ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Musical Acoustics
Session 1aMU: String Instrument Measurements

1aMU1. Eigenvalue shapes compared to forced oscillation patterns of guitars
Malte Muenster*, Rolf Bader and Jan Richter

*Corresponding author's address: Institute of Systematic Musicology, Hamburg University, Neue Rabenstr. 13, Hamburg, 20354, Hamburg, Germany, m.muenster@arcor.de

32 guitars were measured geometrically and acoustically. The geometries of the top plate with its bracing as well as its thickness, the back plate with all bracing, the ribs, and rims are transferred to a CAD model. The top plate and the back plate of these guitars are measured using a 121-microphone array, back-propagating the sound field onto the top and back plates. Therefore, the guitars are once driven by impulses at the guitar bridge, once by plucking all notes on all strings up to the 12th fret to reconstruct the forced-oscillation patterns. Large differences are found with respect to the basic modes between the different guitars in terms of frequency and shape of their eigenmodes. Comparing the measured and calculated eigenvalues with the forced-oscillation modes driven by the strings, it appears that the eigenmode shapes often differ from the forced-oscillation patterns considerably.

Published by the Acoustical Society of America through the American Institute of Physics
Background

For our investigations on how construction influences the sonic behaviour of Classical guitars we developed certain methods to measure a multitude of acoustical parameters specific to them. Our sonic survey includes impulse tests and sound pressure field recordings of every single note in an anechoic environment to evaluate various radiation patterns. To interpret these and to define discrepancies in construction, photometric and manual measurement data of the individual geometries of the in- and outside of the examined guitar bodies were taken into account, determining their geometrical distinguishing marks. Likewise wood specific material data like sonic speed and Youngs-modulus of top, rib and bottom of each guitar were measured.

Our project’s target is to frame an answer on all constructional questions which influence the sound of Classical guitars, to show the whole extent of interaction between the measured parameters and to determine which one to change to achieve any aural imagination of guitar makers and players. A computer program is to be written to provide our conclusions to all professional instrument makers. It will be able to compute sounds out of geometry changes and geometries out of sound spectra changes. Not least this represents an inverse problem which is to be solved.

Method

The sample mostly consists of professional, concert grade, master build, illustrious branded guitars such as Ramirez, Contreras, Hanika, Wichmann, Herb, Imai and many more. Comparative some few industrial and fancy experimental models were part of our examination as well. Our goal is to frame an answer on all constructional questions which influence the sound of Classical guitars and to show the whole extent of interaction between the measured parameters and to determine which one to change to achieve any aural imagination of guitar builders and players. Eventually a computer program is to be written to provide our conclusions to all professional instrument makers. The tests are described in detail as follows.

At first sound pressure fields of all notes from open strings to 12th fret of all guitars were recorded with our acoustic camera. It was build at the Institute of Systematic Musicology at Hamburg University. The acoustic camera is able to record sound fields with 128 linear electret condenser microphones at 16Bit/48kHz simultaneously. The temporal resolution provides enough headroom to cover the whole hearing range up to 20kHz. The need to build a special array is owed by complex sounds and radiation patterns produced by musical instruments. There is a need to find a solution to cover all aspects of vibrating and radiating behaviour. To study radiation patterns unprocessed pressure field data has been used widely. But further and by far more fascinating scientific findings can be made by back propagation of recorded sound field data to the source. For this purpose a mathematical Method is needed which reconstructs the sound pressures back to the surface of musical instruments. To represent complex radiation patterns within a musical range, respectively very low and very high frequencies, special methods had to be tested and lastly developed. Here the minimum energy method or alpha-method comes into its own (Bader (2010)).

\[ \Gamma = r(1 + \alpha(1 + \beta)) \]  

The pressure measured at several points represented by the microphone array is presumed as a superposition of monopole radiating pressures. Hereby a radiation matrix can be generated,
containing phase relations and amplitude drops between radiating points and microphones. The representation of amplitude drops includes the direction angle $\beta$ of the radiation and the decisive parameter $\alpha$; which represents the directivity of radiation. If it tends to or equals zero a monopole source is present. With an increase of $\alpha$ it is getting more and more narrow. So $\alpha$ represents an overall directivity value for radiation. The Solutions are calculated by defining matrices and vectors. A linear equation solver is now able to reconstruct the specific pressure values on the radiating surface itself out of the measured pressure values. for this purpose the value of $\alpha$ has to be assumed. The assumption has to be arbitrary, as we do not know the directivity $\Gamma$. But it appears that the correct value of alpha is the one which minimizes the reconstruction energy. The energy is proportional to the squared reconstruction pressures. Trying several $\alpha$ values and performing reconstructions a curve $\alpha$ versus reconstruction energy can be plotted were its minimum where the correct reconstruction is achieved indeed.

