer matrix approach. The flow resistivity was assumed equal to 103,779 Nsm$^{-1}$.

FIGURE 2. Random absorption for a sound absorption wall with a 1 cm width slot wall and a back air cavity of 10 cm; difference between this absorption and that of a reference solid wall without slots; predicted sound absorption with the transfer matrix method and using the double porosity model.
As it is evident the transfer matrix method is unable to predict the second resonance at 1.6 kHz, while the frequency resonance in the cavity is well estimated considering a normal sound field in the slots. However, the double porosity model shows particularly adapted in these circumstances. In particular, the prediction of the second resonance represents a strong advantage of this method.

For other tufa walls, theory predictions well estimated the frequency of the first resonance, and the increasing absorption with the slot width. The corresponding graphs will be shown at the conference.

CONCLUSIONS

This paper has reviewed existing theory about sound absorption properties of panels with slot perforations. A synthetic review of theoretical models has been done, focusing on different methods: the simple single-layer method, the transfer matrix method, the modes method and the sub-millimeter slot method.

A comparison of these models has allowed showing difference between these methods. The paper focused on the transfer matrix method as this allows taking into account surface, hole and cavity impedances. In particular, the paper has shown formula to express the surface impedance by the sum of a resistance term, a mass term and characteristic impedance. The expressions of these terms have been reported. The paper has also reported a few measurements of sound absorption of slotted walls which work as an assembly of Helmholtz resonant absorbers. Comparisons with theoretical models have been carried out to explain the different mechanisms of sound absorption.

Further research is necessary to test the difference between methods and experimental results. Moreover, an important contribution in this field could be the ability to combine different methods to establish design rules for complex multi-layer structures.

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