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4aAAa7. A diffusion equation model for investigations on acoustics in coupled-volume systems
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Coupled-volume rooms remain as one of the most exciting and challenging research lines in room acoustics. Their benefits lie on their multiple-slope energy decay profiles, being of interest in many current concert halls. However, so far there is no consistent predictive model being able to help architects and acousticians in selecting appropriate design parameters. This work is devoted to studying effects of aperture-size and source/receiver positions on the energy decay characteristics. For this purpose, a diffusion equation model is used to model a coupled-volume system, providing an effective tool for analysis. The diffusion equation model is first validated by experimental investigations using scale models. Bayesian energy decay analysis is applied to the results of both the acoustical scale model and the diffusion-equation model to provide deeper insight in the energy decay characteristics and their dependence on the aperture sizes and the sound source/receiver positions.
INTRODUCTION

This paper presents investigations on effect of coupling aperture sizes in acoustically coupled-volume systems. This paper will discuss the experimental investigations using Bayesian decay analysis based on both acoustical scale-models and the diffusion equation models. Few attempts of studying coupled-volume models when aperture sizes are modified [1, 2]; however, these previous studies have not yet been able to articulate the physical changes of the sound energy decays when changing the aperture sizes.

Billion et.al. [3] applied a diffusion equation model to study the coupled-volume systems, their primary focus was on the feasibility and applicability of the diffusion equation model rather on the aperture sizes. Other authors [4, 5] also applied the diffusion equation model to investigate the coupled-volume systems to reveal the sound energy flows across the coupling aperture.

This work relies on experimentally measured results achieved in acoustical scale models of two coupled rooms when changing the aperture sizes and the sound source/receiver positions relative to the coupling aperture. The results can further validate the diffusion equation models with respect to the varying aperture sizes and other aspects. Using the experimentally validated diffusion equation model, this work further investigates a widely varied configurations of the diffusion equation models in terms of natural reverberation times and the source/receiver positions. Bayesian energy decay analysis described in the recent work [6] is applied to both acoustical scale model and diffusion-equation model results to provide deeper insight in the energy decay characteristics and their dependence on the aperture sizes, the sound source/receiver positions.

INVESTIGATION MODELS

Eighth scale models with automatic scanning system

This work employs an eighth scale model of two adjacent rooms of different volumes, separated by a movable wall for the purpose of adjusting the coupling aperture sizes. Apart from the top wall and the movable wall between the two rooms for varying the aperture sizes, all other walls are featured with diffusely reflecting surfaces. Table I lists the dimension of two rooms given in original (scaled-up) sizes.

| Table 1: Dimensions, absorption and natural reverberation times of two different experimental models. |
|-----------------|-----------------|
| **Primary Room** | **Secondary Room** |
| Dimension (l, w, h) | (4.88, 6.32, 7.20) m | (7.60, 9.76, 7.20) m |
| Volume | 222.06 m³ | 534.07 m³ |
| Absorption | 0.412 | 0.218 |
| Mean Free Path | 3.98 m | 5.36 m |
| Natural RT | 0.469 s | 1.17 s |

The diffusely reflecting surfaces are designed to achieve as much diffuse reflections as possible for a wide frequency range. Almost all the interior surfaces are treated with the diffusors [7]. The diffusion coefficients higher than 0.5 in 1 kHz (oct.) are considered as highly diffusely reflecting.

After the movable wall is completely closed, two rooms are considered to be acoustically separated. The reverberation times in the separate rooms respectively are termed natural reverberation times.
beration times. The natural reverberation times are estimated from five room impulse responses measured in each room at spatially significantly separated receiver positions. Figure 1 illustrates the experimentally measured results of natural reverberation times, both average and deviation. For the octave bands between 250 Hz and 2 kHz the averaged natural reverberation times are well separated in values, and their spatial variation is small, which is the consequence of highly diffusely reflection surfaces of the rooms.

