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1pEAb8. Velocity control with class D amplifiers
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A SONAR array's radiation pattern is affected by the acoustic interactions, which may limit the effectiveness of beamforming algorithms when transmitting. A negative feedback system with a velocity sense signal fed back to the power amplifier can mitigate the array interactions proportional to the loop gain, and be effective across a broad frequency range without requiring a priori knowledge of the input signals. Motional current velocity control and its tradeoffs are summarized. Class D switching amplifiers can achieve greater than 90% efficiency, and are increasingly being used to drive SONAR arrays. When a velocity control system is used with a Class D amplifier, feedback stability becomes a significant concern due to obtaining the feedback signal after the amplifier's LC output filter. The array equations are still obtained by converting the amplifier into a Thevenin equivalent force and output impedance, which now includes the amplifier's output filter and the synthesized impedance due to the velocity control loop. Sample beampatterns both with and without velocity control will be presented, concluding that velocity control is well suited for use with complex dynamic transmit beamforming.
INTRODUCTION

Achieving a required acoustic level and beampattern using an array of transducers across a wide frequency range is a non trivial problem due to the acoustic interaction between elements. Digital signal processing now enables real-time dynamic beamforming, resulting in the ability to introduce time-varying complex shading coefficients for each array element, but the effectiveness of beampatterns requiring precise shading coefficients may be limited by the acoustic interactions. Figure 1 shows a piezoelectric transducer driven by voltage $V_{load}$ and with a surface velocity of $u$. The mechanical impedance may be lumped together into the frequency dependent complex quantity $Z_m^E$. The acoustical load includes the self radiation impedance $Z_{acii}$, and the interaction force $F_{aciN}$.

![Diagram showing single degree of freedom (SDOF) piezoelectric transducer model and mechanical and acoustical parameters transformed onto the electrical domain with self radiation impedance lumped into the mechanical branch inductance $M_m$ and resistance $R_m$.](image)

$F_{aciN}$ is the sum of the forces projected onto the $i$th element from the other elements in the $N$ element array, and may be represented as a coupled mechanical impedance relative to the $i$th transducer’s velocity $u_i$

$$Z_{aciN} = \frac{F_{aciN}}{u_i} = \frac{1}{u_i} \sum_{l \neq i}^N F_{cil} = \sum_{l \neq i}^N Z_{cil}$$

(1)

where $F_{cil}$ is the force acting on the $i$th element due to the $l$th element, and $Z_{cil}$ is the corresponding coupled radiation impedance of the $i$th element due to the $l$th element. The total radiation impedance of the $i$th element is the self radiation impedance plus the interaction impedances $Z_{acli}$

$$Z_{aci} = Z_{acii} + \sum_{l \neq i}^N Z_{acil}$$

(2)

The coupled radiation impedance depends on both the array geometry and the elements’ relative velocities

$$Z_{acil} = z_{acil} u_i / u_j$$

(3)

The mutual radiation impedance $z_{acil}$ is based only on the element geometry and spacing relative to the wavelength, and by reciprocity $z_{acil} = z_{aci}$. For sources with uniform velocity distributions, the solution takes the form of a double surface area integral

$$z_{acil} = \frac{i \rho c k}{2 \pi} \iiint e^{-jkR} \frac{dS_i dS_l}{R}$$

(4)
where \( \rho \) is the impedance of the medium, \( c \) is the sound speed, \( k = 2\pi/\lambda \) is the wavenumber, \( R \) is the distance between points of integration, and \( dS \) are the surface areas of integration. The interaction forces therefore change the transducer’s total load impedance and resulting surface velocity. This may degrade the array’s beampattern, particularly if precise shading coefficients are required. In extreme cases, the radiation resistance for some elements may become negative, forcing the amplifier to receive more power than it is transmitting, typically resulting in catastrophic amplifier failure.\(^4\) Carson concluded that the impedance variations due to acoustic interactions could be mitigated by parallel tuning and adding a large impedance in series with the transducer to effectively turn the power amplifier into a current source.\(^5\) This approach of passive control is usually referred to as "Carson’s velocity control," and has a very limited bandwidth around mechanical resonance, where the transducer’s tuned electrical impedance is high impedance and \( u_i \equiv I_i \).

**ACTIVE VELOCITY CONTROL**

Carson also noted that a negative feedback loop using velocity feedback to the power amplifier may be used to mitigate the interaction affects.\(^5\) The loop attempts to maintain a constant velocity regardless of frequency and acoustic interactions. The velocity of the transducer’s radiating surface may be obtained by integrating an accelerometer’s output,\(^6\) or by differentiating a displacement sensor’s output. Such a system was described for tonpilz transducers in 1967.\(^7\) Woollett later outlined the “motional current” method of velocity feedback control for piezoelectric transducers, where the velocity sensor is the estimated motional current, calculated as the total current flowing into the transducer minus the estimated current flowing into its parallel electrical branch.\(^8\) This approach requires current and voltage feedback at the transducer, and a priori knowledge of the transducer’s electrical branch admittance. Aronov et. al. extended the use of the motional currents used in estimating mutual radiation impedances to demonstrate a calibration method for SONAR transducers and arrays based on impedance measurements.\(^9, 10\)

Figure 2 illustrates motional current velocity control. The forward gain \( G \) includes the open-loop signal gain \( G \), the power amplifier gain \( G_{amp} \), and a generalized matching and tuning network \( G_t \). \( V_{out} \) is at the amplifier output and \( V_{load} \) is at the transducer. The feedback term \( H \) is notionally broken up into a fixed scaling factor \( H_{scale} \) that has units of impedance to convert the sensed currents to voltages, and a frequency dependent portion \( H_{comp} \). The electrical admittance estimator \( Y_e \) is used to estimate \( I_e \). This architecture is applicable to any transducer that has parallel electrical impedance, but it is not applicable to transducers where the electrical impedance is in series with the mechanical circuit parameters, such as voice coil devices.

FIGURE 2. Velocity control using motional current feedback for an electromechanical load transformed onto the electrical domain. The transducer parameters are mechanical branch impedance \( Z_m = jX_m + R_m \), parallel electrical branch impedance \( Z_e = jX_e + R_e \), and acoustic interactions \( V_{acN} \). For the SDOF piezoelectric load in Figure 1, \( X_m = aM_m - 1/\omega C_m, R_m = R_m, X_e = -1/\omega C_e, \) and \( R_e = R_{el} \).

For piezoelectric transducers with high electrical quality factor, the electrical admittance \( Y_e \) is dominated by the blocked capacitance \( C_e \). Velocity control accuracy is therefore limited by the estimation of \( C_e \), which is dependent on voltage, pressure, and temperature.\(^8\) The transducer’s mechanical branch is a combination of its self-impedance and the acoustic interaction. The motional current is

\[ I_m = I_{load} - I_e = (V_{load} - V_{acN}) Y_e. \]  

(5)
The electrical equivalent parameter $Y_m$ includes the mechanical impedance $Z_m^E$ and self radiation impedance $Z_{aci}^E$, both reflected onto the electrical domain, but does not include the mutual radiation impedance $Z_{aciN}$. The forward gain $G$ and feedback gain $H$ may be treated as a single block, so the loop equation from Figure 2 may be written as

$$V_{load} = G \left[ V_{in} - H \left( I_{load} - V_{load} Y_e' \right) \right]. \quad (6)$$

Before continuing, it is helpful to define a shorthand term for the loop gain as

$$J = GH \left( Y_m + Y_e - Y_e' \right). \quad (7)$$

and the portion of the loop gain due to estimation error is

$$J_e = GH \left( Y_e - Y_e' \right). \quad (8)$$

All of the components in $J$ and $J_e$ are frequency dependent complex quantities. If only one transducer is being driven, or the impedance between the amplifier output and the transducer is zero, then the load voltage may be solved by superposition of the loop’s response to the input signal $V_{in}$ and the interaction forces $V_{aciN}$, or equivalently by substituting equation 5 into equation 6. However, if there is any series impedance between the amplifier output and the transducer, then the matching and tuning network’s gain $G_t$ will be dependent on the acoustic array interactions. A Thevenin equivalent approach may then be used to determine the outputs. Figure 3 illustrates a Thevenin equivalent circuit of the system in Figure 2, where the parameters $F_{th}$ and $Z_{th}$ include everything except for the acoustic interaction force $F_{aciN}$.

**FIGURE 3.** Thevenin equivalent circuit of Figure 2, with parameters transformed onto the mechanical domain.

The Thevenin force $F_{th}$ is the mechanical force across the load if the acoustic interaction impedance was infinite. The force may be obtained from equation 6, using the substitutions $F_{th} = N_{em} V_{load}$, $I_{load} = V_{load} Y_e$, and $G = G_{oc}$, where $G_{oc}$ is the forward gain if the transducer’s mechanical branch impedance were infinite. $G = G_{oc}$ if any only if there is no series impedance between the amplifier output and the transducer. The thevenin force is

$$F_{th} = \frac{N_{em} GV_{in}}{J_e + G/G_{oc}}. \quad (9)$$

$Z_{th}$ is the Thevenin impedance seen from $F_{aciN}$ looking back through the transducer’s acoustic load towards the amplifier with $V_{in}$ shorted out. Because the control system uses feedback, this is not a simple passive impedance calculation, so it is convenient to first find the Norton velocity $u_n$, which is the acoustic velocity when the acoustic interaction impedance is zero

$$u_n = \frac{G N_{em} V_{in}}{(J + 1) \left( Z_m^E + Z_{aci} \right)}. \quad (10)$$

The Thevenin impedance $Z_{th}$ is equal to $F_{th}/u_n$, which is
\[
Z_{th} = \left( Z_m^E + Z_{act} \right) \left( \frac{J + 1}{J_e + G/G_{oc}} \right).
\] (11)

From Figure 3, \( u = (F_{th} - F_{aciN})/Z_{th} \), so using the definitions in equations 9 and 11, the output velocity is
\[
u = \frac{N_{em} \, G \, V_{in} - F_{aciN} \left( J_e + G / G_{oc} \right)}{\left( Z_m^E + Z_{act} \right) \left( J + 1 \right)}.
\] (12)

It is clear from equation 12 that the effects of acoustical interactions \( F_{aciN} \) are reduced by one plus the loop gain. Motional current velocity control is optimal when the estimate of the electrical branch is perfect, corresponding to \( Y_e = Y_e^* \) and \( J_e = 0 \). The feedback sensor is then equivalent to an ideal massless accelerometer, \( J = G P H \), and the motional current \( I_m \) for a single transducer without interactions asymptotically approaches \( 1/H \) as the loop gain becomes much larger than unity. Therefore ideal velocity control is approached if \( Y_e = Y_e^*, J \gg 1 \), and sufficient voltage can be generated by the amplifier and handled by the transducer.

**ARRAY EQUATIONS WITH VELOCITY CONTROL**

When multiple transducers are driven in an array, then the mutual radiation impedances affect the resulting output velocities. The array equations are a system of equations in matrix form that solve the steady state outputs of the array, given that the elements are all being driven at the same frequency. Because each term is frequency dependent, numerical analysis must be performed for each frequency to be analyzed. The input voltages required to achieve a desired output may also be obtained from the array equations. The solution for an open loop amplifier has been given by Sherman and Butler. The array equations with motional current velocity control may be obtained from equation 12, replacing \( F_{aciN} \) with equation 1, and replacing \( Z_{aciN} \) with equation 3. Casting the equations in matrix form and solving for the acoustic velocity then yields
\[
u = \left[ \left( Z_m^E + Z_{ii} \right) (J + 1) + \left( J_e + \frac{G}{G_{oc}} \right) Z_{mc} \right]^{-1} \, G N_{em} \, V_{in}.
\] (13)

The terms \( \nu \) and \( V_{in} \) are Nx1 matrixes representing the transducers’ acoustic velocities and amplifiers’ electrical input voltages respectively. \( Z_m^E, Z_{ii}, J, J_e, G, G_{oc}, \) and \( N_{em} \) are NxN matrices that are zero for each off-diagonal term, and the parameters for the \( i \)th amplifier and transducer are on the diagonal terms. \( I \) is the NxN identity matrix. The NxN matrix \( Z_{mc} \) is the mutual radiation impedance as computed in equation 4, and is zero for all diagonal terms. The output velocities are therefore calculated by inverting an impedance matrix and multiplying by the amplifier gain, electromechanical transformation coefficient, and the input signal matrix. The impedance matrix consists of the transducer’s mechanical and self-radiation impedance on the diagonal terms, and mutual radiation impedances on the off-diagonal terms. The loop gain multiplies only the diagonal terms, so the contribution due to \( Z_{mc} \) on the impedance matrix is effectively reduced by one plus the the loop gain, which agrees with the results from equation 12.

**MOTIONAL CURRENT LIMITATIONS**

Motional current velocity control has fundamental limitations. First, if the acoustic interactions cause some amplifiers to be “net absorbers” of power, a velocity feedback control system cannot prevent catastrophic failure. It will result in different array outputs than the same array driven by open loop amplifiers, but this does not inherently prevent the possibility of negative radiation resistances. Second, the accuracy of the output is limited by the accuracy of the blocked capacitance estimate, even when the loop gain is large. Third, the maximum stable loop gain is limited by the transducer parameters and the estimation accuracy. This section will quantify the maximum stable loop gain. The results are presented as absolute limits that may be approached, with the understanding that practical systems will have even lower loop gain in order to achieve better gain and phase margin stability.

