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3aEA5. Calculation of characteristics of nonlinear normal waves in plates of lithium niobate for the designing of acousto-electronic devices.
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The research of anharmonic effects is essential for the design of nonlinear acousto-electronic devices. Such effects involve the generation of nonlinear second harmonics in propagation of normal electroelastic waves in crystal plates. Thereby the analytical and numerical technique of the analysis of small nonlinear anharmonic effects in distribution of normal electroelastic waves in the layer of a trigonal piezocrystal of lithium niobate with thin short-circuited electroinductive coverings of sides has been developed. The research is based on the model of physically and geometrically nonlinear electroelastic deformation with finite deformations and Gibbs's function that includes quadratic and cubic components on deformations and characteristics of intensity of quasistatic electric field. The analysis of nonlinear wave effects is build on the representation of characteristics of a normal electroelastic wave in the form of the sum of summands which are proportional to the powers of the small parameter. The analytical form has been received for the representations of functions of the elastic displacements, intensity, induction of quasistatic electric field in nonlinear second harmonics for the studied waves from the different modes of the dispersive spectrum. Quantitative estimates have been researched for the amplitude levels of second harmonics for normal electroelastic waves with variable frequencies.

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INTRODUCTION

Design and development of several types of acousto-electronic devices are based on knowledge bases on the properties of low-energy nonlinear electroelastic waves in piezocrystal bodies, in particular in piezocrystal plates. Properties of waves of interest to designers can be almost completely determined on a basis of the concepts of generation of nonlinear highest harmonics (nonlinear anharmonic perturbations) for normal linear electroelastic waves in the corresponding waveguides. Therefore, the development of theoretical numerical and analytical methods of the description of the second harmonics of normal waves in plates of different slices of piezocrystals remains an urgent task of ultrasonic nowadays.

This paper describes a technique to study the characteristics of the nonlinear second harmonics of coupled electroelastic waves in piezocrystal plates of monocrystals of class 3m trigonal system.

ANALYTICAL SOLUTION

Consider piezocrystal plate (layer) of thickness 2h. Its flat edges are rigidly fixed and covered with thin conducting shorted electrodes. In the system of dimensionless normalized coordinates Ox₁x₂x₃ the layer occupies the region

\[ V = \left\{ \begin{array}{l} -\infty < x_2, x_3 < \infty, \left| x_1 \right| \leq h \end{array} \right\}. \]

(1)

Considered model of geometrically and physically nonlinear electroelastic deformation is based on the tensor representation of Gibbs energy function in the form:

\[
G_2 = \frac{1}{2} c_{ijkl}^E \varepsilon_{ij} \varepsilon_{kl} + \frac{1}{6} c_{ijklmn}^E \varepsilon_{ij} \varepsilon_{kl} \varepsilon_{mn} - \frac{1}{2} e'_{ij} E_j E_j - \frac{1}{6} e''_{ijkl} E_i E_j E_k - e_{ijkl} \varepsilon_{jk} - \frac{1}{2} d_{ijkl} E_i E_j \varepsilon_{kl} - \frac{1}{2} f_{ijklmn} \varepsilon_{ij} \varepsilon_{kl} \varepsilon_{mn} \varepsilon_{n}\]

(2)

Relations for the tensor components of mechanical intensities and the vector component of the electric displacement (induction vector of the electric field) follow from (2), and have the form

\[
\sigma_{ij} = c_{ijkl}^E \varepsilon_{kl} - e_{ij} E_k + \frac{1}{2} c_{ijklmn}^E \varepsilon_{kl} \varepsilon_{mn} - \frac{1}{2} d_{ijkl} E_k E_l - f_{ijklmn} \varepsilon_{ij} \varepsilon_{kl} \varepsilon_{mn} \varepsilon_{n}\]

(3)

\[
D_i = e'_{ij} E_j + e_{ij} E_k + \frac{1}{2} e''_{ijkl} E_i E_j E_k + d_{ijkl} E_j \varepsilon_{kl} - f_{ijklmn} \varepsilon_{ij} \varepsilon_{kl} \varepsilon_{mn} \varepsilon_{n}\]

Representations (2), (3) contain constants of the second and the third orders for piezactive material. Constants e''_{ijkl} characterize the nonlinear optic and electrooptic effects. Constants d_{ijkl} characterize the elasticoptic and electrostrictive effects. Constants f_{ijklmn} describe electroacoustic effects (changes in the sound velocity in the applied electric field).

