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1aMU2. Calculating guitar sound radiation by forward-propagating measured forced-oscillation patterns

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The radiation patterns of 32 guitars are investigated. Therefore the top and back plates are measured using a 121-microphone array, back-propagating the recorded sound field onto the guitar top and back plates. Both, the eigenvalues and the forced oscillation patterns are measured, the latter by plucking the guitar strings for all possible notes. For each note the forced-oscillation radiation pattern is calculated for 20 partials up to 4 kHz. These radiation patterns are then forward-propagated into the surrounding space around the guitar. Considerable differences appear between the different guitars within the same frequency region in terms of shape and intensity.

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INTRODUCTION

Within one of the current research projects at the Institut of Musicology in Hamburg classical guitars where investigated. The concern is to detect how the sound of acoustic guitars is influenced by different parameters like shape of body or bracing style. To have an adequate base for later findings, the whole geometry for each instrument is measured accurately. