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4pEAa12. Harmonic hydro-mechanical movement
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The current theoretical physics tools used to describe the acoustic phenomena that occur outside the ear, such as the specific impedance, the acoustic impedance and the mechanical impedance are not applicable to describe the cochlear mechanics. For this reason this study uses the hydro-mechanical impedance concept. The latter is only applicable to a harmonic hydro-mechanical systems, which consist of a rigid recipient, filled with liquid and two elastic windows that relate the system with a sound environment, considering that the distance between them should be much smaller than the sound wavelength. This system could be considered as the most primitive model inner ear to build. The movement of the contained fluid in this system has particular characteristics that differentiate it from the wave motion and from a simple mass-spring-damping vibration system. In order to demonstrate the existence of the harmonic hydro-mechanical movement, was modeled and built an equivalent harmonic electrical system, which results corresponded with the ones from the theoretical mathematical model.

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1. INTRODUCTION

The motion of the cochlear liquid is responsible for the motion of the cilia, which governs the quantum of chemical mediator that each ciliated cell releases into the synopsis with the bipolar neuron of the Corti’s ganglion [1].

In this paper, cochlear physiology will be addressed from the particular perspective offered by the hard sciences, based on the specific laws of acoustical physics, in addition to those that govern fluid dynamics. The difficulty that the study of the inner ear derived from direct observation presents, lies in the fact that the osseous case is not just a continent for the fluid and the protector of the membranous structures immersed in it, but a determinant factor of the responses that it presents to the different stimuli; so any solution of continuity produced both in the bone as in its windows, alters in greater or lesser extent the quality of responses. This is why addressing the cochlear physiology without the substrate of a suitable framework, through which the results can be interpreted, becomes insufficient due to the distortion of what makes possible the observation. The sound wave that affects the external side of the eardrum, is responsible for the vibration of the footplate in the oval window, which transfers to the cochlear content an harmonic movement that cannot be defined strictly as a vibration, nor as wave motion [2].

The harmonic hydro-mechanical motion is a specific category of harmonic movement, as it shares common properties with the vibratory motion of the mass-spring-damping system and with the undulatory motion of the propagated wave in an isotropic, homogeneous and infinite medium. But the harmonic hydro-mechanical motion also has characteristics of its own that differentiate it from the previous two. That is why the impedances that are useful in the study of the vibration and wave propagation, are not suitable for the study of the harmonic hydro-mechanical motion [2].

Both the mechanical \( Z_m \) as the acoustic impedance \( Z_a \), and the specific impedance \( Z_s \), are all tools of the acoustic physics, and have no use in the analysis of the motion of the cochlear content. Through the hydro-mechanical impedance \( Z_h \) it is possible to perform the Eulerian analysis of such motion [2].

2. METHODS

a. Theoretical Model

With the purpose of simplifying the analysis of the harmonic hydro-mechanical motion, the simple harmonic hydro-mechanical system is defined as a rectangular recipient, with rigid walls, two circular windows of equal areas placed in opposite walls from each other and closed by two different membranes with equal elasticity and resistance, only if it meets the five restrictions listed below [2]:

- The distance between windows must be much smaller than the stimulus’s wavelength. \( l \ll \lambda \)
- The square root of the window’s area must be much smaller than the stimulus’s wavelength. \( \sqrt{A} \ll \lambda \)
- The cube root of the 95% of the liquid in motion must be much smaller than the stimulus’s wavelength. \( \sqrt[3]{0.95V} \ll \lambda \)
- The windows area must be smaller than the area of the walls in which they are carved.
- The displacement or volumetric elongation must be small.

It is essential to describe the liquid’s motion in this system, in order to understand how the cochlea works. But this description is not enough to decipher the totality of the complex cochlear functionality.

In this system, the inertia is provided by the fluid, and the elasticity and resistance are provided by the windows [2]. This system can be modeled like an electric R-L-C system as shown in Figure 1.

