1pEAa6. Effect of standoff distance on the reconstruction of in-duct velocity field and regeneration of pressure field

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Identification of in-duct acoustic source characteristics is essential in the design of fluid machinery system for reducing and predicting the flow-generated noise. To this end, the inverse estimation method can be employed by using the measured sound field and matrix formulation for wave propagation within a duct. In this paper, the effect of the distance between source and measurement plane is investigated. At each standoff distance, pressures are measured at three planes with two different spacings to widen the estimation frequency range, and measurements are conducted with three different standoff distances. Modal decomposition is applied to estimate modal amplitudes, and the result is used to reconstruct the velocity field at the source plane and to obtain the regenerated pressure field at the measurement planes. It is shown that the modal amplitude identified by measured pressure field at the short standoff distance, i.e., at nearfield, can yield an accurate reconstructed velocity field of the source and regenerate the pressure field with smaller error, which is similar to the other inverse techniques such as equivalent source method and nearfield acoustical holography. A field reduction example by suppressing some parts of source velocity field is shown for demonstrating the effectiveness of the method.

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INTRODUCTION

The in-duct source identification at source plane is essential in the prediction of the radiated noise from the duct opening and in the establishment of the low noise source design. For the identification procedure, in general, in-duct pressures fields are measured by multiple microphones at positions apart from the source plane and the measured data are put together to inversely calculate the source information [1-5]. In this paper, among many other factors that affects to the accuracy of source identification procedure, the effect of standoff distance to the reconstructed velocity field at the source plane is studied. The pressure fields in a hard-walled duct are measured at several positions of different standoff distances, changing the influencing amount of evanescent waves in the measured signals. The velocity field at the source plane is reconstructed, and the result is observed depending on the standoff distance. To show the usefulness of the method in this work, major noise part of velocity field is suppressed, and the resulting radiated sound power is simulated and compared with the original one.

WAVE PROPAGATION IN DUCT

The assumptions for the sound propagation within a straight duct with a rigid wall are: 1) Time–harmonic wave, 3) homogeneous and nonviscous medium, 4) presence of mean flow, 4) constant temperature, and 5) small amplitude of wave. The general pressure, \( p \) and axial velocity, \( u_z \), distributions in cylindrical coordinated are given as [6]

\[
\begin{align*}
 p(r, \theta, z, t) &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \psi_m^n \left( C_{m+} e^{-y_{m+} \rho_{m+} z} + C_{m-} e^{y_{m-} \rho_{m-} z} \right) e^{i \nu x}, \quad (1a) \\
 u_z(x, y, z, t) &= \frac{1}{\rho_0 c_0} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \psi_m^n \left( \frac{k_{m+}^z}{k_0 - M k_{m+}^z} C_{m+} e^{-y_{m+} \rho_{m+} z} - \frac{k_{m-}^z}{k_0 + M k_{m-}^z} C_{m-} e^{y_{m-} \rho_{m-} z} \right) e^{i \nu x}. \quad (1b)
\end{align*}
\]

Here, \( \psi_m^n \) is the cross-sectional mode of pressure, \( k_{m+}^z \) the transmission wave number in the propagation direction, which depends on the sectional shape, \( C_{m+} \) and \( C_{m-} \) the modal amplitudes for two progressive waves moving in opposite directions, \( k_0 \) the wave number (\( = \alpha_0 / c_0 \)), \( M \) the Mach number (\( = U / c_0 \)), \( U \) the mean flow speed, and \( \rho_0 \), \( c_0 \) the density and sound speed at the atmospheric temperature, respectively.

Representing Eq. (1a) in a matrix form and using the Fourier transformation, Eq. (1a) becomes

\[
 p = M c.
\]

(2)

Here, \( p \) denotes the pressure vector, \( M \) the modal matrix, \( c \) modal amplitudes vector. The modal amplitude can be obtained by multiplying the pseudo inverse of matrix \( M \) to the measured pressure distribution. Then, the obtained modal amplitude can be inserted into Eq. (1) to obtain the pressure and velocity fields at any plane.

The sound power can also be calculated by using the modal amplitudes as [7]

\[
 W_s = \sum_{n=1}^{N} \frac{\pi \rho_0 c_0}{2} C_n^2 \left( \frac{1 - M^2}{1 + \alpha_{m+} M} \right)^2,
\]

(3)

where \( N \) denotes the number of propagating modes, and \( C_n \) is the modal amplitude of existing propagating modes. The parameter \( \alpha_{m+} \) is related with the wave number of each radial and circumferential modes, \( k_{m+} \), by

\[
 \alpha_{m+} = \sqrt{1 - \frac{k_{m+}^2}{k_0^2}} (1 - M^2).
\]

(4)

By using the foregoing equations, pressure field, velocity field, and sound power at any point or section can be calculated with the aid of modal amplitudes.
EXPERIMENT SETUP

A duct is excited by a compression driver, and sound propagation within a duct is measured in a section. Anechoic termination is employed at the end of a duct to suppress the reflected wave component from the opposite end of the source plane. The conceptual setup is shown in Fig. 1. The measurement rig is composed of standard acryl ducts having circular cross section with inner diameter 250 mm, thickness 10 mm, and length 510 mm. Pressure distributions are measured at 3 different standoff distances of 0.1, 0.61, and 2.14 m. At the each standoff distance, sound pressures are measured at 3 planes, which are apart from a plane at a standoff distance by 15 and 100 mm respectively, thus the total measurements are conducted at 9 planes.

