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3pEA4. The effect of a middle layer on ultrasonic wave propagating in a three-layered structure
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In this paper the focus is on the effect of an elastic middle layer on the propagation of ultrasonic waves. Systematic parametric studies are conducted to quantify the effects of the middle layer upon the ultrasonic wave propagation, including its thickness and acoustic impedance. We treat this problem analytically and numerically. The three-layered structure is also used to investigate the influence of the imperfect interfaces between two outer layers and a middle layer on the ultrasonic wave propagation. The theoretical analysis considers successive reflections of waves radiated by the transmitting transducer. The output signal is a superposition of successive reflections. Our results demonstrate clearly that there is significant influence of the middle layer in our three-layered problem. Various aspects of our approach are discussed and numerical examples are used to illustrate the suitability of our approach. Some details about the numerical methods employed are also given. The results are presented and discussed.

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INTRODUCTION

A one-dimensional frequency-domain method is derived to model ultrasonic propagation in a tri-layered system composed of isotropic of a middle layer sandwiched between two similar layers. The treatment of three-layered structure is crucial because multi-layered structures occur in bounded plasmas, external helicopter fuel tank wall and elsewhere. The propagation of elastic waves in a multi-layered structure has been studied by a few authors. Ultrasound is used for material nondestructive testing and characterization mainly because of its safety and accuracy. However, reverberation can cause artifacts in ultrasonic images due to multiple reflections among different interfaces. These artifacts can interfere with the accuracy of measurement which may lead to errors in parameter estimation and thus misinterpretation of the results may occur. Two strategies to manage the reverberation problems have been proposed. The first strategy proposed is to detect the presence of reverberating echoes, in order to avoid errors in parameter estimation, namely, the bonding condition and evaluation. The second strategy which has been identified is the reduction of reverberation and a dual-frequency subtraction imaging.

Numerical simulations have been employed to show the effect of the middle layer thickness for slight variations of its magnitude. The potential advantage of numerical simulation tools over experimental approaches is the determination of the influence of each layer property independently. Studying the effect of thickness on the ultrasonic wave propagation in terms of reflection coefficient, transmission coefficients, spatial coordinate and relative intensity is of particular interest.

In the present study, a three-layered structure model is proposed for assessing the effect of the nature of inner layer on the ultrasonic wave propagation in a tri-layered structure.

ONE-DIMENSIONAL TRI-LAYER STRUCTURE MODEL

The approach employed to determine the effect of the inner layer is described below. For our purpose, one-dimensional model of ultrasonic wave propagation in a tri-layered structure is aimed at the investigation of the influence of an inner layer on ultrasonic wave propagating in this structure. Layers of homogeneous materials with perfect and imperfect flat parallel interfaces are assumed. In this section a case is identified in which the rubber-aluminium-rubber forms a tri-layered structure.

Consider the case where an ultrasonic wave is incident on a layer, generating both reflected and transmitted waves as shown in Figure 1. Figure 1 shows the model of an inner layer with thickness \(d\) sandwiched between two layers. Incident ultrasonic waves penetrate real contact areas and intermediate films at interfaces of the materials. The frequency of the ultrasonic wave affects the properties of reflection, transmission and relative intensity at the contact interfaces. This effect varies from frequency to frequency.

\[
\begin{array}{c|c|c|c}
1 & 2 & 3 \\
\hline
\text{Reflected wave} & \text{Reflected wave} & \text{Reflected wave} \\
\text{Incident wave} & \text{Transmitted wave} & \text{Transmitted wave} \\
\hline
\rho_1 & \rho_2 & \rho_3 \\
\rho_1 & \rho_2 & \rho_3 \\
k_1 & k_2 & k_3 \\
\hline
x = 0 & x = d \\
\end{array}
\]

**FIGURE 1.** Schematic representation of the three-layer, one-dimensional wave propagation of the materials.

The wave propagation in each layer \((i=1, 2, 3)\) is a function of the material density \(\rho_i\), speed \(c_i\), layer thickness \(d_i\) and input signal frequency \(f\).
Pressure Reflection and Transmission of Waves in a Tri-Layered System

In the simulation, a pressure source is positioned on the face of the first layer and the excitation signal is a Gaussian pulse with a center frequency of 6 MHz. At both interfaces between rubber layers and middle layer, the boundary conditions are taken into account. Due to the fact that the impedance of the middle layer is lower than that of rubber, we will call the middle layer a ‘soft layer’. The process in Figure 1 is described as follows.