**Figure 1:** Natural reverberation times measured in two separate scale-model rooms (converted into full scale). The variation range represents the estimation spreading from five spatially significantly separated receiver positions.

### Diffusion Equation Model

The acoustic diffusion equation may be considered as an analytical approximation of the acoustic radiative transfer equation [8], also termed transport equation [9] in the acoustics literature, where the propagation of radiation through a medium is mainly affected by absorption, emission and scattering processes. The resulting model assumes that the variations in energy density and energy flow remain small over one mean-free path [10]; therefore, reflected energy must dominate over absorption and the reflection events must be predominantly diffuse [8].

The diffusion equation model for the sound energy density $w(r, t)$ at position $r$, and time $t$, defined on a domain $V$, with a sound source term $P(t)$ located at position $r_s$, can be expressed by a partial differential equation with mixed boundary conditions [12, 11],

$$
\frac{\partial w(r, t)}{\partial t} - D \nabla^2 w(r, t) + cmw(r, t) = P(t)\delta(r - r_s) \text{ in } V, 
$$

$$
-D \frac{\partial w(r, t)}{\partial n} = A(r, \alpha)c \, w(r, t) \text{ on } \partial V. 
$$

Equation (1) represents an inhomogeneous parabolic partial differential equation, where $\nabla^2$ is the Laplace operator and $n$ represents the unity normal outwards vector to the boundary surface. In proportionated enclosures $D = \lambda c/3$ is the so-called diffusion coefficient with $c$ being the speed of sound.

This diffusion coefficient takes into account the room geometry through its mean free path $\lambda = 4V/S$, with volume $V$ and total interior area $S$, whereas the term $m$ is the absorption coefficient of air. Finally, Eq. (2) represents a mixed boundary condition that models the local effects on
the sound field by absorption on surfaces. This absorption factor $A(r, \alpha)$ takes different values according to different existing approaches [3, 12]. In this paper, the modified absorption factor is adopted to perform the simulations since it has been shown the widest range of applicability in terms of overall absorption in the room under investigation [12, 8].

Finally, some details about the implementation are needed to mention. A Dufort-Frankel finite difference scheme of the diffusion equation model has been proposed to be used in a diffusion equation model [13]. In this numerical technique, both time and space are discretized. This scheme is considered unconditionally stable, meaning any discretization value might be chosen. However, in order to ensure that the predictions converge to a fixed value with low error, a certain relation between the space and time sampling must be accomplished (details can be found at Ref. [13]).

**Model adjustment**

Using the dimensions and the acoustical properties of the scale model of the two-coupled rooms, the diffusion equation model of the two-coupled rooms is also established. The model adopts the same geometries of the scale model rooms. When considering each room separately the boundary conditions inside the diffusion equation model take absorption coefficients in order to achieve the same natural reverberation times as those measured in the separate scale-model rooms, taken from 1 kHz octave frequency band. Through this adjustment of absorption properties exact averaged natural reverberation times at 1 kHz octave band measured in each scale-model room can be achieved in the diffusion equation models. Finally, two rooms of the diffusion equation model are then coupled together by assigning the aperture boundary condition.

**EXPERIMENTAL VALIDATIONS**

This section discusses the comparison studies in terms of both measurement results taken in the scale models and the diffusion-equation modeled results for a systematically varied aperture widths (sizes) as well as the source and receiver positions.

In the scale-models, the aperture size is changed systematically from wider to narrower widths. The sound source in the primary room is a miniature dodecahedron of with approximately omni-directional characteristics within frequency ranges of interest, particularly between 6 kHz and 11 kHz at eighth scale model frequencies. When given for the full-size, it corresponds well to 1 kHz octave frequency band.

Two sound source positions are of interest as shown in Fig. 2. One is at the lower-left corner of the primary room [Fig. 2], while a grid for the sound receiver of three rows and ten columns is defined towards the upper left corner of the primary room. For every aperture sizes (from 20 cm wide down to 4 cm in the eighth scale models) at the given sound source position, thirty room impulse responses are measured. The sound receiver (microphone) is moved to the center point of each grid as shown in Fig. 2.