The dynamical equations that describe the propagation of nonlinear coupled normal electroelastic waves (NEW) in this plate and the boundary conditions on its flat edges look

\[
\rho \ddot{u}_i = c_{ijkl}^E \varepsilon_{kl,j} - e_{ij} E_k E_j + \frac{1}{2} c_{ijklmn}^E \varepsilon_{kl,j} \varepsilon_{mn} + \frac{1}{2} c_{ijklmn}^E \varepsilon_{kl} \varepsilon_{mn,j} - \frac{1}{2} d_{ijkl} E_k E_l \]

(4)
\[-\frac{1}{2} d_{kli} E_k E_{i,j} - f_{mij} E_{m,j} e_{kl} - f_{mijkl} E_{m} e_{kl,j},\]

\[e^E_{ijk} E_{j,k} + e^E_{i} E_{j,k} + \frac{1}{2} e^E_{ij} E_{j,k} + \frac{1}{2} e^E_{ijk} E_{j,k},\]

\[\delta \varphi = \varphi^{(1)} + \delta \varphi^{(n)} \]

Formulated problem investigation is based on the concept of the presentation of characteristics of nonlinear NEW as binomial expansion segments in powers of the dimensionless small parameter \( \delta \). This parameter is the ratio of the maximum amplitudes of the studied waves in the linear approximation to the half of the plate thickness \( \tilde{\delta} = u_\infty / h \). Thus, presentations \( \bar{\varphi} = \bar{\varphi}^{(1)} + \delta \bar{\varphi}^{(n)} \), \( \bar{\varphi} = \varphi^{(1)} + \delta \varphi^{(n)} \) are introduced for the vector of wave movements and for the potential of the quasistatic electric field. The first terms in these decompositions correspond to the vector of the wave elastic movements \( \bar{\varphi}^{(1)} \) and to the potential of quasistatic electric field \( \varphi^{(1)} \) for linear NEW. By small parameter method, the first linear harmonics of NEW are determined from the homogeneous spectral boundary problems

\[ \rho \bar{u}^{(1)} - c^E_{ijkl} \bar{u}_{k,j} - e^E_{ijkl} \bar{U}_{j} - u^{(1)} = 0, \quad c^E_{ijkl} \bar{u}_{k,j} + e^E_{ijkl} \bar{U}_{j} - \rho \bar{u}^{(1)} = 0, \]

\[ \left( \bar{u}^{(1)} \right)_{x_j = \pm h} = 0. \]

Functions \( \bar{u}^{(n)}(x), \varphi^{(n)}(x) \) characterize the nonlinear second harmonics (NSH) (small nonlinear anharmonic perturbations) for NEW. By small parameter method, they are determined by the solutions of inhomogeneous boundary problems of the form

\[ \rho \bar{u}^{(n)} - c^E_{ijkl} \bar{u}_{k,j} - e^E_{ijkl} \bar{U}_{j} = \left( c^E_{ijkl} e^E_{kl,j} - e^E_{k,j} E_{k} \right) - \frac{1}{2} e^E_{ijklmn} e_{mn,j} - \frac{1}{2} e^E_{ijklmn} e_{mn,j}, \]

\[ e^E_{lij} E_{j,k} + e^E_{ij} E_{j,k} + \frac{1}{2} e^E_{ij} E_{j,k} + \frac{1}{2} e^E_{ij} E_{j,k} = \]

\[ \left( \bar{u}^{(n)} \right)_{x_j = \pm h} = 0. \]

It is an assumption that the linear NEW of mode \( q \) of corresponding dispersion spectrum propagates in the plane of considered piezocrystal layer. The direction of propagation of NEW is arbitrary and defined by the vector \( \hat{n} = (n_2, n_3) \). Functions of the wave elastic movements and of the potential of quasistatic electric field for linear NEW are determined from the homogeneous spectral problem (6), (7) and have the structure

\[ \bar{u}^{(aq)} = \alpha^{(q)}_{ip} \exp \left( \lambda^{(q)}_p x_1 \right) \exp \left( -2i \left( \omega t - k^{(q)} \left[ n_1 x_1 + n_2 x_2 \right] \right) \right), \]
\[ \varphi^{(i,q)} = \beta^{(q)}_p \exp\left(\lambda^{(q)}_p x_1\right) \exp\left(-2i\left(\omega t - k^{(q)}(n_1 x_1 + n_2 x_2)\right)\right) \quad (i = 1,3; \ p = 1,8). \]

Here \( \omega \), \( k^{(q)} \) - are the angular frequency and wave number of the free linear NEW of mode \( q \) respectively; \( \lambda^{(q)}_p = \lambda_p (\omega, k^{(q)}) \) - are the roots of the characteristic polynomial equation of degree eight. The coefficients \( \alpha^{(q)}_p \), \( \beta^{(q)}_p \) in (10) are obtained in analytical form up to a constant amplitude factor.

