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Session 2pAAb: Dah-You Maa: His Contributions and Life in Acoustics

2pAAb11. Active noise control in rooms
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Maa published his ideas and achievements in the early days of the new branch of noise control based on using secondary sources to reduce the sound level of the primary sources by negative interference. He concentrated on active control of the reverberant sound in a room by a loudspeaker placed in one of the room corners and a microphone in its close vicinity. He examined the role of the complete solution of the wave equation consisting of a direct wave radiated from the sources and the reverberant field created by room modes. His work resulted in deriving a general formula for possible noise reduction that is independent of the shape, size and the content of the room. He has shown noise reduction of 8 dB for a noise band centered at 100 Hz.

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Professor Maa, who studied in the USA spent his life in China and eventually became the member of the Academy of Sciences. I was lucky to meet him at some acoustical meetings. His book could be called a collected works on acoustics. Although he lived in isolation he followed new developments and made fundamental and original contributions to many subfields. One of them is the active control of sound fields, a new area that has been explored relatively recently.

Active control of sound, applied usually on noise reduction is one of the difficult acoustic assignments. In general, active control requires solving two separate problems. The first consists of finding the type and locations of secondary acoustic sources and microphone locations that would permit to implement the desirable control of the sound field of the primary sources. The second problem consists of constructing the electric circuitry that would transform the signal from the controlling transducer into the signal that feeds the secondary sources. The first problem is usually more difficult and requires suitable analyses of the sound fields, radiation analyses, sound interference etc. Once and if the first problem is solved, the signal processing can be usually implemented.

Prof. Maa published an article “Sound Field in a Room and its Active Noise Control” [1], which presents a fairly general analyses and a proposal where to place and how to control the sound field of a secondary source to reduce the sound field of the primary source at low frequencies in a reverberant room.

Today, the most often used system to reduce the sound levels generated by a source located somewhere in the room is based on the reduction of sound power radiated by this source. This is achieved by placing the secondary sources close to the primary sources so that the power reduction is achieved by mutual coupling. In this way, the monopole can be converted into a multipole. However, this may not be feasible for larger and complicated primary sources.

Maa’s idea is different. His system can control the modes in the whole room by a secondary source in the corner of a rectangular room that can excite all modes. The controlling microphone is close to the secondary source. The noise reduction is thus achieved by the interference of the modes.

**Normal Mode Response in a Room**

In order to correctly set up the equations for the noise reduction by the source in the corner and the controlling microphone close to this source, as shown in Fig.(1), the basic equations for the sound field in the room must be extended by a term for the direct field of the secondary source.
Fig. 1 Active noise control system

The complete solution of the wave equation consists of two parts: the direct field of the source and the reverberant field due to the modes. The complete solution is

$$p(r \mid r_0) = R \frac{j \rho f Q}{2 \mid r - r_0 \mid} \exp[(j \omega(t - 1 \mid r - r_0 \mid/c))] + \rho c^2 \frac{Q}{V} \sum_n \frac{\Phi_n(r) \Phi_n(r_0)}{\Lambda_n} \sum_n \frac{\Phi_n(r) \Phi_n(r_0)}{\Lambda_n} \exp(j \omega t)$$

where $\rho$ is the air density, $c$ the speed of sound, $Q$ the source strength, $V = l_x l_y l_z$ the room volume of the rectangular room, $f_n$ the frequency of the mode, $r$ the location of the field point and $r_0$ of the source location. The eigenfunction $\Phi_n(r)$ of the room is

$$\Phi_n(r) = \cos(n_x \pi x / l_x) \cos(n_y \pi y / l_y) \cos(n_z \pi z / l_z)$$

and

$$\Lambda_n = \iiint_V \Phi_n(r) \Phi_n^*(r) dV$$

The first term in Eq. (1) represents the direct wave and the second term are the standing waves summed over all modes within the frequency band of interest. The expression for $\Lambda_n$ is real due to the complex conjugate term. Its magnitude
depends on $n$ that can be zero or some non zero integer. The constant $R$ depends on the position of the sound source. The mean square sound pressure is

$$p^2(r \mid r_0) = \frac{1}{8} \left( \frac{R \rho f Q}{|r - r_0|} \right)^2 + \left( \frac{\rho c^2 Q}{V} \right)^2 \sum \frac{\omega^2}{2 \Lambda^2_n (2 \omega_n k_n)^2 + (\omega^2 - \omega_n^2)^2} \Phi_n^2(r) \Phi_r^2(r_0)$$

(4)

For a narrow band sound source of frequencies $\frac{\Delta \omega}{2\pi}$ that is wide enough to contain many normal modes the mean square sound pressure from Eq.(4) is

$$p^2(r \mid r_0) = \frac{1}{8} \left( \frac{R \rho f Q}{|r - r_0|} \right)^2 + \left( \frac{\rho c^2 Q}{V} \right)^2 \frac{1}{\Delta \omega} \sum \frac{\pi}{8 k_n \Lambda_n} \Phi_n^2(r) \Phi_r^2(r_0)$$

(5)

The summation in Eq. (5) is over the normal modes within the selected band of noise. The instantaneous sound pressure of this narrow band noise with the average frequency $\Delta \omega / 2\pi$ is

$$p(r \mid r_0) = R \frac{j \rho f Q}{2 |r - r_0|} \exp[(j \omega(t - |r - r_0|/c)] + \sum p_n(r \mid r_0)$$

