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3pEA2. Numerical investigation of the functionally graded materials by the interaction of the plate guided waves with discontinuities and cracks
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This work intends to give a better comprehension of the guided wave interactions with damage in a functionally graded material (FGM). For this purpose, a two dimensional finite element (FE) method is used to analyse the near field surrounding the damage. Then, the expansion of the solution into sums of guided modes enables the determination of the reflection and transmission coefficients of each existent mode. The determination of the dispersion curves is ensured by the way of the so called semi-analytical finite element (SAFE) method applied to the one dimensional inlet and outlet cross-sections. Results are obtained by solving the global system of the 2D hybrid FE-SAFE method. The latter has the benefit to study an arbitrary shape-like damage in an infinite structure having the same shape by translation in the propagation direction in a fast and efficient way. The work aims to predict the propagation and interactions of plate guided waves with discontinuities in a functionally graded material composed of ceramic and metal mixture. Different symmetrical and asymmetrical notches are studied and so for cracks. Results are obtained and discussed for a FGM and compared to those obtained for an isotropic material.

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INTRODUCTION

Functionally graded materials (FGMs) are known for their ability to separate or isolate two different mediums from each other. They are mainly used where large temperature gradients are observed and are used in many fields as for instance aerospace or semi-conductors industry. The FGMs are generally made of two isotropic materials however, they are microscopically heterogeneous.

The propagation of guided waves in FGM materials have been studied recently by Cao et al. [1]. They have used the power series to study the influence of the gradient coefficients. Yu et al. [2] have determined dispersion curves for a functionally graded viscoelastic plates by solving the Legendre orthogonal polynomial series. Gravenkamp et al. [3] have used however, a scaled boundary finite element method to predict the dispersion curves avoiding numerical problems and instabilities. Other methods can be found in the literature to overcome the determination of dispersion curves in such materials and will not be presented here. On the contrary, the interaction of guided waves with defects present in plates made of FGM still not studied nor presented, at the knowledge of authors. This motivates the present work to highlight the interaction phenomena and their application in nondestructive testing and evaluation.

This work aims to predict the interaction of guided waves with damage in a FGM plate. For this end, a hybrid numerical method involving the classical finite element method (FEM) and the semi-analytical finite element (called SAFE) technique is used (see Ref. [4] for the three dimensional study applied to cylinders).

The first section is devoted to the characterisation of FGMs varying from a metal-rich surface to a ceramic-rich surface. Hence, dispersion curves of phase and group velocities for a FGM made of chrome and ceramics are given. These curves are first validated by results found in the literature then used to study wave interactions.

The second section deals with the interactions of the fundamental guided waves with different kinds of damage (notches and cracks), different symmetries (symmetrical and asymmetrical) and different depths. Notches have been studied by authors in previous works [5, 6] for an isotropic material in transient regime. Obtained results are shown first for notches then cracks. comparisons and discussions are carried out in these two subsections. Finally, conclusions are given in the last section.

PROPAGATION OF GUIDED WAVES

The determination of the dispersion curves is ensured by using the SAFE technique. As known, this method reduces the analysis, in this study, of two dimensional waveguides to one dimension by using a spatial Fourier transform along the propagation direction (u(x,z) = u(z)e^{jkx}, where k is the wavenumber and x is the propagation direction).

In what follows, two isotropic materials are considered: chrome (Cr) and ceramics (Cer). The density (ρ), the Young’s modulus E, the Poisson’s coefficient ν, Lamé’s coefficients λ and μ and longitudinal cL and shear cT velocities for these materials are reported in table 1.

<table>
<thead>
<tr>
<th>Material</th>
<th>ρ (kg/m^3)</th>
<th>E (GPa)</th>
<th>ν</th>
<th>λ (GPa)</th>
<th>μ (GPa)</th>
<th>cL (m/s)</th>
<th>cT (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chrome</td>
<td>248.04</td>
<td>7190</td>
<td>0.21</td>
<td>74.2</td>
<td>102.50</td>
<td>6231.7</td>
<td>3775.6</td>
</tr>
<tr>
<td>Ceramics</td>
<td>299.86</td>
<td>3900</td>
<td>0.27</td>
<td>138.0</td>
<td>118.11</td>
<td>9795.4</td>
<td>5503.2</td>
</tr>
</tbody>
</table>

The FGM is obtained by a chrome-ceramics mixture. The characteristics of this mixture depends on the thickness of the plate and are obtained as follows:
\[ g^{FGM} = (g^{Ce} - g^{Cr}) \left( \frac{2z + h}{2h} \right)^{p_p} + g^{Cr}, \]

where \( g \) is a characteristic of the material which takes the values of \( \rho, E \) or \( \nu \) and \( p_p \) is a gradient coefficient. In this work, \( p_p = 1 \) for linear functions.