A corresponding problem determining complex functions of elastic wave movements \( u^{(n,q)}_k \) and quasistatic electric field potential \( \varphi^{(n,q)} \) in NSH for NEW of mode \( q \) takes the form

\[ \rho u^{(n,q)}_{i,j} - c_{ijkl} u^{(n,q)}_{k,ij} - e_{ij} \varphi^{(n,q)} = g^{(q)}_{pm} \exp\left(\left(\lambda^{(q)}_p + \lambda^{(q)}_m\right) x_1\right) \exp\left(-2i\left(\omega t - k^{(q)}(n_1 x_1 + n_2 x_2)\right)\right), \quad (i = 1,3; \ m, p = 1,8). \]  

By methods of computer algebra the solution of inhomogeneous boundary problem (11) - (12) was obtained in an analytical form

\[ u^{(n,q)}_i = n^{(q)}_{pm} \exp\left(\left(\lambda^{(q)}_p + \lambda^{(q)}_m\right) x_1\right) \exp\left(-2i\left(\omega t - k^{(q)}(n_1 x_1 + n_2 x_2)\right)\right), \quad (13) \]

\[ \varphi^{(n,q)} = \eta^{(q)}_{4,pm} \exp\left(\left(\lambda^{(q)}_p + \lambda^{(q)}_m\right) x_1\right) \exp\left(-2i\left(\omega t - k^{(q)}(n_1 x_1 + n_2 x_2)\right)\right). \]

Complex analytic representations were obtained for all the coefficients in (13).

**NUMERICAL RESEARCH**

Built analytical solution allows analysing the basic regularities that are typical for the phenomenon of generation of NSH for different modes of linear NEW. In the research of nonlinear effects primary interest is the estimation of amplitude levels of the anharmonic nonlinear perturbations. In numerical studies it was used experimental data on the physicomechanical constants of the first and second orders for the piezoelectric crystal of lithium niobate [1 - 3].

Frequency dependencies presented in Fig. 1 describe values of the maximum normalized wave movements and maximum normalized indicator for the potential of quasistatic electric field by thickness layer

\[ U_i = \max_{x_1} \left| u^{(n,q)}_i \right| / \left( \alpha^{(q)}_{48} \right)^2, \quad \Gamma = 10^{-10} \max_{x_1} \left| \varphi^{(n,q)} \right| / \left( \alpha^{(q)}_{48} \right)^2 \quad (14) \]

in NSH for moving NEW from the second mode with non-zero frequency locking for the direction that forms a \( \pi / 4 \) angle with the positive direction \( O x_2 \) and \( O x_3 \).
FIGURE 1. Frequency distribution of values $U_i (i = 1, 3)$ and $\Gamma$ in the NSH for NEW of the second mode in a layer of lithium niobate.

Regularities on Fig. 1 given specific values of $\delta$ allow estimating the relative amplitudes of movements and energy potential in the NSH. A nature of these dependencies point to the effects of the resonant nonlinear second harmonics generation under specific conditions of synchronism.

Characteristic of evaluation of nonlinear effects is the detailed analysis of the frequency dependencies $U_j(\omega)$, shown on fig. 1. Note that most of the values of $U_j(\omega)$ lie within the range of $U_j \in (0.13, 57.75)$ in the case of propagation of linear NEW in the layer of lithium niobate in the frequency range $\omega \in (5825, 8000)$. Comparative analysis of the distributions $U_j(\omega)$ shows that wave movement $U_3(\omega)$ dominate in the NSH in the considered layer. In the frequency range $\omega \in (5825, 8000)$ the indicator $\Delta_{31} = U_3 / U_1$ that characterizes typology of NSH decreases and lies in the range $1.11 \leq \Delta_{31} \leq 5.15$. Indicator $\Delta_{32} = U_3 / U_2$ increases and has limits $1.49 \leq \Delta_{32} \leq 3.12$ in this interval $\omega$. Comparative analysis of values $U_1(\omega)$ and $U_2(\omega)$ shows that $U_2(\omega)$ dominate in the frequency range $\omega \in (5825, 6590)$. Indicator $\Delta_{21} = U_2 / U_1$ decreases and has limits $1 \leq \Delta_{21} \leq 3.44$ in that frequency range. There is an opposite character ratio $U_2 / U_1$ in the frequency range $\omega \in (6590, 8000)$ and $\Delta_{12} = U_1 / U_2$ changes in the range $1 \leq \Delta_{12} \leq 2.8$. 

CONCLUSION

With the use of computer algebra there was received an analytic form of representations of functions of the elastic displacement and induction of a quasistatic electric field in NSH for NEW from different modes of dispersion spectrum, propagating along an arbitrary direction in plane of the trigonal piezoelectric crystal of lithium niobate. There was provided a quantitative assessment of the levels of nonlinear anharmonic perturbation of NEW of the second mode of the linear dispersion spectrum.

REFERENCES