4pAAa5. Inversion of a room acoustics model for the determination of acoustical surface properties in enclosed spaces

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Acoustic consultants are often in charge of treating spaces to fix problems or improve their room acoustics. To assess the situation and to find a solution, it is common practice to perform computer simulations. This technique is well established, cheap and effective. But it requires a CAD model of the room as well as properties of its boundaries, such as absorption and scattering coefficients. The CAD model is usually easy to obtain by asking the architect or measuring yourself, but quantifying the absorption and scattering coefficients of every single wall is a challenging task. This contribution presents a method that automatically matches absorption coefficients for every single wall by applying an inverse room acoustics model which bases on geometrical acoustics. The inversion is done numerically using a non-linear least-squares optimization process in MATLAB. The independent variables are all absorption coefficients and the goal is to minimize the error between measured and simulated impulse responses at all measured positions in the room. In addition to the acquisition of absorption and scattering coefficients, the goal after the optimization process is to perform interactive binaural auralizations that have a high perceptual congruence with the existing space.

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INTRODUCTION

The nowadays most widely used programs for room simulations base on the principles of classical geometrical acoustics (GA). The roots of computer-aided acoustics range back to the 1960s when Schroeder and Krookstad were the pioneers in this field with ray tracing (RT) simulations of specific room shapes [1, 2]. Since then the algorithms of GA have continuously and extensively advanced, so that modern software can handle complex wave effects such as diffraction [3]. It was observed that major uncertainties in current simulations were rather due to the user input than to the pure algorithms [4]. This aimed mainly at the description of the acoustical properties of room surfaces, which is commonly done using frequency-dependent absorption $\alpha$ and scattering coefficients $s$. The definition of these coefficients is obligatory for any of the known prediction tools and can have prominent effects on the computation results even when only slightly changed. This makes it crucial to apply absorption coefficients with the highest possible degree of accuracy. Unfortunately, the necessary accuracy is often not reached with state-of-the-art methods [5, 6]. Thus the possibilities of computer simulations are limited by the available quality of input data.

While it is not too hard to accurately measure the dimensions and shape of a room\(^1\), it can be much more demanding to get accurate data of the absorption and scattering of all room surfaces. For projects which deal with existing spaces that are subject to modifications it makes sense to overcome the currently prevalent and unsatisfactory situation that unexpected simulations results are often due to uncertain input coefficients.

In this contribution a novel method for the acquisition of acoustical coefficients in-situ is proposed. The idea is to invert a room acoustics prediction model, so that it works in reverse mode. The traditional calculation of impulse responses (IRs) by using a configuration of surface materials as input data is turned into a calculation which takes measured IRs of a real room and returns acoustical coefficients of the surface materials.

STATE-OF-THE-ART IN THE MEASUREMENT OF ACOUSTICAL COEFFICIENTS

It is well known that it is common practice to estimate the type of material and apply coefficients from a comparable material listed in one of the available databases [7]. Sometimes parameters such as scattering coefficients $s$ are guessed on a purely empirical basis, as they are usually not contained in these databases. As this obviously limits the possibilities of modern simulation tools, it is recommended to provide data of actual measurements of the material. The reflective behavior of surfaces can be simplified for simulation purposes and therefore divided into the amount of sound energy that is absorbed by the material, the amount that is reflected specularly and the amount that is scattered in other directions.

Measurement of Absorption Coefficients

Common absorption measurement are done by taking samples and running standardized measurements in either a reverberation room after ISO 354 or an impedance tube after ISO 10534. These two methods differ mainly in the sample sizes and radiant exposure, as small samples ($< 0.5m^2$) are used for the impedance tube with sound incidence from normal direction and large samples ($> 10m^2$) are used for the reverberant chamber with diffuse sound incidence. These contrasts are convenient for the different applications, i.e. the reverberant chamber is suited for large complex structures with irregular or rough surface texture whilst the impedance tube is well suited for flat homogeneous materials. The impedance tube measurements are

\(^1\)To the experience of the author, mistakes in the exact shape of the room are much less problematic and have much less effect on the results than mistakes in the assumed absorption coefficients.
considered to be precise (if applicable), but it turned out that standardized reverberation room measurements might still not be reliable enough to provide accurate results, which was discovered in round robin tests using the same sample in different measurement facilities [8].

