1pEAA10. The threshold of the difference between a mathematical model applied to active noise control and data recorded

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Nowadays, there are many methods used to obtain mathematical models applied to active noise control, especially, when the transfer function is required. Inside rooms, the global active sound control has bad results due to the reflections and the diffuse field. Then, authors have applied system identification to find more complex mathematical models based on measured data. Also, the number of system identification methodologies is increasing and it carries to having many unexplored models. In order to know which models are useful for global active noise control inside rooms, a relationship between the sound pressure level decreased and the error of the mathematical model is presented. First, the concept of "a useful mathematical model" is defined under any context based on an analysis of the error (FIT). In addition, this concept is delimited to the active noise control context. Finally, an example is presented.

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INTRODUCTION

Control systems need a way to predict the effect of the input and an actuator in the output system. Usually, it is found through a mathematical model. This concept applied to active noise control is verifiable when the control algorithms are applied. The Least Mean Square (LMS) filter assume that the output can be expressed as a FIR filter, sum of many delayed and attenuated copies of the input [1]. The same case is proposed by the control through RLS filter [2]. Also, there are many more complex models as neural network (nonlinear system), FxLMS, etc.[3, 4, 5]. Furthermore, some control methods require to know which is the transfer function of the system and another methods could be applied to obtain another mathematical models.

Therefore, in acoustics has begun to apply methodologies to "find" the mathematical models [5, 6, 7, 8]. Some of them are known as system identification, and every day new methodologies have been created [9, 10, 11, 12, 13].

Nonetheless, the models usually are limited to some conditions and some variables, but they are not only the variables or conditions that determine the value of the output, e.g. the background noise. Then, there is a difference between the real output and the estimated output (obtained through the mathematical model). This paper deals with the problem to know when a mathematical model can be applied to an active noise control system or it cannot.

APPLICABILITY OF A MODEL TO A PROBLEM

The concept that a "wrong" model can solve some problems can be explained through two examples:

• **First example:** The context is shown by the figure 2. The aim is to make that a person who listens to music in a concert does not feel the two line array systems as two separated sources. According to the Haas effect, the sound of two sources (emitting the same sound) has to come to the receiver delayed less than 30 seconds, to make that it can be perceived as only one sound [14].

![Diagram for examples one and two.](image)

In order to solve this problem, the input signal of nearest line array (Line array 2) is delayed a time \( t_d \). Then, the value of \( t_d \) depends on the next model:

\[
P_r(t) = P_{s1}(t-t_1) + P_{s2}(t-t_d-t_2)
\]

(1)

Where \( P_r(t) \), \( P_{s1}(t) \) and \( P_{s2}(t) \) are the sound pressure at the receiver and emitted sound by the line array 1 and 2; \( t_1 \) and \( t_2 \) is the delay produced by the distance between the line array 1 or line array 2 and the receiver; \( t \) is the time.
According to the aim and the model, the idea is to get this condition: $t_1 = t_d + t_2$. Then, the value of $t_d$ depends on the speed of sound $c$. According to [15], $c$ can be calculated, as a function of the temperature $T$, using the next equation:

$$c = 331.5 \left(1 + \frac{T}{273}\right)^{\frac{1}{2}} \tag{2}$$

If the value of $T$ is unknown, an approximation can be used, assuming $T = 20^\circ C$. If the real temperature is $15^\circ C$, then the difference between the speed of sound, estimated and real, makes that two generated sounds (from line array 1 and 2) comes to the receiver delayed. Using the estimation can be obtained a values for $t_1$, $t_2$ and $t_d$, as it is shown by the table 1.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$c$</th>
<th>$t_1 = \frac{10}{c}$</th>
<th>$t_2 = \frac{10}{c}$</th>
<th>$t_d = t_1 - t_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$15^\circ C$</td>
<td>339.97</td>
<td>0.32355</td>
<td>0.02941</td>
<td>0.29414</td>
</tr>
<tr>
<td>$20^\circ C$</td>
<td>342.91</td>
<td>0.32078</td>
<td>0.02916</td>
<td>0.29162</td>
</tr>
</tbody>
</table>

Nevertheless, the difference between this two values of $t_d$ (for each condition) could be ignored due to it is smaller than the 50ms, then it could not be perceived.

