2pEAb12. Back scattering attenuators (silencers)

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Back scattering silencers are a kind of "sonic crystals". The frequency range of "sonic crystals" modeling includes the whole audio range and ultrasound, up to phonon waves. Here we concentrate on applications in the audio domain. However, the present investigation is applicable at the higher frequencies as well, because of the high scalability of the system. The paper defines and analyses 2-D and 3-D back scattering silencers, made of arrays of rigid or soft obstacles (cylinders, spheres or prolate/oblate spheroids for example), in order to attenuate plane waves by multiple scattering along a wave guide. This effect is strong especially at certain band-gaps along the frequency domain. Each obstacle reflects secondary waves that are partially reflected. Thus, along each row of the array, the sound waves lose a certain amount of the energy that adds to the total amount of attenuation. Specifically, the paper analyses back scattering silencers built of meshes of either cylinders or prolate spheroids, where each obstacle is located symmetrically within a fluid cell and each cell is identical to the others.

Published by the Acoustical Society of America through the American Institute of Physics
INTRODUCTION

The paper defines and analyses 2-D and 3-D back scattering silencers, made of grids of rigid or soft obstacles (cylinders, spheres or prolate/oblate spheroids, for example), in order to attenuate plane waves by multiple scattering along a wave guide. Here we concentrate on applications in the audio domain. However, the present investigation is applicable at the higher frequencies as well, because of the high scalability of the system. For example, scattering by arrays of piles to block underground vibration waves in front of a building and attenuation of sound by using arrays of trees of graded heights, acoustic silencers at high temperature for machines, effects of cylindrical grids at nuclear plants, scattering by aerosols and many other applications.

Lamb (1898) analyzed the scattering of long acoustic waves by a grating of small circular cylinders of spacing 2h and diameter 2a, assuming kh<<1 and a/h<<1, with the wave number k = 2π/λ. λ is the wave length. (See Lamb, 1932, §307). Using a method similar to the matched asymptotic expansions (Datta and Sabina, 1986) he obtained a simple formula for the reflection coefficient factor. This method became popular, but it was observed by Twersky (1956, 1962) that the resulting reflection coefficient for normal incidence of waves on a grating of small circular cylinders is incorrect. Martin and Darymple (1988) modified Lamb's formulation, giving a correct answer. The domain kh<<1, which is Rayleigh scattering (Rayleigh, 1919) - see figure 1, allows for some analytical simplifications in the analysis of the silencer. See also Kleinman and Senior (1986).

![Figure 1: Zones of scattering](image)

The paper illustrates quantitatively the use of a back scattering silencer, based on an array of cylindrical obstacles and spheroids. The acoustic energy lost to a plane wave by a back scattered secondary wave suggests the development of a silencer based on transmission of plane waves through a grid of obstacles through coupled scattering. The results stimulate further research on the subject for different types of obstacles (rectangles and different surface properties such as elastic, sound absorbing, etc). Within the wave guide the walls of the wave-guide are acoustically hard (Neumann boundary conditions) and it is possible to use the method of image scattering. When the wave-guide includes lattices of equally separated relatively small obstacles (see figure 2), the back scattering of the plane wave, the "T matrix method" (TMM) can be used to calculate the back scattering sound attenuation caused by the grid.

In the case of scattering by multiple-bodies system the mutual scattering by the obstacles should be added to the original wave and the result is the "effective field".
AN ARRAY OF OBSTACLES - AN OVERALL VIEW

Plane wave propagation through an array of obstacles - "black box" approach

Say, there is a random distribution of obstacles of a uniform density in a two dimensional domain. The scheme for a line source is shown in figure 3a. Each obstacle has a circular cross section of the area: \( A = \pi a^2 \). If the initial intensity, in front of the array is \( I_0 \) due to a line source, and the number of obstacles per unit depth of the array is \( n \), then the intensity of sound as a function of location in the array is assumed to be:

\[
I = I_0 \exp(-\eta n A x),
\]

(1)

with \( \eta \) as the characteristic decay coefficient.

Example: Say, \( x = 1 \) m, \( A = 10 \) cm\(^2\), \( n = 1000 \), and \( \eta = 1 \); then \( n A x = 1 \), and \( I = 0.3 I_0 \).