**FIGURE 1:** Dispersion curves of (a) phase and (b) group velocities vs. dimensionless wavenumber and dimensionless frequency, respectively of plates made of: chrome (---), ceramics (----) and FGM (--).

Figures 1a and b display dispersion curves of guided waves in a plate with different materials: chrome, ceramics and FGM. Figures 1a represents the phase velocity vs. the dimensionless wavenumber \( k^2h \) (where \( 2h \) is the plate thickness). These curves agree with those found in the literature [1]. Dispersion curves of the group velocity versus dimensionless frequency \( \Omega = \frac{\omega c_T}{2h} \), where \( \omega \) is the angular frequency). From this figure, the chosen lowest cut-off frequency is taken to be equal to 3. This choice permit to have only fundamental modes in order to make easier interpretations. In what follows, only the dimensionless frequency is used.

**DAMAGED PLATES**

The study of a damaged plate is ensured by using a hybrid FEM-SAFE method. The description of the method for a three dimensional waveguide can be found in reference [4]. Two different kinds of damage are considered: notches and cracks. In addition, two different symmetries are investigated: symmetric and asymmetric.

**FIGURE 2:** Geometries and meshing of the studied structures: symmetric (a) and asymmetric (b) notches and symmetric (c) and asymmetric (d) cracks.

Geometries and meshes are shown in figures 2a - d (the number of elements is devised by three for clarity). Different depths are investigated: 83.33%, 66.67%, 50.00%, 33.33% and 16.67% of the plate thickness. Figures 2a and b depict a symmetrical and asymmetrical notches, respectively. The width of the notch is fixed and equal to the plate thickness \( 2h \). Figures 2c and d
present a symmetrical and asymmetrical cracks, respectively. The width of these cracks, contrary to the schematic representation, is very small. The inlet section for the generation and the reflection of guided waves is situated on the left \( L \) and the outlet section for the transmission waves is on the right \( R \). The power reflection \( R \) and transmission \( T \) coefficients are given by [4]:

\[
R_{mn} = \frac{P_{\text{r}n}}{P_{Lm}},
\]

\[
T_{mn} = \frac{P_{\text{t}n}}{P_{Lm}},
\]

where \( m \) is the incident mode, \( n \) is the reflected or transmitted mode, \( P \) is the power flow and the superscripts \((+\) and \((-)\) designate the positive and negative mode directions, respectively. The obtained results are shown for notches and cracks in the following subsections.

**Notches**

![Graphs showing reflection and transmission coefficients](image)

**FIGURE 3:** Reflection (a and c) and transmission (b and d) coefficients when the \( A_0 \) (a and b) and \( S_0 \) (c and d) modes interact with symmetrical notches with different depths in a plate made of a chrome (---), ceramics (--·) and FGM (---).

The reflection and transmission coefficients when the \( A_0 \) and \( S_0 \) modes interact with symmetrical notches with different depths are depicted in figures 3a - d. In these figures, three materials: chrome, ceramics and FGM are compared. As expected, no mode conversions are observed for the isotropic materials (chrome and ceramics). However, insignificant mode conversions are revealed for the FGM but not presented here for the clarity of figures. These mode conversions are due to the material asymmetry.

When the \( A_0 \) mode is launched (Figs. 3a and b), reflection and transmission coefficients differ slightly when the isotropic materials are used. However, the difference with those obtained for
FGM are clearly identified, especially for some frequency ranges.

For the case of a launched $S_0$ mode, one can also observe that for isotropic materials, the behaviour of the power coefficients is the same. However, the FGM presents a significant difference in some frequency ranges especially when the dimensionless frequency tends to 3. Furthermore, it is shown that the power reflection coefficients (see Fig. 3c) for the chrome is greater than that of the ceramics below a certain frequency (about 2). Above this frequency, the phenomenon is inverted. The behaviour of the power coefficients for a FGM is the same as that for ceramics. In addition, the same behaviour can be observed for the transmission coefficients (Fig. 3d).

![Graphs](image-url)

**Figure 4:** Reflection (a and c) and transmission (b and d) coefficients when the $A_0$ (a and b) and $S_0$ (c and d) modes interact with asymmetrical notches with different depths in a plate made of a chrome ($\neg$) and FGM with: top ($\neg\neg$) and bottom ($\neg\cdot$) crack surface.