- **Second example:** the conditions are the same as the previous example except by the problem. The problem now is how to get the same energy in all frequencies at the receiver position. If the same model is applied, the delay generate a comb filter shown in the figure 2. Then, it needs another model that produce less error, for example if the delay is so short time such that the comb filter is produced only in unperceived frequencies.

![FIGURE 2: Transfer function of the two copied signals, one of them delayed.](image)

The previews examples show that some mathematical models can be applied to solve some problems, but they not necessary solve another problems. It does not matter if they are under the same conditions. Each problem accept some error from the model and it can be measure using the concept known as fit [16].

The FIT is “how much similar” is the model to the reality. And, it can be quantified for dynamical models using the next expression:
\( c(t) = G(y(t), \hat{y}(t)) \)  \hspace{1cm} (3)

Where \( G(\cdot) \) is a transformation, \( y(t) \) is a measure data set of the variable of interest, and \( \hat{y}(t) \) is the estimation of \( y(t) \) using the mathematical model.

Then, the model can be applied if the next condition is verified: \( c(t) > E \) when \( t = 0, \cdots, \infty \); where \( E \) is the limit error accepted by the problem.

**CONCEPT APPLIED TO ACTIVE NOISE CONTROL**

The aim of the mathematical model in active noise control is to predict the sound at the receiver location \( R \), as it is shown by the figure 3. One or many sensors can measure the noise source or sources \( S \), and a transformation \( F(\cdot) \) can relate the measured sound to the sound at the receiver location. Furthermore, another transformation \( H(\cdot) \) allows to get the effect of a controled sound source (the actuator \( A \)). Indeed, the functions \( F(\cdot) \) and \( H(\cdot) \) cannot be obtained, then they are estimated and called \( \hat{F}(\cdot) \) and \( \hat{H}(\cdot) \) respectively.

**FIGURE 3:** Common diagram for active noise control.

Undoubtedly, the actuator cancel the noise if:

\[ F(u(t)) + H(x(t)) = 0 \]  \hspace{1cm} (4)

Where \( u(t) \) is the sound measured by de microphone, and \( x(t) \) is the sound emitted by the actuator (see the figure 3).

The condition expressed by the equation 4 can be ensured if the actuator generate the correct sound \( x(t) \). The transformations \( F(\cdot) \) and \( H(\cdot) \) are unknown, then the processor \( P \) use estimations. Thus, the error \( c(t) \) is produced, and affects the control system.

\[ F(u(t)) = \hat{F}(u(t)) + \epsilon_1(t) \]  \hspace{1cm} (5)
\[ H(x(t)) = \hat{H}(x(t)) + \epsilon_2(t) \]  \hspace{1cm} (6)
\[ \hat{F}(u(t)) + \hat{H}(x(t)) = -\epsilon_1(t) - \epsilon_2(t) = c(t) \]  \hspace{1cm} (7)

The equation 7 shows the effect of the error in the condition, \( c(t) \) is the minimum noise the control system can ensure. In order to check if the model is appropriated to the control, let compare the energy of \( u(t) \) to the energy of \( c(t) \)and check if the noise is controlled enough.
\[ N_r > 10 \log \left( \frac{\int_{t=0}^{\infty} F(u(t))^2 dt}{\int_{t=0}^{\infty} \epsilon(t)^2 dt} \right) \]  

(8)

Where \( N_r \) is the attenuation required for the noise. In discrete time \( n \), using \( k \) samples, the equation can be changed by:

\[ N_r > 10 \log \left( \frac{\sum_{n=0}^{k} F(u(n))^2}{\sum_{n=0}^{k} \epsilon(n)^2} \right) \]  

(9)

It is important to note that the model can change if the problem change, it depends on the noise reduction desired.