**Comparison of energy decay characteristics**

Figure 3 illustrates comparisons of double-slope decay characteristics between acoustically measured decay functions in scale models and those from the diffusion equation models. Varied aperture widths are arranged from 30, 40, 60, 80, 120 to 160 cm (given in full-size). The sound energy decay characteristics are quantified using Bayesian decay analysis [6]. Figure 3 (a) shows the comparison results of two decay times ($T_1, T_2$) of the double-slope energy decays evaluated for each aperture width, respectively. The variation bars represent the estimation spreading analyzed over 30 receiver positions. The averaged decay times estimated from both experimen-
**FIGURE 2:** Top view of scale models, showing the sound source position, the receiver grid definition, and the aperture location. (a) Sound source finds itself in the lower-left corner of the primary room, the far most away from the coupling aperture. For the receiver positions, a grid of three rows and ten columns is defined towards the upper left corner of the primary room.

...tally measured and the diffusion-equation modeled sound energy decay functions demonstrate good agreements across all the aperture sizes (widths). With increasing aperture sizes/widths the two decay times ($T_1, T_2$) decrease slightly. The natural reverberation times measured in each separate rooms are also plotted for ease of comparison.

**FIGURE 3:** Comparisons of double-slope decay characteristics between acoustically measured decay functions in scale models and ones from the diffusion equation models. the aperture widths change from 30, 40, 60, 80, 120 to 160 cm. (a) Two decay times ($T_1, T_2$) of the double-slope decays. A top view of the models showing the source location, the aperture location, and the receiver grid is also included. The natural reverberation times are also plotted for ease of comparison. (b) Level differences (c) The time instance of the turning points.

Figure 3 (b) shows the comparison results of the level differences. Figure 3 (c) shows the com-
parison results of the turning points from the initial decay (the first slope) to the late decay (the second slope) of the double-slope decay functions. Figure 4 conceptually illustrates the definition of the level difference $\Delta L$ and the turning point $(\tau_t, \tau_L)$. The level differences and the turning point are defined in Ref. [4]. The turning point is quantified by the turning point time, $\tau_t$, and the turning point level $\tau_L$. As illustrated in Fig. 4, the two decay times and one turning point, or the level difference can adequately describe the double-slope decay characteristics observable in two coupled rooms, when the decay times of the secondary room are clearly longer than those in the primary room. Given the good agreements of the two decay times between both models [Fig. 3(a)], the consistent discrepancies of the level differences and the turning point time hint at the inherent nature of the diffusion equation models.

The measured averaged turning point times in the acoustical scale models, are clearly larger than those estimated from the diffusion equation models. The turning point time is defined from the starting of the direct-sound. The overall averaged time difference amounts to 85.6 ms which is 7.36 times of the mean-free time (11.6 ms) in the primary room. In other words, the diffusion equation model seems to be valid to provide accurate reverberation process predictions after a certain number of mean-free times.

It is worth to mention that recent works have also reported on this phenomena [4]. Those works reported that the diffusion equation model can be considered as valid after at least 2 or 3 mean-free path times. In the previous investigations [4], the overall surface absorption is significantly lower ($\alpha < 0.5$) than those used in the scale models recently developed for the current investigations, although the dimensions of the scale model rooms are also quite different, which can be summarized by mean-free-path lengths.

The measured turning point times when compared with those predicated by the diffusion equation reveal that the accurate prediction of the diffusion equation models can also be considered valid at a later time instance. Figure 4 conceptually illustrates the observed agreement between experimentally measured energy decay function and that predicated by the diffusion equation model. The comparison between the experimentally measured results and the diffusion equation modeled results in Fig. 6 suggests that the diffusion equation model is valid in a late time instance, since the experimentally measured results include the direct sound, early reflections, and the reverberation tails while the diffusion equation models only diffuse sound.
field in the room which develops after the early reflections. Table I also suggests that the time instance for the diffusion equation models to be valid depends on the overall absorption inside the room under investigation. The lower the overall absorption in the room, the earlier the diffusion equation models become valid, because it more efficiently facilitates the mixing process of the multiple reflections bouncing back-and-forth inside the enclosure. In opposite, enclosures with higher overall surface absorption will less efficiently promote the mixing process.