When a prolonged harmonic stimulus, with a specific frequency and intensity is applied to the oval window of an harmonic hydro-mechanical system, it transfers to the liquid content between both windows, a stable heterogeneous harmonic pressure field. The round window provides the system with the necessary elasticity to make the liquid’s inertial motion possible. This window replaces the compression efforts, which are usually very high in liquids, by shear efforts which are negligible. For these reason, although it is valid to replace the R-C blocks as just one series of equivalent block in the electric analogy, such replacement is not possible in the hydro-mechanical model because it would lose the quality provided by the round window. Therefore the round window determines the quality of response as the pressure field that the sound stimuli transfers from the oval window to the liquid content [2].
As mentioned in [2] the pressure applied to the oval window is not responsible for the liquid's inertial motion; the actual responsible for this motion is the pressure between windows, because if pressure and frequency are applied in the same phase to both windows, the hydro-mechanical flow disappears, but if it is applied in counterphase, the flow increases.

In fluid mechanics, a stream line is a line everywhere tangent to the velocity vector at a given instant [3]. Thus, a stream tube is the limited space between streamlines that passes through a surface contour [3]. This surface is
affected by the same pressure, so it is called “equal-pressure surface”. The pressure’s field determines liquid’s harmonic inertial motion in which is specified the ninety-five per cent of flow as a “virtual harmonic streamtube” [2]. This “virtual harmonic streamtube” is sufficiently separated from the case, so it avoids adhesion forces which alter the fluid motion. In this system the viscosity is considered negligible due to the molecular forces adhesion are minimum [7].

![Diagram](image)

**FIGURE 3.** Two-dimensional graphic representation of: (a) a three-dimensional “virtual harmonic streamtube” through which occurs 95% of the flows and (b) its electric analogy [2].

As defined in [2], the streamtube on Figure 4 shows the same quantity of streamlines as “equal-pressure lines”. The amount of flow between consecutive streamlines is called “flow elements” (\(\Delta \phi\)) and is the same in every case. These streamlines are perpendicular to the “equal-pressure surfaces”. “Elemental streamtubes” are shaped between consecutive streamlines because through each of them flows a “flow element”. There is no flow-loss in the total streamtube neither in each “elemental streamtube” in spite of not being confined by solid walls but by fluid surfaces [3], this is possible because all along of each “equal-pressure line” there is no pressure gradient (\(\nabla p\)). The gradients that are being transferred to an harmonic hydro-mechanical system are perpendicular to the equal-pressure surfaces, therefore tangent to the streamlines. While the pressure drop is the same for each streamline, the average minus pressure gradient (\(-\nabla p\)) is not. This is because their lengths are not equal. Also, each streamline segment between successive equal-pressure surfaces are subjected to equal pressure drop, but as they have different lengths (\(-\nabla p\)), the transferred gradient on each of them is different. The velocity module of the particle does not depend on the pressure to which it is subjected; it depends on the minus gradient (\(-\nabla p\)) that affects it, which not only gives the module to the acoustic particle’s velocity, but also gives direction to the particle [2].

The hydro-mechanical impedance (\(Z_h\)) is defined as the quotient between the pressure drop applied to the windows (\(\Delta p\)) and the flow generated between them (\(\phi\)). This definition enables the study of the effects of vibrations or waves applied to the oval window of an harmonic hydro-mechanical system [2].

\[
Z_h = \frac{\Delta p}{\phi} \tag{1}
\]
FIGURE 4. Two-dimensional graphic representation of: (a) a three-dimensional harmonic flow divided in six “flow elements” and (b) its electric analogy [2].

Analogously to the electrical impedance of a RLC circuit, where:

\[ Z_e = \frac{v}{i} \]  \hspace{1cm} (2)

The general expression of \( Z_h \) derives from applying the action and reaction principle to the harmonic hydromechanical system. According to this principle, the pressure drop applied to the windows and the \( \Delta p \) that the different elements of the system offer are in equilibrium [2].

\[ \Delta p = R_h \phi h + m_h a_v + k_h X_v \]  \hspace{1cm} (3)

Where: \( R_h \) = Hydromechanical resistance; \( m_h \) = Hydromechanical inertance; \( k_h \) = Hydromechanical elasticity; \( a_v \) = Volumetric acceleration; \( X_v \) = Volumetric elongation.