FIGURE 3. A scheme for (a) plane wave propagation through obstacles of radius "a" and (b) cylindrical wave propagation obstacles of radius "a".
Cylindrical wave propagation through an array of obstacles-"black box" approach

A scheme for a point source is presented in figure 3b. It is assumed that the decay is proportional to the square of the distance from the source. Instead of \( \frac{dI}{dx} = -\eta I \), we write:

\[
\frac{d(r^2I)}{dr} = -\eta (r^2I).
\]

Hence:

\[
I = I_0 \frac{r_0^2}{r^2} \exp(-\eta n Ax)
\]

The ratio between the intensity due to the line source and the point source, having the same intensity at a prescribed distance \( r_0 \), becomes:

\[
\chi = \frac{I_{\text{line source}}}{I_{\text{point source}}} = \frac{r_0^2}{r^2}
\]

Multiple scattering of sound waves from a group of objects

The back scattering analysis of multiple scattering of sound waves from a group of circular cylinders has been developed by Lin and Raptis (1985) which results in the “effective wave”

MODELING THE BACK SCATTERING SILENCER

As a model of silencer, consider a wave guide with a square cross section with hard walls (Neumann’s boundary conditions).

Since all the scatterers are at the same situation concerning the wave propagation, we can describe imaginary walls located in the middle of the distance between two close lines of scatterers, satisfying thus the Neumann’s boundary conditions. This is due to the fact that in this case the gradient normal to the boundary, which is the sum of the effects of the scattered fields, is equal to zero.

A similar approach was used by Brekhovskikh (1980) for the field created by a source in a plane wave-guide.

When analyzing the balance between the backward and forward scattering energies, unfortunately we could not obtain an exact result. Thus, we have used an approximate approach, to make the solution easier. We considered only a propagation of a plane wave (the zero mode) and since we have an infinite lattice with a limiting period of less than half wave length, only a plane wave is obtained.

An Illustration

Given a plane sound wave encounters a grid of obstacles in a host fluid (say, air), as in figure 4, then by using the approximation presented here,

\[
P_{\text{trans}} = P_0 - \left( P_{\text{backward scattered}} - P_{\text{forward scattered}} \right)
\]

\( P_0 \) – the original plane wave power, \( P_{\text{trans}} \) – transmitted power, \( P_{\text{backward scattered}} \) – the power scattered forward, \( P_{\text{forward scattered}} \) – the power scattered backwards.

If the original wave loses \( \Delta L(1) = 12 \% \) of its power by each layer of the grid, then 100-12 = 88\% of the total power moves downstream, and the total power lost by \( n=100 \) layers of the grid becomes:

\[
\Delta L(n) = (1 - \Delta L(1))^n \Rightarrow (1 - 0.12)^{100} = 2.8 \times 10^{-6}
\]

\[
\Delta L_w(n) = 10 \log_{10} \left( 2.8 \times 10^{-6} \right) = 57 \text{ dB}
\]

Which is the noise attenuation by the grid.
FORMULATION FOLLOWING THE T-MATRIX METHOD FOR SPHEROIDS SILENCER

The T-Matrix

The T-matrix method is based on the theoretical acoustics principles of scattering as can be found in Morse and Ingard (1968), Pierce (1981) and Chew (1990) among others. Sound wave scattering in a certain medium occurs when it strikes a body of properties that differ from those of the host medium. Then some secondary part of the original wave is spread out in different directions than those of the original wave as scattered wave. This is the difference between the actual and the original wave power. Consequently, the original sound wave is attenuated. Each row of obstacles in Figure 2 attenuates the propagating plane wave by a certain amount of power, related to the original power that impinges it. The transition matrix or "T-matrix" can relate the scattered wave amplitude to that of the undisturbed wave. The T-matrix method was first introduced by Waterman (1965, 1969, and 1971) who first dealt with T-matrix for acoustics and later referred to electromagnetic scattering.

A relatively comprehensive review of T-matrix use for many years is given in Varadan, Lachtakia and Varadan, (1988) including 150 references, and more recently Mischenko, Travis and Mackowski (1996, 2010) added new information. Varadan and Varadan (1980) collected the papers of a conference that took place in 1979, where the most important theoretical and numerical aspects of the T-matrix approach appeared. After 1979, Waterman (2009) summarized the T-matrix method, adding improvements and modifications to the technique. Two main consequences of these reviews are significant. The first one is the diversity of uses of the T-matrix method, since, when changing the form of the scatterer, only the equation that describes this form has to be changed accordingly, but not the algorithm of the solution as a whole. The second consequence is the advantage of the T-matrix due the fact that its form for the problem of a single body can be used for the formation of solutions to other more advanced problems, such as scattering by many bodies, by layered media, at wave-guides, etc.

