1pEAa4. Low frequency sound absorption of resonators with flexible tubes

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Classically, passive acoustic liners, used in aeronautical engine nacelles to reduce radiated fan noise, have a Quarter-wavelength behavior, thanks to perforated sheets backed to honeycombs (SDOF, DDOF). So, their acoustic absorption ability is naturally limited to medium and high frequencies because of constraints in thickness. To drastically improve their capabilities to the lowest frequencies, the combination with active control systems or the using of foam architecture have shown an interest but the industrial application is tricky (ie. problems of fouling, robustness). A possible approach is to carry out a perforated panel resonator with flexible tube bundles to shift the resonance frequency to a lower frequency by a prolongation of air column length. This paper describes theoretically this concept that allows a significant change in the acoustic impedance due to the large thickness of the resistive and reactive material and the coupling with the surrounding cavity. Applied to aeronautical configurations, the resonance frequency decreases considerably compared to a conventional resonator (factor of about 1/5) but with a reduction of the maximum absorption when the tubes fill the cavity. Experiments in impedance tube validate the theoretical approach.
INTRODUCTION

Locally reacting liners, as those used in aeronautical engine nacelles, are generally “sandwich” resonators with a perforated plate linked to an honeycomb material above a rigid plate. Their absorption behavior can be described approximately with the principle of an Helmholtz resonator. The frequency range of absorption is so essentially controlled by the thickness of the honeycomb cavity (“Quarter-wavelength” behavior). The small size of holes (mostly from 0.4 to 2 mm according to industrial needs), absorbs the energy (thanks to the acoustic boundary layer applied at the internal walls) when a wave is propagated through the resonant cavity [1-2]. The impedance can depend non linearly on the incident particle velocity level (or Sound Pressure Level) [1]. So, acoustic “vortices” of particle velocity can occur at the resonator surface thus modifying the impedance. Many studies, since Ingard in the 50s [3], have tried to determine the influence of various parameters on the impedance and the absorption of holes. Gaeta & Ahuja [4] show in particular that to increase the perimeter of the hole with the same surface allows to increase the absorption with low magnitudes of particle velocity (< 1 m/s) but has not a significant effect for higher velocities. Above a threshold value of the ratio \( \frac{v_o}{v^*} \) (acoustic or particle velocity / friction velocity of acoustic boundary layer) the hole behavior becomes non linear [5]. It appears that the nonlinear dissipation mechanism of vortex shedding is crucial for noise level higher than 120 dB [6], value much lower than in an aircraft engine. Chandrasekharan et al. [7] lead impedance measurements in a tube and compared results with classical laws of Hersh, Kraft and Candrall & Melling. It is shown that an increase of ratio \( \frac{l_p}{d} \) (plate thickness / hole diameter) increases the frequency band on which there is linear behaviour of the plate with the sound level (between 100 and 150 dB until 6,4 kHz), which allows in particular to choose a liner characteristic more easily. Finally, in order to enlarge the frequency range of absorption, different types of SDOF liners can be piled up to constitute 2DOF or 3FOF liners. As for SDOF, the increase of SPL increases their resistance and decreases their reactance [8]. Nevertheless, their physical law is not suited to an absorption to the lowest frequencies, as needed for future Ultra High Bypass Ratio (UHBR) engines with shorter and thinner nacelles (frequencies around 500 Hz). A possible approach could be to link the perforated panel with flexible tubes introduced in the cavity, as proposed by Lu et al. [9], thus to shift the resonance frequency to a lower frequency by a prolongation of air column length. The interest of this concept has been proved experimentally but the authors have used a mathematical model based on a thin perforated plate (i.e. thin tubes) to describe the phenomena.

The aim of this paper is to describe mathematically this concept without assumption of thin tubes, to validate the theoretical approach thanks to experiments (with a variation of SPL), led in impedance tube with samples satisfying aeronautical dimensions. A comparison is achieved finally with a classical liner.

DESCRIPTION OF RESONATOR

The resonator is composed of a perforated plate whose holes are connected to hollow flexible tubes, inserted in a cavity and opened at the end (Figure 1).