In any harmonic motion the first derivative of the elongation is the flow or volumetric velocity and the second derivative is the volumetric acceleration [2], [4].

\[ \phi h = j \omega X_v \]  \hspace{1cm} (4)

\[ a_v = j \omega \phi h \]  \hspace{1cm} (5)

The result of replacing (4) and (5) in (3) is.

\[ \Delta p = R_h \phi h + m_h j \omega \phi h + \frac{k_h \phi h}{j \omega} \]  \hspace{1cm} (6)
The general expression of $Z_h$ is obtained from the division between expression (6) and the hydro-mechanical flow.

$$Z_h = \frac{N}{\phi_h} = Rh + j(mh,\omega) \frac{k_h}{\omega}$$  \hspace{1cm} (7)

[2]

The stimulus of any hydro-mechanical system is the pressure drop applied to its windows, which does not present changes of density in the liquid. As described in [2], the harmonic flow ($\phi_h$) established between both windows can be defined as the volume of moving fluid through each of the explicit equal-pressure surfaces. This motion is considered as an inertial type of motion because the divergence of the acoustic particle’s velocities that defines the hydro-mechanical flow is negligible. As previously stated, the inertance in this system is provided by the fluid, and is called hydro-mechanical inertance ($mh$), which cannot be weighted through the mass, because it is a density with geometry. Density is the common element between mass and inertance. The difference is that the mass is the result of the multiplication between density and mass ($m = \delta V$) or ($m = \delta l.A$), while the inertance is the result of the multiplication between density and the division between length and area ($mh = \delta \frac{l}{A}$) [8].

- Thence, it is possible to signalize three physical parameters about the liquid of an harmonic hydro-mechanical system.
- Real mass is the result of the multiplication between density and recipient volume.
- Apparent mass is the result of the multiplication between density and volume of the liquid that acquires inertial motion.
- Hydro-mechanical inertance is the result of the multiplication between density and the division between averages length and area of moving liquid.

Then, the harmonic streamtube represented on Figure (3), through which ninety-five per cent of the flow passes, can be considered as an hydro-mechanical inertance [2].

In solid dynamics, force is related to the acceleration through the mass ($m = \frac{F}{a}$), while in fluid dynamics, the applied force to a particular volume is called minus pressure gradient ($-\nabla p$), and relates to the acceleration through the density ($\delta = \frac{-\nabla p}{a}$) [8].

On figure (4), shaped geometries between streamlines are “parallel elements” of hydro-mechanical inertance as shown in the hatched area on figure (5) [2].

$$\Delta mh = \frac{\delta I_p}{A_p}$$  \hspace{1cm} (8)

[5]

When: $I_p =$Average large; $A_p =$Average area.

Each “flow element” elapses through each “parallel element” of hydro-mechanical inertance. Each “parallel element” of hydro-mechanical inertance is identified by a sub-index number showing the “flow element” to which it belongs [2]. Then:

$$\frac{1}{mh} = \frac{1}{mh_1} + \frac{1}{mh_2} + \frac{1}{mh_3} + \frac{1}{mh_4} + \frac{1}{mh_5} + \frac{1}{mh_6}$$  \hspace{1cm} (9)

[2]
On the figure (2), the geometry between two consecutive equal-pressure surfaces constitutes a “series element of hydro-mechanical inertance”, as shown in grey color on figure (5), which is identified with the superscripts of equal-pressure surfaces that form it [2].

\[ mh = \Delta nh^{a-a} + \Delta nh^{a-b} + \Delta nh^{b-c} + \Delta nh^{c-d} + \Delta nh^{d-e} + \Delta nh^{e-f} \]  \hspace{1cm} (10)

The totality of the hydro-mechanical flow elapses through each element of hydro-mechanical inerance. “Elements of net inertance” are formed between the consecutive streamlines and equal-pressure lines. The corresponding value of each element of net inertance is equal to the total value of the inertance.

**FIGURE 5.** Two-dimensional graphic representation of the net inertances of (a) a simple harmonic hydro-mechanical system and (b) its electric analogy [2].

Each “net element of hydro-mechanic inertance” can be identified by sub-indexes numbers corresponding to the “flow element” and supra-indexes corresponding to the equi-pressure surfaces that conform it. The “net inertance” (colored in Figure (5)) is identifiable in the:

\[ \Delta nh_{b-c}^{b-c} = \frac{\delta l_3}{A_{b-c}} \]  \hspace{1cm} (11)

The quotient between the pressure drop that the stimulus transfers to the hydro-mechanical system’s liquid and the volumetric acceleration that it acquires, results in the hydro-mechanical inertance [2].

\[ mh = \frac{\Delta p_{mh}}{a_v} \]  \hspace{1cm} (12)