A longitudinal wave impinges on interface a through layer 1, which causes reflected longitudinal wave and transmitted longitudinal wave. The letters a and b represent interfaces of the tri-layered structure. The wave that propagates in layer 2 causes both reflected longitudinal wave and transmitted longitudinal wave that propagates in layer 3. The wave reflected from interface b impinges on interface a and causes reflected longitudinal wave that propagates in layer 2 and transmitted longitudinal wave that propagates in layer 3. Thus, the wave field in layer 1 consists of a single and multiply reflected wave in layer 2. Parameters, $c_i, \rho_i, k_i, (i=1, 2, 3)$ are the longitudinal wave velocity, density and wavenumber respectively. The densities, speed and wave impedance of each layer are shown in Table 1.

### Table 1. Material properties of rubber and aluminium.

<table>
<thead>
<tr>
<th>Material</th>
<th>Longitudinal wave ($\times 10^3$ m/s)</th>
<th>Density (kg/m$^3 \times 10^3$)</th>
<th>$\eta$ (kgm/(m$^2$s)$\times 10^6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber</td>
<td>1.70</td>
<td>1.55</td>
<td>26.19</td>
</tr>
<tr>
<td>Aluminium</td>
<td>6.42</td>
<td>2.70</td>
<td>17.33</td>
</tr>
</tbody>
</table>

For the sake of simplicity, the attenuation was not considered in the following expressions for the velocities. Let the velocities of these waves in their directions of propagation be expressed in the form

$$c_{i1} = \frac{p_{i1}}{\eta_1} = \frac{p_{i1}}{\eta_1} \exp\left(j(\omega t - k_i x)\right)$$ (1)

$$c_{r1} = -\frac{p_{r1}}{\eta_1} = -\frac{p_{r1}}{\eta_1} \exp\left(j(\omega t + k_i x)\right)$$ (2)

$$c_{r2} = \frac{p_{r2}}{\eta_2} = \frac{A \exp\left(j(\omega t - k_2 x)\right)}{\eta_2}$$ (3)

$$c_{r2} = -\frac{p_{r2}}{\eta_2} = -\frac{B \exp\left(j(\omega t + k_2 x)\right)}{\eta_2}$$ (4)

$$c_t = \frac{p_t}{\eta_3} = \frac{p_t}{\eta_3} \exp\left(j(\omega t - k_3 x)\right)$$ (5)

where $p_i$ is the incident pressure wave in layer 1, $p_r$ is the reflected pressure wave in layer 1, $A$ and $B$ are the amplitudes of the pressure waves in layer 2 and $p_t$ is the pressure of the transmitted wave in layer 3. The wave impedances in the tri-layered structure are known to be $\eta_i = \rho_i c_i (i=1, 2, 3)$ and wave numbers $k_i = \omega / c_i$ are indexed the same way. For the layer reflection and transmission coefficients, the following equations for the reflection and transmission coefficients may be written.
\[
R = \left(1 - \frac{\eta_1}{\eta_3}\right) \cos(k_2d) + j\left(\frac{\eta_2}{\eta_3} - \frac{\eta_1}{\eta_2}\right) \sin(k_2d)
\]
\[
T = \frac{4}{2 + \left(\frac{\eta_1}{\eta_3} + \frac{\eta_2}{\eta_3}\right) \cos^2(k_2d) + \left(\frac{\eta_2^2}{\eta_1\eta_3} + \frac{\eta_1\eta_2}{\eta_3^2}\right) \sin^2(k_2d)}
\]

where \(k_2 = \omega / c_2 = 2\pi f / c_2\) and \(d\) is the thickness of the middle layer. The variation of the thickness of the middle layer is given by

\[
d = d + \delta
\]

where \(\delta\) is the change in thickness of the middle layer. The layer reflection and transmission coefficients are functions of frequency because they contain all the waves that bounce back and forth in the layer and emerge into adjacent layers.

**Spatial Coordinate**

The spatial coordinate \(x_i(t)\) is formulated as

\[
x_i(t) = c_{av} \frac{t_i}{2}
\]

where \(c_{av}\) is the averaged velocity in a three-layer structure and \(t_i = 2d_i / c_i\ (i=1, 2, 3)\) is the two-way travel time for the ultrasonic wave to propagate in the three layers. The averaged ultrasound velocity is calculated according to the ultrasound velocities, \(c_1\), \(c_2\) and \(c_3\) as follows

\[
c_{av} = \frac{2d}{\frac{2d_1}{c_1} + \frac{2d_2}{c_2} + \frac{2d_3}{c_3}} = \frac{dc_1c_2c_3}{d_1c_2c_3 + d_2c_1c_3 + d_3c_1c_3}
\]

where \(d = d_1 + d_2 + d_3\) is the total thickness of the tri-layer structure.