**EXAMPLE**

This section presents an example gotten through a simulation. It is based on the source-image method shown by [17]. First, two sound sources are simulated separately inside an enclosure (a noise source and an actuator), including the effect of another sources. Second, the signals obtained are used to estimate a model through the empirical transfer function estimate (ETFE) [18]. Next, an intuitive method to get \( x(t) \) is proposed. Finally, the equation 9 is applied.

The enclosure is a cube, the position of the two sources (\( S \) and \( A \)) and the receiver (\( R \)) are positioned as it is shown in the figure 4. For this practical case, \( F(\cdot) \) and \( H(\cdot) \) applies the source-image method with all surfaces with sound absorption coefficient equal to 0.9, the impulse response is based on only one or two reflections on each surface.

**Figure 4:** Scheme of the system of the example.

After to get the simulated input and output data, \( F(\cdot) \) and \( H(\cdot) \) are assumed as independent systems with input \( u(t) \) is related to the output \( F(u(t)) \) and another input \( x(t) \) is related to the output \( H(x(t)) \); and they are estimated separately. In order to apply the estimator ETFE, the next procedure is followed:

1. Take the input, and output vector of the systems, it means sufficiently long vector of \( u(n), F(u(t)), x(n) \) and \( H(x(t)) \).

\[ u_N = [u(0), u(1), \ldots, u(N-1)]^T \]  

(10)

\[ y_N = [F(u(0)), F(u(1)), \ldots, F(u(N-1))]^T \]  

(11)
\[ x_N = [x(0), x(1), \ldots, x(N-1)]^T \]  
\[ z_N = [H(u(0)), H(u(1)), \ldots, H(u(N-1))]^T \]  

(12)  
(13)

The operator \( T \) denotes the transpose matrix.

2. Then, these vectors are used to estimate the transfer function of these two systems \( \tilde{F}_N(\omega) \) and \( \tilde{H}_N(\omega) \) as follows:

\[
\tilde{F}_N(\omega) = \frac{Y_N(\omega)}{U_N(\omega)}
\]  
\[
\tilde{H}_N(\omega) = \frac{Z_N(\omega)}{X_N(\omega)}
\]  

(14)  
(15)

With,

\[
Y_N(\omega) = \sum_{n=0}^{N-1} y(n)e^{-jn\omega}
\]  
\[
U_N(\omega) = \sum_{n=0}^{N-1} u(n)e^{-jn\omega}
\]  
\[
X_N(\omega) = \sum_{n=0}^{N-1} x(n)e^{-jn\omega}
\]  
\[
Z_N(\omega) = \sum_{n=0}^{N-1} z(n)e^{-jn\omega}
\]  

(16)  
(17)  
(18)  
(19)  
(20)

The transfer functions are verified using the equations 5 and 6. The signals used for the equation 5 are shown by the figure 5. While, the figure 6 shows the signals used in the equation 6.

**FIGURE 5:** Signals used in the application of the equation 5. \( F(u(t)) \) is in color blue, \( \tilde{F}(u(t)) \) is in color green and \( e_1(t) \) is in color red.

It is important to note that the error signal is very small in amplitude respect to the output signals. Then, a good noise reduction is expected. Nonetheless, the condition shown by the equation 9 shows that the reduction, for this case, will not be more than 2.31 dB. Then, the \( x(t) \) value to obtain the best reduction is found and the reduction was 2.26 dB, only 0.05 dB is the difference.
**CONCLUSIONS AND FUTURE WORKS**

This paper showed the relationship between the mathematical model and the problem. Then, this concept was applied to the active noise control and the fit of mathematical model was related to the noise reduction. Finally, a simulation of the problem was according to the proposed limit by the theory.

This is the first step to find effective methodologies to model acoustic inside rooms and make active noise control getting high reduction levels. The second step is to find models that allows to get high values of the expected noise reduction, specially with multiple input - multiple output systems. Finally, Apply this concept to the virtual sensors.

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