Feedback loop instability occurs when the loop gain is -180° and the magnitude exceeds unity. Stability may be evaluated by plotting the loop gain on a Nyquist diagram and determining if the curve encircles the point \([-1, 0]\) or...
by plotting the loop gain on a bode plot and checking for positive gain and phase margins. The bode plot approach will be used here. Stability may be evaluated both with and without array interactions.

From Figure 3, the forward gain $G$ consists of signal gain $G_f$, amplifier gain $G_{amp}$, and the matching and tuning stage $G_t$, which is the transfer function from the amplifier output to the load and may include a step up transformer and series or parallel tuning. Velocity control is most effective when there is no series impedance between the amplifier and the transducer, so parallel tuning and transformer matching may be employed, but series tuning and transformers with large leakage inductances should be avoided. For large loop gain the feedback term $H$ controls the closed-loop output level, and is notionally broken up into a fixed scaling factor $H_{scale}$, and a frequency dependent portion $H_{comp}$. If $H_{comp}(\omega_m) = 1$ and the amplifier can deliver $K$ times more voltage than is required at mechanical resonance for ideal velocity control, then $H_{scale}$ should be

$$H_{scale} = \frac{KR_m}{G_f(\omega_m) \times G_{amp} \times \max V_{in}},$$  \hspace{1cm} (14)$$

where $\max V_{in}$ is the maximum input level to the amplifier, and $G_f(\omega_m)$ is the matching and tuning transfer function at mechanical resonance and with no interactions. The loop gain magnitude at mechanical resonance is then set by adjusting the forward signal gain $G_f$

$$|J(\omega_m)| = \frac{KG_f}{\max V_{in}},$$  \hspace{1cm} (15)$$

Using these definitions, the loop gain is

$$J = |J(\omega_m)| \times \frac{G_t H_{comp}}{G_f(\omega_m)} R_m (Y_m + Y_c - Y_c').$$  \hspace{1cm} (16)$$

The loop gain error term is

$$J_e = -|J(\omega_m)| \times \frac{G_t H_{comp}}{G_f(\omega_m)} \left( j \frac{\omega \alpha_b}{\omega_m M} + \frac{\alpha_c R_m}{R_{ch}} \right).$$  \hspace{1cm} (17)$$

The remainder of this section will assume that there is no series impedance between the amplifier and the transducer so $G_f = G_f(\omega_m)$, and the effect of the estimate error $\alpha_b$ is negligible. If $GH$ contains no poles or zeros, then at high frequencies the loop gain magnitude asymptotically approaches a slope of $+20\text{dB/decade}$ unless the blocked capacitance is perfectly estimated. Therefore when $|Y_c - Y_c'| \gg |Y_m|$, $J_e$ dominates the loop gain, which approaches

$$J \equiv -j |J(\omega_m)| \times H_{comp} \frac{\omega \alpha_b}{\omega_m M},$$  \hspace{1cm} (18)$$

where $M$ is a Figure of Merit defined as

$$M = \frac{Q m k^2}{1 - k^2} = \frac{1}{\omega_p R_mC_e}.$$  \hspace{1cm} (19)$$

The loop gain is illustrated in Figure 4 for various $\alpha_b$. 

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FIGURE 4. Loop gain with no feedback poles or zeros, with $k_{\text{eff}} = 0.56$, and $Q_m = 3.0$. Curves are for different values of $\alpha_b$.

At high frequencies, the phase response asymptotically approaches $-90^\circ$ if the capacitance is exactly estimated or overestimated, and asymptotically approaches $+90^\circ$ if the capacitance is underestimated. In either case, any fixed delay in the system will cause the phase to cross $-180^\circ$ and the high frequency gain will cause the system to be unstable. One way to ensure stability is to add a pole to $H_{\text{comp}}$ at $\omega_p$, and then scale $H_{\text{comp}}$ so it has unity magnitude at resonance

$$H_{\text{comp}} = \frac{\sqrt{1 + \omega_m^2 / \omega_p^2}}{1 + j\omega / \omega_p}.$$  \hspace{1cm} (20)

Using the $H_{\text{comp}}$ in equation 20, then for $\omega \gg \omega_p$ the loop gain at high frequencies asymptotically approaches

$$J = -\left| J(\omega_m) \right| \times \frac{\alpha_b}{M} \sqrt{\omega_p^2 / \omega_m^2 + 1}.$$  \hspace{1cm} (21)

For the system to be stable, the high frequency response must be less than unity. From equation 21, the maximum stable loop gain is therefore fundamentally limited by the transducer’s figure of merit, the accuracy of the blocked capacitance estimate, and the dominant pole frequency.
\[ |J(\omega_m)| < \frac{M}{\alpha_p \sqrt{\omega_p^2 / \omega_m^2 + 1}}. \]  