Theory

First, a brief explanation to scattering of a plane sound wave by a finite number of scatterers is given. The effective incident field, which impinges the jth obstacle, includes the contribution of waves scattered from all the obstacles but the jth one – see figure 5.

Hence:

\[ \Phi_j^{\text{eff}} (\vec{r} - \vec{d}_j) = \Phi_j^{\text{inc}} (\vec{r} - \vec{d}_j) + \sum_{j' \neq j}^{N} \Phi_j^{\text{scat}} (\vec{r} - \vec{d}_{j'}) = \sum_{n} \Psi_n (\vec{r} - \vec{d}_j) e_j^{n} \quad (5) \]
The coefficients of expansion of the "effective field" in the second equality can be related now by the T-matrix for the obstacle $j$, to the expansion coefficients of the field scattered from obstacle $j$, which means:

\[ f_n^{(j)} = \sum_{n'} T_{nn'}^{(j)} c_n^{(j)} = \sum_{n'} c_n^{(j)} T_{nn'}^{(j)}; \quad j = 1, \ldots, N \]  

(6)

It is necessary to translate the basis functions in the fields scattered from all the obstacles into a common set, with the center at a given obstacle. Translations for spherical basis functions are implemented by the orthogonal matrix transformations $\sigma$ as follows:

\[ \Psi_n(\mathbf{r} - \mathbf{d}_j) = \sum_{n'} \sigma_{nn'}(\mathbf{d}_j) \Psi_{n'}(\mathbf{r}) \left| \mathbf{d}_j \right| > |\mathbf{r}| \]  

(7)

The last condition must be carried out for convergence of the T-matrix method. By using the matrix transpose of coordinates following Lim and Hackman (1992), we have:

\[ \sum_{n} \Psi_n(\mathbf{r} - \mathbf{d}_j) c_n^{(j)} = \sum_{n} \Psi_n(\mathbf{r} - \mathbf{d}_j) \left\{ a_n + \sum_{j=1}^{\infty} \sum_{n''} \sigma_{nn''} \left[ - (\mathbf{d}_j - \mathbf{d}_j') T_{n''n'}^{(j)} c_{n''} \right] \right\} \]  

(8)

\[ \mathbf{d}_j \] constitutes the vectorial description of the distances from the origin of the coordinates system to the centers of the scatterers.

The coefficients $c_n^{(j)}$ are identical for all scatterers, independently of the number $j$ of the scatterer in issue in the case of an infinite lattice of scatterers and identical scatterers. In this case, the T-matrices are identical, too. Consequently, equation (8) can be presented in a more convenient form:

\[ \sum_{n} \Psi_n(\mathbf{r} - \mathbf{d}_j) c_n^{(j)} = \sum_{n} \Psi_n(\mathbf{r} - \mathbf{d}_j) \left\{ a_n(\mathbf{r}_j) + \sum_{n''} \sum_{j=1}^{\infty} \sigma_{nn''} \left[ - (\mathbf{d}_j - \mathbf{d}_j') T_{n''n'}^{(j)} c_{n''} \right] \right\} \]  

(9)

Using a matrix notation for the last equation we get:

\[ c = \left( I - T \sum_{j=1}^{\infty} \sigma \left[ (\mathbf{d}_j - \mathbf{d}_j') \right] \right) a \]  

(10)

The coefficients $c$ give us the full incident field, including the source and the effect of scattering by other obstacles. The scattered field at an obstacle is obtained by equation (6). The method for calculating the transposition matrix $\sigma$ for the lattice follows Lim and Hackman (1992). See also an extension of that formalism to the problem of multiple scattering by many bounded obstacles arbitrarily distributed in a plane stratified acoustic medium as presented in Lim (1992). For more details see Rosenhouse (2001).
Analysis

If the half wave length of a wave propagating along a silencer is larger than the lateral
dimension of the silencer, only the null mode can propagate a long distance along the duct. In this
case the problem is of scattering of a plane wave propagating in a direction perpendicular to the
lateral plane of the lattice. Using a close analogy, also for the scattered wave we assume propagation
of only the null mode. Thus, only a forward and backward scattering from the lattice are taken into
account.