**Receiver distance dependence**

Figure 5 illustrates averaged values of decay times and the level differences over three rows as function of receiver positions evaluated from the experimental results of the scale models. Receiver position 1 corresponds to the very right column, close to the aperture, while receiver position 10 corresponds to the far-left column, far away from the aperture [see Fig. 2 (a)]. For clarity, only three aperture widths (30, 60, and 120 cm) are shown. With the receiver position going away from the aperture, the decay times ($T_1, T_2$) change slightly. A slight increase of the level difference (2 - 3 dB) with an increasing distance to the side wall is observed as illustrated in Fig. 5 (b). Overall, with the narrow slit-shaped aperture the double-slope profiles of the reverberant energy decays change only slightly over the distance to the aperture.

![Figure 5](image)

**Figure 5:** Receiver distance dependence from the aperture for three aperture widths of 30 cm, 60 cm, and 120 cm. The receiver position is numbered from 1 for the closest distance to the aperture, to 10 for the farthest distance. (a) Decay times ($T_1, T_2$). (b) Level differences.

**Aperture size effect**

In order to achieve distinct double-slope decay profiles, the natural reverberation times $T'_1$ of the primary room is assigned to be 0.56 s while $T'_2$ of the secondary room to be 4.1 s by adjusting the surface absorptions. Figure 6 (a) shows the changes of the energy decay functions when the aperture width decreases from 8 cm down to 1.3 cm. With decreasing of the aperture width the two decay slopes change slightly. Significant changes are observed in the level differences or the starting level of the second slop decay. For clarity, Fig. 6 (b) magnifies three decay curves taken from Fig. 6 (a), for aperture width of 1.3 cm, 1.8 cm and 3 cm. With decreased aperture width, the decay time $T_1$ changes from 0.56 s, 0.55 s, to 0.54 s, while the decay time $T_2$ changes from...
Figure 6: Trend of energy decay profiles predicted by the diffusion equation model when decreasing the aperture sizes. The natural reverberation times are adjusted to be ($T'_1 = 0.46\text{s}, T'_2 = 4.1\text{s}$). (a) Energy decay functions when the aperture width changing from 10 cm down to 1.3 cm. (b) Energy decay functions at aperture width of 1.3 cm, 1.8 cm and 3 cm, taken from (a) in magnified presentation.

3.77 s, 3.65 s, to 3.34 s. The slight changes in decay time values of the second slope decay are illustrated in the decay curves by three nearly parallel decay slopes, but the second-slope decay starts at three different levels, reflected by the level difference of 13.9 dB, 20.4 dB and 26.6 dB.

Discussions

This paper gives a step further on coupled-volume system modeling with application to architectural acoustics. This step is given by a thorough comparison between a experimental measurements by using a scale model and computational simulations by using a diffusion equation model. By varying parameters on both scale model and computer model, it is observed a considerable agreement on the behavior of the computer model regarding to the experimental acoustic measurements, and establishing a diffusion equation model as a powerful tool for architectural acoustic design. This paper is mainly devoted to analyze the effects of aperture changes and relative position between source and receivers (listener areas).

The results show a considerable agreement in double-slope energy decay parameters, allowing to define a diffusion equation model as a well-establish method for acoustic room simulation. However, inherent limitations in terms of turning point time are shown, as expected, since it is well-known a diffusion equation model only is valid for the later part of impulse responses. However, this work provides a significant advance on determining which is the turning point time delay. Only (multiple) double-slope profiles can support the ‘time-delay’ measurement. But the discussion about the observations can be also generalized in single-space simulations.

References