However, both established methods have in common that they need to extract a sample from the original material. Dealing with finished buildings or complex compounds, e.g. in cars, it is often not possible to get appropriate samples in this (often destructive) way. The most convenient option is to measure the materials in their natural built-in state. The prevailing way to gather the absorption properties on-site is to use the surface impedance technique with a PU-probe [9], which turns out to be sensitive to objects in the close proximity and also limited to a frequency range of ca. 300 Hz to 10 kHz. But also impedance tube measurements are limited in its frequency range, e.g. around 200 Hz to 6 kHz, dependent on the tube’s diameter.

Measurement of Scattering Coefficients

The acquisition of scattering coefficients can be conducted either in free-field condition or in a reverberation room, which is documented [10] and standardized in ISO 17497. The principle is to separate the amount of specularly reflected energy \(E_{spec} = (1 - \alpha)(1 - s)\) from the sum of energy that is scattered in all other directions \(E_{scat} = (1 - \alpha)s\). This definition is useful for geometric room acoustic models, which typically have separate algorithms dealing with specular and scattered components. The scattering coefficient defines the proportion of energy that is not reflected specularly and carries no further information on the direction of the scattered reflections. Different proposals for the definition and measurement of scattering coefficients and also the related diffusion coefficients exist and were recently summarized by D’Antonio [11]. In general the measurements have a) to be done for individual material configurations and b) are more complicated and far less transferable to different situations compared to absorption data. As an attractive alternative, the scattering is mainly dependent on the structural shape and can also be simulated using the boundary element method [12].

ROOM ACOUSTICS SIMULATION

The applied room acoustics model is part of the RAVEN framework [13], which bases on GA methods. It combines the image source model (ISM) for the realistic representation of early specular reflections with a stochastic raytracing to model the diffuse, scattered reflections in the late part of the room impulse response (IR). Sound sources are modeled by inclusion of their directivity and free field sensitivity. At defined receiver positions, the simulation returns energy decay curves and IRs.

The simulation model was extended to export log files with the information of all occurring surface reflections. For each frequency band and each single particle, or image source (IS) respectively, the number of intersections with each material is stored. This extensive list will then be used to build a mathematical description of the room acoustics model. This mathematical function is a closed solution of the room acoustics model and therefore suited for inversion. The model is defined in Equations 1-3 and discussed in detail in the following section.

INVERSE ROOM ACOUSTICS MODEL

A traditional room acoustics model \(T\), predicts the energy decay over time \(E(t)\), at a certain position \(x_r\), in a provided room, defined by a set of N polygons \(R(n)\), for a certain acoustical parameterisation \([a(n, f), s(n, f)]\), of the N surface materials and frequency \(f\), after the room being excited at \(t = 0\) by a sound source at position \(x_s\). Considering the geometrical data, i.e. the 3D room model \(R\) and the source and receiver positions \(x_s\) and \(x_r\), as known and fixed, the
inverse transformation $T^{-1}$ of this prediction model will transform any provided energy decay process $E_{meas}(t)$ into acoustical coefficients $a(n, f)$ and $s(n, f)$ of the N surface materials.

**FIGURE 1:** The traditional room acoustics model $T$ predicts energy decay $E$ using acoustical coefficients $a$ and $s$ while its inverse function $T^{-1}$ provides reversed functionality.

**TABLE 1:** Simulation model $T$ in matrix form $M$ for a single frequency band. The entries describe for each time interval how often a single ray has hit a certain surface of the room before arriving at the receiver.

<table>
<thead>
<tr>
<th>Ray Index</th>
<th>Time Slot</th>
<th>Surface Index</th>
</tr>
</thead>
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<td>1 5 4 ... 3</td>
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<td>2</td>
<td>1 4 2 ... 5</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
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<td>...</td>
</tr>
<tr>
<td>P</td>
<td>1</td>
<td>4 3 8 ... 2</td>
</tr>
</tbody>
</table>

**Room Acoustics Simulation Model as a Mathematical Function**

To facilitate the inversion the complex GA simulation model was condensed into a simple mathematical transformation function, in this case into a matrix form as shown in Table 1. This matrix $M$ describes the complete propagation, reflection and absorption of sound in a room. It is constructed during the RT simulation by storing the information how often a ray hits each single surface. The arrival times of a single ray at a receiver are therefore quantized and each time interval $\Delta t$ of each ray is represented by a single row in the matrix $M$. The total number of rows is equal to the number of rays, $P$, times the number of time intervals, $I$. The number of columns is equal to the number of surfaces of the room, $N$.

The matrix $M$ has to be built for each individual frequency, usually for the center frequencies of octaves or one-third octaves. Its dimensions easily exceed the available memory of commodity hardware by far. But most of the entries are zero as not every particle hits the receiver in every time interval after hitting every wall. Thus, by further condensing $M$ into a sparse representation, the memory footprint is reduced to only a few percent.

The temporal energy decay $E(t)$ for the fixed source receiver combination is then calculated using Equation 1.

$$E_{RT}(t, \alpha, f) = \sum_{p=1}^{P} E_{0,RT}(p) \prod_{i=1}^{N} (1 - a_i(f)) M(f, t/\Delta t, p, i) e^{-m_{air} ct}$$

For a frequency $f$, it uses the initial energy $E_{0,RT}$ of each ray $p$, the simulation matrix $M(f, t/\Delta t, p, i)$, where $t/\Delta t$ represents the correct time interval index at a time $t$, the frequency-dependent absorption coefficients $a_i$ of the i-th surface, and the air absorption $e^{-m_{air} ct}$ for the travel distance $ct$ of the ray after a time $t$ with speed of sound $c$. The initial energy of each particle $E_{0,RT}$ is a function of the total number of launched particles $P$, the source directivity and the size of the detection sphere around the receiver.

It is known that a RT simulation is fairly accurate in modeling the whole energetic reverberation process, but does not model strong early reflections (ERs). Therefore it can be beneficial to include the ERs matrix $M_{IS}$, which is calculated by the ISM similar to the RT matrix $M$, into the energy decay $E(t)$. The contribution $E_{IS}(t)$ of Q ISs to the decay is calculated by Equation 2.
\[ E_{IS}(t, \alpha, f) = \sum_{q=1}^{Q} E_{0,IS}(q) \prod_{i=1}^{N} [(1-s_i(f))(1-\alpha_i(f))]^{M_{IS}(t/\Delta t, q, i)} e^{-m_{air} r_q} \]  

(2)

In the ERs part, the spherical spreading $1/r$ must be taken into account for an IS with distance $r$, as well as the energy loss on the specular path due to sound scattering according to the frequency-dependent scattering coefficients $s_i(f)$ for the $i$-th surface.

The energy contributions of the ray tracer $E_{RT}(t)$ and the IS $E_{IS}(t)$ can be superposed to build the total energy decay $E_{total}(t)$, as described by Equation 3.

\[ F = E_{total}(t, \alpha, f) = E_{RT}(t, \alpha, f) + E_{IS}(t, \alpha, f) \]  

(3)

This equation describes the whole reverberation process only in dependency of the time $t$, frequency $f$ and the absorption coefficients $\alpha$. It can be calculated in milliseconds, once the matrices $M$ and $M_{IS}$ are built.