Power coefficients when the $A_0$ and $S_0$ modes interact with asymmetrical notches with different depths are displayed in figures 4a - d. As before, chrome, ceramics and FGM are compared. In the special case of FGM, two notch positions are carried-out: a top crack (ceramics face) and a bottom crack (chrome face). As expected, significant mode conversions are observed but not presented nor discussed here for conciseness. These mode conversions are mainly due to the damage asymmetry.

When the $A_0$ mode is launched (Figs. 3a and b), reflection and transmission coefficients differ slightly when the isotropic materials are used. For this reason, only results for the chrome are shown here. However, the difference with those obtained for FGM are clearly identified, especially for some frequency ranges. In addition, one can clearly differ between top and bottom crack positions. Hence, power coefficients for the top crack (ceramics face) differ outstandingly.

When the $S_0$ mode is launched (Figs. 3c and d), the same remarks as mentioned before can be established. Furthermore, one can observe that the difference between curves increases as
the frequency increases. From an NDT point of view, the best mode to characterise the studied symmetric and asymmetric notches in this frequency range is the $S_0$ mode if it can be easily generated experimentally.

**Cracks**

![Graphs showing reflection and transmission coefficients](image)

**FIGURE 5:** Reflection (a and c) and transmission (b and d) coefficients when the $A_0$ (a and b) and $S_0$ (c and d) modes interact with symmetrical cracks with different depths in a plate made of a chrome (−), ceramics (−·) and FGM (−−).

Figures 5a - d show a comparison between obtained power coefficients when the fundamental Lamb modes interact with symmetrical cracks of different depths. Also, no mode conversions are observed for both modes. The discrepancies between curves obtained for different materials increases as the frequency increases. However, the case of 16.67% of depth is insensitive to the interaction phenomena due to its small depth.

A special attention can be given to the case of 83.33% of depth when the $A_0$ mode is launched (Figs. 5a and b). Indeed, the behaviour of the power coefficients is very different from those obtained for the other depths. The reflection (transmission) coefficient decreases (increases) unlike other cases. In fact, this phenomena starts from the depth of 66.67%.

Interesting results are found when the $S_0$ mode is launched (Figs. 5c and d). Curves are not overlapped for all depths and almost all the studied frequency range. One can differentiate clearly between results obtained for different depths and different materials except for the two smallest depths i.e. 33.33% and 16.67%. This observation can be helpful to characterise such a damage in these conditions.

The last and more realistic studied damage are asymmetric cracks. Obtained reflection and transmission coefficients, when the $A_0$ and $S_0$ modes are launched, are depicted in figures 6a - d.
In figure 6a, reflection coefficients cannot be exploited in some frequency ranges due to overlapping curves of the different depth cases. Hence, the differentiation between different depths is a hard task, if not impossible. The same phenomenon can be shown for the transmission coefficients (Fig.6b). Overall, $A_0$ mode is not a good indicator to evaluate this kind of damage. The case of the converted $S_0$ mode will not be discussed in this work.

Contrary to $A_0$, the $S_0$ mode is a good candidate to differentiate between plates made of isotropic material and FGM and containing asymmetrical cracks. In addition, the top and bottom surface cracks in a FGM can be clearly distinguished (Fig. 6c and d). However, one case remains insensitive to the propagation of the $S_0$ mode (mode conversion is not discussed here).

It has been verified that the power balance is satisfied with an excellent precision for all results. The discussed cases and the given observations must be further investigated to highlight the interaction phenomena. However, first results show accuracy and stability of the hybrid numerical method to study such a problem and its application to the nondestructive testing and evaluation field.

**CONCLUSION**

The semi-analytic finite element technique is used to determine the dispersion curves of chrome, ceramics and a functionally graded material made of chrome and ceramics mixture. Results are compared successfully to those obtained in the literature. A numerical hybrid method is used to study the interaction of the fundamental Lamb modes with different kinds of dam-
Symmetrical and asymmetrical notches then cracks are investigated, respectively. Results show a good sensitivity of the fundamental Lamb modes to the symmetrical and asymmetrical notches and cracks. It is also shown that the best mode to characterise these damage is the $S_0$ mode. Notice that mode conversions are not taking into account in discussions. The interaction phenomena behaviour is almost the same for plates-made of isotropic materials especially for the fundamental symmetrical mode when it interacts with cracks. The interaction of the $A_0$ and $S_0$ modes with damage in a FGM show the ability to distinguish between top surface and bottom surface notches and cracks.

**References**