**The Relative Intensity**

The reflection intensity of the ultrasonic pressure is determined by using Equation 7. Thus, the relative ratio of a reflected ultrasonic wave is expressed in the form
where \( I \) represents the maximum intensity of an ultrasonic wave reflected at the interface, \( I_0 \) stands for the maximum reflection intensity for the total reflection of the ultrasonic wave at the interface and \( T \) is determined by using Equation 7.

**The imperfect interface between layers**

An imperfect interface concept between dissimilar layers has been introduced by Delsanto and Scalerandi and applied by Small to model degraded bonds or delaminations between dissimilar layers. The letter \( Q \) is used to represent a contact quality and is used for each interface between two layers. If the value of \( Q \) is equal to zero then there is no bond between the layers and when \( Q=1 \), the bond is perfect. According to Small, the reflection and transmission coefficients for the imperfect interface are given by

\[
R = \frac{\eta_1 - \eta_2 - i\omega \eta_1 \eta_2}{\eta_1 + \eta_2 - i\omega \eta_1 \eta_2} \left(1 - \frac{i\omega \eta_1 \eta_2}{K(\eta_1 + \eta_2)}\right)
\]

(12)

\[
R = \frac{2\eta_1}{\eta_1 + \eta_2 - i\omega \eta_1 \eta_2} \left(1 - \frac{i\omega \eta_1 \eta_2}{K(\eta_1 + \eta_2)}\right)
\]

(13)

where

\[
K = \frac{h\omega^2}{2} \left(\frac{\rho_1 \rho_2}{\rho_1 + \rho_2}\right) \left(\frac{Q}{Q - 1}\right)
\]

(14)

with \( \eta_i = \rho_i c_i \) (\( i = 1, 2, 3 \)) being the impedance, \( \omega = k / c \) representing the angular frequency and \( h \) is the distance between particles in the interface. When \( Q \to 0 \) and \( K \to 0 \), the reflection and coefficients of Equations 12 and 13 are

\[
R = 1
\]

(15)

and

\[
T = 0
\]

(16)

**NUMERICAL RESULTS AND DISCUSSION**

The model considered was a rubber-aluminium-rubber system of the middle layer which is soft because its impedance is less than that of rubber and is frequency dependent. Various values of thickness of the middle layer have been assumed in order to calculate the reflection coefficients, transmission coefficients, spatial coordinate and relative intensity. To examine the nature of the middle layer, we used MATLAB to calculate and plot the results of the reflection coefficients, transmission coefficients and relative intensity as a function of frequency ranging from zero to 6 MHz.
The effect of thickness on the reflection, transmission, spatial coordinate and relative intensity is described as follows. Figures 1, 2 and 3 show the influence of the middle layer’s thickness on the reflection, transmission, spatial coordinate and relative intensity.

**FIGURE 1.** (a) The effect of thickness by using the reflection method. (b) The effect of thickness by using the transmission method.

The middle layer has minimum reflection coefficients from 2 MHz to 3 MHz. From Figures 1 and 3, it is clear that for the wave propagation studies, the frequency dependence for the middle layer should be opted. The significance of thickness of the middle layer on the reflected coefficients, transmission coefficients, spatial coordinate and relative intensity can be clearly seen in Figures 1, 2 and 3.
The calculated averaged velocity for the rubber-aluminium-rubber structure is $c_{av} = \frac{2.25 \times 10^3 m/s}{s}$ for the inner layer thickness of 20 mm. With the lower thickness of the inner layer, Figure 3 shows a very interesting trend of the relative intensity. On increasing the frequency, the relative intensity increases steeply high and levels between 2 MHz and 3 MHz and then smoothly decreases to a minimum relative intensity at 5 MHz.

CONCLUSION

The presence of the elastic middle layer and its influence on the propagation of longitudinal waves in the rubber-aluminium-rubber system, have been investigated theoretically and numerically by proposing a tri-layered structure model. The reflection coefficients, transmission coefficients, spatial coordinate and relative intensity have been calculated for selected thickness values of the inner layer. The systematic parametric studies have been carried out to quantify the effects of the middle layer in terms of its thickness. According to our study we conclude that the thickness of the middle layer in a three-layered structure with slight variations affects significantly the propagation of the ultrasonic waves.

REFERENCES

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