**Numerical Inversion by Iterative Least-Mean Squares Optimization**

Due to the occurrence of the entries of the matrix $M$ in the exponents of the energy decay calculation, the acoustics model $T$ is a non-linear function, which means that it cannot be inverted straightforward in a closed solution. Therefore a pseudo-inverse $T^{-1}$ is defined and calculated using a non-linear least-mean squares estimation, as described by Equation 4 with a cost function $f_c(\alpha)$ that is defined in Equation 5. This optimization is a numerical iterative process which uses differentials of the absorption coefficients and is highly accelerated due to the fact that a new combination of surface materials can be evaluated within milliseconds.

\[ \min_{x} \| f_c(x) \|_2^2 = \min_{x} (f_1(x)^2 + f_2(x)^2 + ... + f_n(x)^2) \]  

(4)

Unfortunately, an infinite number of solutions exist to this inversion problem. To reduce the number of possible solutions for a certain room under investigation, multiple source and receiver positions should be included so that each of the transfer functions $T$ uses the same common shared absorption coefficients, which are then adjusted with increased stability. Through the simulation matrix $M$, the optimized energy decays are directly linked to individual surfaces of the room, so that the absorption coefficients are adjusted right in the correct place.

**Processing of Measured Room Impulse Responses**

The target function for the least-mean squares optimizer is to minimize the deviations between the simulated energy decay and the measured energy decay at multiple positions in the room. For the cost function $f_c(\alpha)$, the measurements must be processed so that they can be subtracted from the simulations, as shown in Equation 5.

\[ f_c(\alpha) = E_{total}(t, \alpha, f) - E_{meas}(t, f) \overset{!}{=} 0 \]  

(5)

This is done by converting the measured IRs $s(t)$ into energy histograms, comparable to the results of a ray tracer. The IRs are band-pass filtered using either an octave or one-third octave filter bank (equal to the frequency resolution of the simulation). The energy of a time interval is then calculated by integration of the squared sound pressure over this interval, as described by Equation 6 for discrete signals.
\[ E_{\text{meas}}(t,f) = \int_{t_1}^{t_2} s_{\text{cont},BP_f}(t_c)^2 dt_c = \sum_{n=N_1}^{N_2} s_{\text{sampl},BP_f}(n)^2 \]  

The subscript \( BP_f \) indicates the band-pass filtering of the signals with a center frequency \( f \), which makes the equation frequency-dependent. The limits \( t_1, t_2, N_1 \) and \( N_2 \) define the continuous or discrete limits of the current time interval, which is defined by the time \( t \), with \( t_2 - t_1 = \Delta t \).

### Estimation of Scattering Coefficients

For simulations, absorption and also scattering coefficients must be defined. Scattering data is not broadly available and difficult to measure, so only in few cases accurate data can be used. However, experiments indicated that scattering sensitivity is limited and approximations might be sufficient without negative effects [14]. This holds true especially in already diffuse conditions, e.g. in rooms with high 'randomization strength'.

Therefore a method shall be introduced to calculate at least estimates for the scattering coefficients in harmony to the presented methods so far. After comparing the ERs in the measured and simulated IRs, the scattering coefficients of involved surfaces can be used to adjust the amplitudes in the simulated ERs.

![Figure 2](image1.png)  
**Figure 2:** Extracted early reflection of a room IR. The scattering coefficient of the originating wall is estimated by adjusting (shown are 0%, 20%, 40%, 60%, 80%) it to the amplitude of the measurement (dotted line).

![Figure 3](image2.png)  
**Figure 3:** For a successful estimation of scattering coefficients it is important to use exact positions of sources and receivers so that the reflections precisely line up. These positions are found using another minimization algorithm, which moves the positions for a best fit.