(22)

In practice, an additional pole in \( H_{\text{comp}} \) will be desired to roll off the gain at high frequency, to increase the gain margin, and to prevent higher frequency modes of vibration from causing the loop gain to increase past unity. Fully describing the tradeoffs involved with a 2\textsuperscript{nd} order pole is beyond the scope of this paper, but in general it will require lower loop gain to retain acceptable phase margin. In addition, if poles in \( H \) cause it to vary in-band, then it will directly affect the closed loop response, which is approximately \( 1/H \) for large loop gain. Lowering the dominant pole frequency may increase the maximum possible loop gain at the expense of the flatness of the closed loop response. Therefore, a system with good gain margin and phase margin, and relatively flat in-band response will have a maximum loop gain lower than the fundamental limit in equation 22.

**VELOCITY CONTROL WITH CLASS D AMPLIFICATION**

Class D amplifiers have extremely high efficiency (>95%) but require an LC output filter to remove the high frequency components resulting from the pulse width modulation. The LC filter consists of an inductor in series with the amplifier output, and a capacitor in parallel with the transducer. This filter introduces some tradeoffs when used with a velocity control system. First, the filter capacitor is in parallel with the transducer's blocked capacitance, which effectively reduces the transducer's figure of merit. For some applications, the LC filter may be designed with little or no \( C_{\text{LPF}} \), and instead rely on the transducer's capacitance \( C_e \) instead. Second, the loop gain is directly proportional to the transfer function \( G_t \), which now includes a 180\(^\circ\) phase shift and a large gain due to the undamped LC filter. The feedback loop will be unstable and will oscillate at the LC filter resonance if it is not compensated for.

One method of making the system stable is to add an inner feedback loop with current feedback. This inner loop synthesize a thevenin equivalent resistance which is in series with the LC filter, in order to artificially dampen the filter's high Q resonance. However, this synthesized impedance is detrimental to velocity control, so \( H_{\text{inner}} \) should have a frequency response that synthesizes this impedance around the LC filter resonance, but not at in-band operational frequencies. Figure 5 illustrates such a system, and also includes a parallel tuning inductor \( L_{\text{tune}} \), chosen so that \( L_{\text{tune}} \times (C_{\text{LPF}} + C_e) = M_m \times C_m \). A matching transformer may also be added between \( C_{\text{LPF}} \) and \( L_{\text{tune}} \), but this may result in additional high frequency resonances due to the transformer's leakage inductance. This may complicate the dampering and frequency requirements of the inner feedback loop, and may also require a minimum \( C_{\text{LPF}} \).

**FIGURE 5.** Motional current velocity control of a SDOF transducer with a Class D amplifier's LC output filter, and an inner feedback loop to synthetically damped the filter's high Q resonance. A parallel tuning \( L_{\text{tune}} \) inductor is also included.

When using an inner and outer feedback loop, stability of each of the two local loops must be evaluated in addition to the global feedback system stability. The system will be stable if and only if all three of the following are stable: first the inner feedback loop; second the outer feedback loop with the transfer function \( G_t \) now dampened by the inner loop’s thevenin equivalent output resistance; and third the global feedback loop including the gain and phase response of both feedback loops summed together.
CONCLUSION

One of the benefits of a velocity control system in an array of transducers is the ability to mitigate the acoustic interaction effects proportional to the loop gain, which should be maximized near mechanical resonance. This may be necessary for beam patterns with precise side lobe requirements. The motional current method has the advantage of not requiring installation of an accelerometer on the transducer, but it has the disadvantage of being highly susceptible to the estimation accuracy of the transducer’s blocked capacitance. The maximum loop gain is fundamentally limited by the accuracy of the blocked capacitance estimate and the transducer’s figure of merit, so transducers with a higher coupling coefficient generally perform better with this approach. As the transducer ages and the blocked capacitance changes, stability margins will be negatively affected if the system is not recalibrated.

Velocity control with Class D amplification is further complicated because of the amplifier's LC output filter, and may require an additional feedback loop to prevent the LC filter resonance from causing feedback instability. The filter capacitor effectively reduces the transducer's figure of merit, but may still be necessary if a matching transformer is used. The inner feedback loop must synthesize an output resistance across a frequency range including the LC filter's resonance frequency, but it should also be designed so it has a minimal effect at in-band frequencies. This may require that the LC filter resonance frequency is much higher than the highest operational frequency, which may limit the filter's ability to remove the high frequency pulse width modulation components.

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