At each measuring plane, a microphone array (B&K 4935, 1/4 inch) is used to measure the sound pressure data at 9 radial positions, and the line array rotated 18 times at each measurement cross-section, so sectional measurement are conducted at 162 points per each section. This set-up is designed to estimate accurate modal amplitude in the Helmholtz number range of $0.38 < kR < 12.8$, which is dependent on both axial distance between measurement plane [5,8] and number of measurement points in a cross-section [9].

![Figure 1. Schematic of measurement set-up.](image)

RESULT

Regeneration of pressure field

Modal amplitudes are obtained from the measured pressures at 3 planes with different standoff distances. The pressure fields at measurement planes are regenerated and compared with the measured pressure field to verify the obtained result. The percentage error between two fields are calculated as

$$e = \frac{\sqrt{\sum |p_{\text{meas}} - p_{\text{reg}}|^2 \times 100}}{\sum |p_{\text{meas}}|} \quad (5)$$

where $p_{\text{meas}}$ is the measured pressure field, $p_{\text{reg}}$ is the regenerated pressure field.

In general, pressure fields are well regenerated regardless of the standoff distances, $d$. When $d = 2.14$ m, the percentage error is calculated and shown at Table 1 at some specified Helmholtz numbers. From the result, for a given number of measurement points, the error increases as the increase of Helmholtz number. The biggest error happens at the Plane #2 at $kR = 11.2$. However, even for this worst case, the regenerated field is in good agreement with the measured field as can be seen in Fig. 2.
Two representative points, one on the center and the other on the peripheral point, are selected, and the pressure spectra are regenerated in the full interested Helmholtz number range at two representative points mentioned above. The result shows a good agreement with the measured spectrum as shown in Fig. 3.

**TABLE I.** Percentage error between measured and regenerated pressure at a standoff distance \( d = 2.14 \) m.

<table>
<thead>
<tr>
<th>( kR )</th>
<th>Plane 1</th>
<th>Plane 2</th>
<th>Plane 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.14</td>
<td>2.59</td>
<td>2.81</td>
</tr>
<tr>
<td>2.5</td>
<td>6.54</td>
<td>8.11</td>
<td>7.88</td>
</tr>
<tr>
<td>4.5</td>
<td>12.47</td>
<td>11.09</td>
<td>13.93</td>
</tr>
<tr>
<td>6.7</td>
<td>16.54</td>
<td>14.60</td>
<td>10.88</td>
</tr>
<tr>
<td>8.9</td>
<td>27.73</td>
<td>34.11</td>
<td>24.66</td>
</tr>
<tr>
<td>11.2</td>
<td>32.28</td>
<td>37.63</td>
<td>23.83</td>
</tr>
</tbody>
</table>

Figure 2. A comparison of the sound pressure distributions at \( kR = 11.2 \) at the 2\textsuperscript{nd} plane with \( d = 2.14 \) m. (a) Measured pressure field, (b) regenerated pressure field.

Figure 3. A comparison of pressure spectra of a center point \#1 and a peripheral Point \#2: (a) position of the Point \#1 and \#2 in a cross section, (b) pressure spectrum at the Point \#1, (c) pressure spectrum at the Point \#2. ——, measured spectrum; -----, regenerated spectrum.
Reconstruction of velocity field

The velocity fields at the source plane are reconstructed by using the obtained modal amplitude at 3 different standoff distances. As depicted in Fig. 2, in this example, the compression driver is mounted at the source plane to excite the duct system, so the active velocity should be observed at the mounting position. Figure 4 shows the velocity field obtained with the mounting position of the compression driver indicated as a broken black circle. Only, at a position of d = 0.1 m case, which corresponds to a nearfield measurement case and the distance is actually equal to 0.68λ for the given Helmholtz number kR = 11.2, shows a good agreement with the actual excitation position.

![Figure 4. Reconstructed velocity field at the source plane at kR = 11.2, a) d = 0.1 m (= 0.68λ), b) d = 0.61 m (=8.9λ), c) d = 2.14 m (=31.2λ).](image)

The result reveals that the agreement quality between the regenerated and measured pressure fields is insufficient as the verification for the preciseness of the obtained modal amplitude, when the backward projection is considered. Therefore, depending on the frequency range of interest, the standoff distance should be carefully decided to capture the evanescent wave component satisfactorily, as like it is utilized in the nearfield acoustic holography (NAH) and equivalent source method (ESM).

**APPLICATION**

It is intended to change the radiated sound power by suppressing the high amplitude parts in the noise source plane of the velocity field. The high amplitude portion of volumetric velocity source, which is indicated by a broken black circle in Fig. 4 (a), is modified to be attenuated by half, and then the modal amplitude due to this modification is calculated. The calculated modal amplitude is inserted into Eq. (3) for sound power calculation. As a result, the radiated sound power at kR = 11.2 changes from 44.0 dB to 41.1 dB. One can predict the effect of design modification before the actual treatment on the source and also one can get an idea about how much change on the source field is required to fulfill a prescribed target value of the radiated sound power.

**CONCLUSION**

The effect of the standoff distance between source and measurement plane is investigated in the reconstruction of velocity field at source plane. Modal amplitudes are obtained by using the measured pressure fields that are excited by an compression driver in a duct system. By using the obtained modal amplitude, pressure fields are regenerated at the measurement plane, velocity field is reconstructed at the source plane. The result shows that even though the regenerated pressure fields, which are obtained by the pressure field measured at farfield, shows a good agreement with the measured pressures, the nearfield measurement is required for the inverse calculation of the source information. A field reduction example, suppressing the major noise source in the velocity field, is shown. By using this technique, the amount of design change for a source for low noise, which usually is inversely proportional to the performance, can be determined to reduce the certain amount of radiating sound power, which can be useful information for establishing design strategy.
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