This method can be understood intuitively which makes it very stable and reasonable. If $\alpha$ is large, it is assumed that each radiating point is only radiating into the microphone opposite to it. Decreasing $\alpha$ leads to sharpening of reconstruction that can be followed visually very easy. If $\alpha$ is calculated correctly the reconstruction is perfect and the minimum energy is reached. If $\alpha$ is decreased even further, the influence of neighbouring radiation points onto the recording of one microphone would be overestimated. We obtain an estimated overall radiation directivity by this value. Indeed a very interesting parameter for musical instrument research (Bader (2010)).

For geometrical measurements pictures of all parts were taken. A plan view photo was used to calculate contour data. To put attention to detail the geometry was additionally picked up manually to get exact positions of bridge, neck, soundhole and all internal bracings, glued to bottom and top with a negligible tolerance. Moreover Guitar tops are all more or less abraded by the guitar maker to emphasise a preferred frequency range or to open up the sound colour. The way they do it is also a part of our examination. We putted a grid on the guitar top and measured the variation of thickness over the whole top plate by a manual magnet field thickness gauge with a tolerance of +/- 0.1mm. These are the main geometrical data showing most obvious visible distinctions between guitars. With these data geometries were generated in C#.

In addition sonic speeds and therefore Younsg-modulus of top, rib and bottom of each guitar were measured, taking wood identification into account. The sonic speed is detected by a rubber mallet strike recorded by a piezo contact microphone into a high resolution, computer based oscilloscope. This had to be done in every three directions for top, rib and bottom, owed by the inhomogeneity of wood having three different grain directions.

Posing the problem

The Classical guitar is generally described as a system of coupled vibrating systems. Strings and Body possess their own characteristic vibrating frequencies. A sonic survey upon guitars means to deal with issues concerning vibration of strings, plucking, body resonances, coupling between string and body and finally of radiation. The damping properties of each part force the guitar body to vibrate with the eigenfrequency of the plucked string. The body is required to radiate sound otherwise put to make the string audible (Bader (2005, 2010)).

The three lowest frequency modes of Classical guitar bodies are described as combination modes (Meyer (2000)). Top plate, back plate and inclosed air are to be taken as one vibrating system, as one body generating resonances. Resonance frequencies are not of the same pitch than the fundamental of the played tone or note. It is exactly what is originally intended, otherwise there would be a strong resonance with huge magnification of the amplitude of the tone compared to the neighbouring. An instrument of this characteristic is supposed to be unplayable. Modes were often been taken as indicator for sonic quality.
The value of the lowest resonance of the top plate change considerably if it is coupled with
the inclosed air. Influences of back plates are also discussed widely. As expected low tuned back
plates lead to lower body resonances. However, its influence on the resonances as a whole is less
than that of the top plate and the inclosed air. Fletcher and Rossing (Rossing (2000)) also reflect
upon lowest modes as being combination modes. In the case of the lowest mode, back and top
plate vibrate out of phase against each other. The air flows out of the sound hole while the
instruments body vibrates inwards. In the case of higher modes top and back plate oscillate in
phase. The inclosed air vibrates partly in phase with the top, partly in phase with the back
plate. Frequency modes of top plates are a well studied object nowadays. As a matter of course
these modes are varying among different guitars. Modes are often been taken as indicator for
sonic quality.

To inquire into mode shapes and their specific radiation patterns the radiation data,
representing sound pressures, were back propagated to the surface of the guitars top as outlined
above. Close equivalence between eigenmodes found out by impulse tests using a rubber mallet
and mode shapes of forced oscillation excited by plucked strings was expected (Fayhe (2007)).
Respectively the correlation amongst eigenmodes and modeshapes are expected to be strongly
interdependent in such a way that modeshapes produced by forced oscillations are mixtures of
nearest eigenmodes or are at least explainable by these. Orthogonality is to be estimated. If the
forced oscillation pattern does is not equal to the corresponding eigenmode. It is supposed to be
a mixed mode (Bader (2005)).

Astonishingly and contrary to general theory, the forced oscillation patterns differ in shape
and frequency from normal mode shapes to a huge extend. We give it a serious consideration
that forced oscillation patterns are not only explainable by a combination of their neighbouring
eigenmodes. A correlation analysis has been accomplished to depict and prove the existence of
the discovered phenomenon. To avoid extrema in the diagrammed curve caused by low but
persistent noise floor induced by the complex signal chain of the acoustic camera the correlation
function is normalised.