(6)

where

$$p_n(r \mid r_0) = \frac{\rho c^2 Q}{2 V \Lambda_n} \left( \frac{\pi}{k_n \Delta \omega} \right)^{\frac{1}{2}} \Phi_n^2(r_0) \Phi_n(r) \exp(j \omega_n t + \theta_n)$$

(7)

The direct sound and the modes have the character of a random signal passing through a narrow band filter. The direct wave has a bandwidth $\Delta \omega / 2\pi$ and the normal mode $n$ has $k_n / \pi$. The average frequencies are $\omega / 2\pi$ and $\omega_n / 2\pi$. Due to the random noise, their amplitudes are modulated with random functions with Rayleigh distributions. Both $R$ and $\theta_n$ are random quantities with the average values of $R$ as in a sinusoidal field and zero for all $\theta$s. The direct sound pressure has a continuous spectrum while the pressures of the modes become the peaks on the spectrum of the reverberant sound.

**Active noise control**

The direct sound that incidents on the controlling microphone that is close to the secondary source in the corner is weak and, therefore, is not considered. However, the direct sound from the corner loudspeaker is strong and affects the system. The reverberant sound due to the noise source incidents on the microphone of sensitivity $M$ consists of slightly fluctuating pure tones of the
Fig. 2 Spectra of sound pickup at increasing distances from a noise source located in a 200 m³ reverberation room.

Modal frequencies. The sound picked up by the microphone is amplified A times. The sound pressure of the primary source at the microphone location d according to the equation (7) is

\[ p_n(d\mid r_0) = \frac{\rho c^2 Q}{2V\Lambda_n} \left( \frac{\pi}{k_n\Delta\omega} \right)^{\frac{1}{2}} \Phi_n(d)\Phi_n(r_0) \exp(j\omega_n t + \theta_n) \] (8)

The sound pressure from the secondary speaker at the microphone location d is

\[ p_s(d\mid 0) = \frac{jR\rho f_{sn}}{2d} \exp[-j\omega_n d / c] + \frac{\rho c^2 q_n}{2k_n V\Lambda_n} \Phi_n(d) \] (9)

where \( q_n \) is the secondary source strength due to the sound pressure at the point d produced by the primary and secondary source. It is

\[ q_n = \frac{\text{MAS}}{Bl} \left[ p_n(d\mid r_0) + p_n(d\mid 0) \right] \] (10)
where the first term $MAS / Bl$ is the loudspeaker constant. Combining equations (8,9 and 10) results in

\[ q_n = \frac{MAS}{Bl} \frac{p_n(d) r_0}{1 + D_n j \exp \left( -j \left( \omega_n d / c \right) \right) + D_n K_n} \quad (11) \]

where

\[ D_n = \frac{MAS R \rho f}{Bl 2d} \quad D_n K_n = -\frac{MAS}{Bl} \frac{\rho c^2}{2 \nu k_n \Lambda_n} \Phi_n(d) \quad (12) \]

Eliminating $D_n$ from Eqs.(12) results in

\[ K_n = \frac{c \lambda d}{R k_n V \Lambda_n} \Phi_n(d) \quad (13) \]

The sound pressure at any location in the room is the sum from both sources

\[ p_n(r) = p_n(r \mid r_0) + p_n(r \mid 10) \quad (14) \]

This expression can be expressed as

\[ p_n(r) = p_n(r \mid r_0) \frac{1 + D_n \left( \sin \left( \omega_n d / c \right) + j \cos \left( \omega_n d / c \right) \right)}{1 + D_n \left( \sin \left( \omega_n d / c \right) + j \cos \left( \omega_n d / c \right) \right) + D_n K_n} \quad (15) \]

The analyses of these expressions reveals that even when the direct wave is neglected the attenuation can be achieved for a negative amplification $A$ or the factor $D$ is increased. The maximally obtainable noise reduction $NR_n$ for a mode $n$ is given by

\[ NR_n = \lim_{A \to -\infty} \left| \frac{p_n(r \mid r_0)}{p_n(r)} \right|^2 = 1 + K_n^2 + 2 K_n \sin \left( \omega_n d / c \right) \quad (16) \]

### Noise Reduction

In the article published in 1994 (1) that is paraphrased here, Maa presents detailed analyses of the noise reduction of the frequency band 90-100 Hz that
contains in a 200 $m^3$ reverberant room 17 normal modes, mostly oblique and
tangential. Only the results of the measurements will be presented here. The
noise reduction defined Eq. (16) that depends on $K_n$ (Eq. 13) is analyzed
considering the parameters of the room and position of the controlling elements.
The results are shown in Fig. (3). One obstacle of obtaining a higher noise
reduction in the room reverberant field is the howling caused by the
microphone loudspeaker room. This can be removed by frequency filtering.
Also, the noise reduction can be substantially increased, if the direct sound from
the secondary source is eliminated.

The noise reduction that acts on the reverberant field, does not depend on
the room shape, size and noise source. It does not on the noise source direct
field.

This idea presented and tested by Maa is one of his original contributions
to acoustics and documents his extreme creativity.

Fig. 3 Spectra of uncontrolled (upper) and controlled (lower) noise.

(1)Dah-You Maa, Sound field in a room and its active noise control, Applied
Acoustics 41 (1994) 113-126