While the total energy in a room decays due to air and material absorption, the scattering does not directly affect the amount of energy, it only redirects the energy flow. However, the energy of the specular ERs is dependent on the absorption \( \alpha \) and scattering \( s \) of the contributing walls, with \( E_{\text{spec}} = (1 - \alpha)(1 - s) \). Thus the appropriate absorption coefficients are found first using the inverse model \( T^{-1} \), then the energy of each ER is corrected by adjusting the scattering coefficient of related surfaces, as shown in Figure 2. Only surfaces that contribute to ERs in one of the source-receiver combinations can be provided with scattering estimates. This is not a major downside due to the reduced importance of scattering in the already diffuse late decay [15].

To ensure that the ERs in the measured and simulated IRs are accurately time-aligned, accurate positions of sources and receivers are essential. These can be found using another optimizer which slightly moves initially given source and receiver positions until the ERs best match, as shown in Figure 3.
APPLICATION AND RESULTS

General applications of the presented methods can be divided into a) the acquisition of surface coefficients via traditional room acoustics IR measurements, e.g. in concert halls, classrooms, factory halls, etc., and b) the dedicated measurement of a single sample, such as in traditional reverberation room measurements but with improved room acoustics model.

(A) On-site measurement using 8 omnidirectional microphones and a dodecuhedron loudspeaker at 2 positions.

(B) CAD room model representation prepared in Trimble SketchUp for computer simulations.

(C) Particle traces of early reflections, simulated using the ISM.

(D) Average absorption coefficients of all surfaces after application of the inverse model.

FIGURE 4: In-situ absorption measurements using the inverse model approach in the chamber music hall of the University of Music in Aachen, Germany.

In-situ Measurement of Acoustical Coefficients

Many rooms contain diverse materials which sometimes results in an inhomogeneous distribution of absorption, e.g. in a concert hall the absorption is usually concentrated in the audience area. Therefore the optimization algorithm accounts for the location of individual faces, so that for any section of the resulting energy decay curve, it is possible to address all surfaces that contribute to the reflections in this time interval.

The proposed method was examined in different test scenarios and room acoustics measurements were performed in the reverberation chamber (Volume $V = 123m^3$) and a corridor ($V = 167m^3$) of the Institute of Technical Acoustics, Aachen, and in the Kammermusiksaal ($V = 706m^3$) of the University of Music, Aachen. These rooms with rather small volumes have plain surfaces and could be measured in an empty state, without obstacles and seating. The concert hall together with its CAD-version is shown in Figure 4.

In both spaces, the acoustic surface properties were estimated using the presented method. After applying the matched absorption coefficients in an evaluation simulation, it can be seen
that the resulting reverberation times (shown in Figure 5b for the concert hall) and energy decay curves (shown in Figure 5a for the reverberation chamber) are well matched between simulation and measurement.

Application in Reverberation Room Absorption Measurements

The highly simplified room acoustics models after Sabine and Eyring are still used in reverberation room measurements, although it is known that insufficient diffusion, which is inevitable when measuring samples with high absorption, violate the basic conditions of ergodicity and make the use of traditional formulae incorrect. It was already proposed by Benedetto in 1984 to replace them with computer simulations to overcome the problems of the non-diffuse sound fields which lead to non-linear decay curves [16].

The inverse model is well able to handle these non-linear decay curves and can calculate the correct absorption coefficients. For the application in reverberation room absorption measurements, the independent variables for the optimization can be constrained to the absorption coefficients of the sample only. Therefore all other surfaces are calibrated in a prior step by inverting the room acoustics model for the empty room, with no sample installed.

CONCLUSION AND OUTLOOK

By means of a hybrid GA room acoustics model inversion it was possible to calculate absorption coefficients after performing in-situ measurements in a room. The frequency-dependent coefficients could be related to individual surfaces. Using the ERs' additional dependence on the scattering coefficient, appropriate scattering coefficients were estimated. The presented method is suited for rooms of any shape, while less scattering and simple geometry will be of advantage. It will be analyzed in upcoming studies how well the algorithm is suited for rooms with extreme shapes and complex structures.

This is a condensed version of a future publication in progress.

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