As stated in [2], the pressure drop that the stimulus transfers to the hydro-mechanics system’s liquid is, in it is totality of the reactive type, so it is in phase with the volumetric acceleration. The pressure drop transfered to the hydro-mechanics system’s liquid depends on the stimulus’s \( \Delta p \) and on the system’s quality factor \( Q \). This means that the \( \Delta p \) module transferred to a system’s liquid can be larger than the stimulus’s \( \Delta p \). For example, if a
system’s $Q$ is equal to one, in resonance the $(\Delta p)$ transferred to the liquid is equal to the stimulus’s $(\Delta p)$; but if a system’s $Q$ is equal to ten, the $(\Delta p)$ transferred to the liquid would be ten times larger than the stimulus’s $(\Delta p)$.

$$P_r = \Delta p_{mh} \cdot \Delta \phi h$$

(13)

Where: $P_r =$ reactive power that is transferred to each net element of hydro-mechanical inertance.

$\Delta p_{mh} =$ Pressure drop applied to each net element of hydro-mechanical inertance.

$\Delta \phi h =$ Flow element in each net element of hydro-mechanical inertance.

The total power provided to the fluid is equal to the power sum provided to each one of the net elements of inertance. [2]

b. Experimental model

With the purpose of proving the existence of a greater pressure drop applied to the fluid than the stimulus pressure drop, an electric R-L-C filter analog to an harmonic hydro-mechanical system with a $Q$ factor of 10 has been simulated. This filter, as shown on Figure (5), is made of 36 net inductances arranged in 6 “parallel elements”, 6 “series elements”, two capacitors and two resistances.

FIGURE 6. Capture of the simulation using software NI Multisim 10.1

3. RESULTS AND DISCUSSION

a. Analysis and discussion

In the simulation:

- The voltage drop applied to the total inductance (100 V in the XMM1 multimeter) is ten times larger than the voltage drop applied to the circuit (10 V). This is due to the quality factor of this particular system. The current is the same in both cases.
In each “elements of series inductance” runs the same current but the voltage drop is the sixth part of the total inductances (16.16 V in XMM1 multimeter).

In each “element of parallel inductance” the current is the sixth part of the total current (166.66 A in the XMM3 multimeter). The voltage drop in each of them is the same as the total inductance (100 V in XMM1 multimeter).

The current that runs by each “elements of net inductance” is equal to the sixth part of the total current (166,66 A in the XMM3 multimeter), and the voltage drop in each one of them is equal to the sixth part of the voltage drop applied to the total inductance (16,16 V in the XMM4 multimeter).

In the hydro-mechanical analogy:

- The pressure drop applied to the total inertance is ten times larger than the pressure drop applied to the model. The flow is the same in both cases.
- In each “elements of series inertance” runs the same flow but the pressure drop is the sixth part of the total inertance.
- In each “element of parallel inertance” the flow is the sixth part of the total flow. The pressure drop in each of them is the same that the total inertance.
- The flow that runs by each “element of net inertance” is equal to the sixth part of the total flow, and the pressure drop in each one of them is equal to the sixth part of the pressure drop applied to the total inertance.

b. Future direction

This work represents the first link between the theoretical demonstration and the existence of the harmonic hydro-mechanical movement inside the cochlea. The complete understanding of this movement furnishes a tool that enables to elucidate different kinds of pathologies that even today remain without a clear solution.

The ultimate aim is to understand the processes which are held by the different systems throughout the cochlear processing, from the vibrating signal emitted by the footplate to the nerve fibers arranged in the apex.

As a continuation of the current study, making the experiment in a real harmonic hydro-mechanical system is proposed. In this way, the geometric patterns of the harmonic hydro-mechanical motion, its velocities and pressure fields would be explained. This experiment will also allow the sketching of the minus pressure gradient map which affects each fluid particle.

Once the harmonic hydro-mechanical system and the movement of each particle of the fluid is fully understood, different kinds of variables impedances in a tonotopic distribution should be introduced thus recreating the frequency selectivity phenomena on the healthy basilar membrane.

4. CONCLUSIONS

Considering the ear as a particular type of harmonic hydro-mechanical system, the laws that apply to the liquid’s motion of a simple harmonic hydro-mechanical system are the same as in a labyrinthine content. Therefore, when the cochlear fluid responds to a stimulus with an inertial motion, is possible to use the hydro-mechanical impedance as a theoretical physics tool, in order to study it. Then, the concept of hydro-mechanical net inertance, constitutes a valid element in the study of the motion of the cochlear content.

5. REFERENCES