The parameters describing the resonator are given below (Table 1).
### TABLE 1. Dimensional Parameters of material

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Plate thickness</th>
<th>Plate porosity</th>
<th>Number of tubes</th>
<th>Inner radius of tubes</th>
<th>Outer radius of tubes</th>
<th>Tube length</th>
<th>Cavity filling factor</th>
<th>Cavity thickness</th>
<th>Cavity radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol</td>
<td>$l_p$</td>
<td>$\sigma_p$</td>
<td>$n$</td>
<td>$r_i$</td>
<td>$r_e$</td>
<td>$l_t$</td>
<td>$\sigma_l$</td>
<td>$h$</td>
<td>$r_c$</td>
</tr>
</tbody>
</table>

An example of samples with different types of tubes is shown in Figure 2 (without cavity).

![Perforated plates with tubes](image)

**FIGURE 2.** Perforated plates with tubes

### THEORY

The propagation of waves along hollow tubes (direction x) can be shown as a linear combination of "propagational", "thermal" and "viscous" modes [10].

We assume that the wavelength of waves is much larger than the inner diameter of a tube.

To simplify the mathematical description, the theory of wave propagation is shown in narrow channels with parallel plates whose the distance equals $2 r_e$.

The pressure field in presence of visco-thermal effects is solution of the classical equation

$$ \nabla^2 p + \left( \frac{\omega}{c} \right)^2 p = 0 $$

and can be expressed for as following:

$$ p(x, r) = A \cos(q_r r)(e^{iq_x x} + e^{-iq_x x}) $$

(1)

with $q$, the propagation constant in the transverse direction of a channel with two parallel rigid walls.
As the associated transverse velocity must vanish at the inner boundaries, the transverse propagation constant (with the assumption of $q, r_i \ll 1$) is given by:

$$q_r^2 = \left(\frac{\omega}{c}\right)^2 = -\frac{(\gamma - 1)F(k_h r_i) + F(k_v r_i)}{1 - F(k_v r_i)}$$

where $F(X) = \tan(X)$ with

$$k_h = \frac{1 + i}{\delta_h} \text{ where } \delta_h = \sqrt{\frac{2K}{\rho C_p \omega}} \text{ (thermal boundary layer thickness)}$$

and

$$k_v = \frac{1 + i}{\delta_v} \text{ where } \delta_v = \frac{2\mu}{\rho \omega} \text{ (viscous boundary layer thickness)}$$

The average axial velocity has the following form:

$$u_v = \frac{A q}{\omega \rho} \left(1 - F(k_v r_i)\right)\left(e^{iqx} - e^{-iqx}\right)$$

As $q^2 = \left(\frac{\omega}{c}\right)^2 - q_r^2$, the complex propagation constant in the axial direction of the narrow channel $q$ is determined simply by:

$$q = \left(\frac{\omega}{c}\right) \sqrt{\frac{1 + (\gamma - 1)F(k_h r_i)}{1 - F(k_v r_i)}}$$

For circular tubes, expressions with Bessel functions can be used instead of the function $F(X)$ and simplifications for thin tubes or perforated plates lead to the Melling formulation [11].

Then, one assumes that:

- the continuity of pressure and mass flow between narrow channels and the surrounding cavity is verified at the end of channels
- the transmitted waves propagate in the rigid cavity, without loss, mainly in the direction of thickness, as for a classical resonator.
The main associated parameters are so:

- the cavity filling factor $\sigma_t$ defined as the total surface of mass flow $n \pi r_c^2$ by the air cavity area $\pi r_c^2 - n \pi r_c^2$.

  Remark: This one is close to the plate porosity, i.e $\sigma_t = \sigma_p$.

- the cavity thickness $h$

We are interested only on the specific reactance of the structure given thanks to the plate porosity:

$$x_s = \text{Im} \left( \frac{Z_t}{\rho c} \right) = \text{Im} \left( \frac{Z_c}{\rho c \sigma_p} \right)$$

with $Z_t$, the impedance at the opening of narrow channels.

**EXPERIMENTATION**

The impedance is obtained in an impedance tube equipped with three microphones for pressure measurements. On the opposite side of the tube, the loudspeaker generates a broadband random noise propagated in plane waves from 100 to 5,000 Hz from 100 to 145 dB.