One way to solve the forward and back scattering problem is to sum the scattered fields from all
the obstacles that exist in the lattice. However, the result of this method is a difficult problem of
summation for a two dimensional infinite lattice of sources, in addition to bad convergence. On the
other hand, since the knowledge of the complete picture of the resulting field is not necessary, and
only the energies that propagate forward and scattered back are of interest, a certain approximation
could be done. We consider the finite area of the lattice for simplicity as a circle, and consequently
find the field radiated from a circular aperture in an infinite rigid plate, with and without a lattice. See
figure 6.

![Figure 6](image)

**Figure 6.** A plane wave $\phi^{(p)}$ within a strip of width $b$, impinging laterally an array of obstacles (or
apertures) of radius "a" each.

We can take as a result for radiation from the aperture, the ratio of amplitude of the far field in
presence of a lattice to the amplitude in absence of the lattice. The field caused by the incident plane
wave, being radiated through the aperture, is estimated as:

$$\Phi = -i\Phi_0 \frac{kS \exp(ikR)}{2\pi R},$$

where $R$ is the distance to the observer, $S$ is the area of the circular aperture and $\Phi_0$ is the amplitude
of the incident wave. For $N$ scatterers under the same conditions we can define $N$ as follows:

$$N = \frac{S}{S_{cell}} \cdot$$

where $S_{cell}$ is the area of a cell in the lattice which equals the cross section of the silencer. The
scattered field radiated from each obstacle of the lattice is:

$$\Phi^S = \sum_n f_n \Psi_n(kR)$$

In the far field this result may be approximated, using the Hankel functions that are included in $\Psi_n
(kr)$:

$$\Phi^S = \frac{\exp(ikR)}{kR} \sum_n f_n h_n^\prime(kR) j_n'(\cos \theta) j_n$$
The scattered field due to N obstacles is obtained by multiplying Φ₅ by N and the propagation coefficient of the lattice will be:

$$\frac{\Phi_5}{\Phi_{\text{plane}}} = \frac{2\pi i}{kS_{\text{cell}}} \sum_n f_n h_n(kR) P_n(\cos \theta) \frac{\gamma_n}{k}$$

Finally, the back and forward scattering can be obtained approximately by multiplying the field scattered from a single obstacle by the coefficient \((2\pi i) / (kS_{\text{cell}})\).

**RESULTS:**

A good backscattering silencer has optimal backscattering and sound energy absorption. In addition, it should acquire an optimal propagation of air flow through it. For this last purpose, the internal scatterers must be smooth and strongly elongated in the direction of the flow. On the other hand, application of the T-matrix method demands that the distance between the scatterers in the lattice will be greater than the radius of the fictitious sphere that circumscribes each scatterer. This condition avoids the use of the T-matrix method for calculating scattering from a lattice where strongly elongated obstacles are located too close to each other, losing optimality. But, the method still allows for the investigation of the main relations involved with the problem. Figure 7 plots of the amplitudes of the back scattering for spheroids with the semi-axes \(a = 0.2\) and \(b = 0.1\) in a silencer with a periodic distance of 0.25 (figure 1) as a function of \(k\). Figure 8 plots of the amplitudes of the back scattering as a function of the periodic distance for spheroids with the semi-axes \(a = 0.2\) and \(b = 0.1\) and \(a=0.3\) and \(b=0.1\) in a silencer.

**FIGURE 7.** Plot of the back scattering amplitude as a function of \(k\)

**FIGURE 8.** Plot of the back scattering amplitude as a function of the periodic distance; \(k=4\)
CONCLUSION

Back scattering energy depends strongly on the dimension of the silencer, when the width of the silencer is taken as a period in an infinite lattice. Effective back scattering is obtained only if the side of the form chosen for scattered body is less than a certain value (here it is 0.3). Under these conditions, the back scattering power is more than 10% of the total. This amount is useful for some practical applications, say, if many layers of grid are applied.

When the wavelength of the wave that propagates through the silencer is of the same order of magnitude as the scatterer the shape has small effect and most of the scattered acoustic energy is defined by the volume of the scatterer and the shape of the scatterer is designed aerodynamically.

If the density of the scatterer is much larger than the density of the host medium, sound absorption over the surface of the scatterer does not have a significant effect on the back- and forward-scatterings from the lattice. However, absorption by walls has a very significant effect on scattering.

AKNOWLEDGMENT

Special thanks to Dr. Sergei Kviatkovskii for introducing me to the T-matrix analysis in 1994.

REFERENCES