To clarify our observation in an empirical way we took the frequencies of every first partial
of each note up to the 12th fret of every examined guitar. This results in 78 different partials
representing forced oscillation patterns of the particular fundamental tone. Their frequencies
were correlated with the first four partials of the impulses of each guitar in a 4x4 matrix at first
with the result of being not orthogonal at all with a small value standard deviation.

Table1 shows the mean values of the processed correlation. The mean was calculated over
all correlations of each guitar of the sample. Value 1 indicates perfect correlation. 1 is reached
only in the case of correlating partials of the same order. Just the correlation of the first partial
of forced modeshapes with the third eigenmode or third partial of the impulse response seems
very low, which has to be discussed. The corresponding standard deviations of the values of
Table 1 are very low, so our results can be taken as valid, see Table 2. Likewise both tables the
values of the modeshapes produced by forced oscillations are horizontally and the values of the
eigenmodes are vertically displayed.

<table>
<thead>
<tr>
<th></th>
<th>1. partial</th>
<th>2. partial</th>
<th>3. partial</th>
<th>4. partial</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. partial</td>
<td>1.</td>
<td>0.78523</td>
<td>0.238119</td>
<td>0.645679</td>
</tr>
<tr>
<td>2. partial</td>
<td>0.78523</td>
<td>1.</td>
<td>0.590996</td>
<td>0.764493</td>
</tr>
<tr>
<td>3. partial</td>
<td>0.238119</td>
<td>0.590996</td>
<td>1.</td>
<td>0.556876</td>
</tr>
<tr>
<td>4. partial</td>
<td>0.645679</td>
<td>0.764493</td>
<td>0.556876</td>
<td>1.</td>
</tr>
</tbody>
</table>
TABLE 2: The corresponding standard deviations of the values of Table 1

<table>
<thead>
<tr>
<th></th>
<th>1. partial</th>
<th>2. partial</th>
<th>3. partial</th>
<th>4. partial</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. partial</td>
<td>6.05555\times10^{-16}</td>
<td>0.116201</td>
<td>0.0841914</td>
<td>0.12203</td>
</tr>
<tr>
<td>2. partial</td>
<td>0.116201</td>
<td>3.88578\times10^{-16}</td>
<td>0.0858811</td>
<td>0.0922858</td>
</tr>
<tr>
<td>3. partial</td>
<td>0.0841914</td>
<td>0.0858811</td>
<td>5.46721\times10^{-16}</td>
<td>0.0554348</td>
</tr>
<tr>
<td>4. partial</td>
<td>0.12203</td>
<td>0.0922858</td>
<td>0.0554348</td>
<td>7.8111\times10^{-16}</td>
</tr>
</tbody>
</table>

FIGURE 1 shows the relations of the correlation. The four vertical lines represent the frequencies of the specific eigenmodes of one guitar. The four curves display the correlation of modeshapes produced by the strings with the eigenmodes. For instance the dark-grey coloured curve represents the correlation values for the fourth partial.

As we can see the correlation between the fourth eigenmode and the fourth partial of the forced modeshape is indisputable high near 370Hz. But there is a lot more going on here. The increase and decrease around this point is expected to be much more steeper which it is not in this case. We discovered this phenomenon on all examined guitars of our sample.

![FIGURE 1: Correlation diagram of modeshapes vs. eigenmodes](image)

ACKNOWLEDGMENTS

One important criterion for guitar quality, likewise for all instruments with a hollow body seem to be the demand of musicians for a smooth resonating behaviour. The amplification of the partials should be even enough to avoid dead spots on the fretboard but not totally flat. The latter is often described as a cold, missing some kind of character. These distinctive deviations from smoothness makes the sound colour of an instrument. This is controlled mainly by size, wood selection, applied tension during assembling of guitars but also many other steps of the production process have to be taken into account.

The instruments were investigated using microphone array techniques and simple statistics to explain our discovery clearly. Some new insights into theory of vibration are given by the fact that forced modeshapes are not only explainable by a combination of eigenmodes of a vibrating system. This points to a complex travelling impulse behaviour of Classical guitars to be studied in more detail in further investigations. As Microphone Arrays are able to measure and backpropagate radiating sources of irreducibly complicated, strong interacting vibrating structures it makes the method suitable for musical instrument acoustics with many meaningful observations waiting for in near future.

REFERENCES