The standard measurement method for two microphones is used in accordance with [12-14]. The three microphones taken by pair allow to satisfy the total frequency range.

First tests have been led for 5 samples (ex. in Figure 2) whose characteristics are specified in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$l_p$ (mm)</th>
<th>$\sigma_p$ (%)</th>
<th>$n$</th>
<th>$r_c$ (mm)</th>
<th>$r_c$ (mm)</th>
<th>$l_t$ (mm)</th>
<th>$\sigma_t$</th>
<th>$H$ (mm)</th>
<th>$r_c$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1</td>
<td>1.92</td>
<td>73</td>
<td>0.35</td>
<td>0.55</td>
<td>Variable: 2</td>
<td>35.9</td>
<td>43.2</td>
<td>10-20-30-60-90</td>
</tr>
</tbody>
</table>

It appears that these materials have a linear behavior according to the incident acoustic pressure level, representative of constant impedance and absorption coefficient (ex. for sample with $l_t$ = 20 mm in Figure 4), while a sample with only the perforated plate (without tubes) generates a large variation of absorption (Figure 5). Non-linearity are due to acoustic vortices around the holes for a high ratio of "$r_c / l_p$" (hole radius / plate thickness) [6, 7]. So, to increase artificially the plate thickness, by the extension of tubes, prevent the presence of vortices.
FIGURE 4. Effect of SPL (dB) for sample "Mansart" with $l_t=20$ mm

FIGURE 5. Effect of SPL (dB) for classical resonator (Perforated layer ($\sigma=4.7\%$, $r=0.55$ mm, $l=1$ mm) + cavity)

One can notice, also, that the frequency range of absorption is very different: around 260 Hz, for the resonator with tubes, vs 1300 Hz for the classical resonator.

The comparison of absorption coefficient for all samples (Figure 6) confirms that the length of tube allows to shift the frequency range of absorption. Nevertheless, an increase of length is associated with a reduction of absorption coefficient.
SIMULATIONS

Simulations of reactance $x$ (cf. equation (5)) are led with previous model for several samples (Figure 7). Despite the approach based on parallel plates with equivalent boundary layers and porosity, comparison with experimental results is satisfying (better than with a Melling formulation), allowing to be confident in the determination of the frequency range of absorption, relative to "0" reactance (Table 3).

The simulation of reactance for tubes satisfying the characteristics of Table 1 but placed in front of a rigid background (Figure 8), i.e without cavity, shows that the maximum absorption is obtained at high frequencies (from 2760 Hz cf. Table 3). It appears that the coupling with the surrounding cavity is predominant to generate an absorption in low frequency range.

![Figure 6](image_url)

**FIGURE 6.** Effect of tube length for tested samples, vs. classical resonator

![Figure 7](image_url)

**FIGURE 7.** Comparison of simulated /experimental reactance for different samples
TABLE 3. Comparison of simulated / experimental frequency of "0" reactance.

<table>
<thead>
<tr>
<th>Material</th>
<th>l = 10 mm</th>
<th>l = 20 mm</th>
<th>l = 30 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. Frequency (Hz)</td>
<td>352</td>
<td>264</td>
<td>224</td>
</tr>
<tr>
<td>Simu. Frequency (Hz) with cavity</td>
<td>376</td>
<td>264</td>
<td>216</td>
</tr>
<tr>
<td>Simu. Frequency (Hz) without cavity</td>
<td>8400</td>
<td>4168</td>
<td>2760</td>
</tr>
</tbody>
</table>

CONCLUSION

This experimental and theoretical study has shown that the introduction of tubes in a cavity of a conventional resonator generates a significant shift of the frequency range of absorption towards lower frequencies, thus by a prolongation of air column length. Despite the simplicity of theoretical approach, the resonance frequency can be determined precisely with dimensional parameters. Nevertheless, a formulation with Bessel functions will be more adequate to simulate the absorption coefficient due to circular long tubes in a cavity. Applied for an aeronautical liner, the resonance frequency decreases considerably compared to the initial resonator (factor of about 1/5). These first results allow to consider that these resonators with linear behavior could be an alternative to classical resonator, on the assumption of an automatic manufacturing process (requirements of robustness and cleaning theoretically satisfied as for classical resonator).

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REFERENCES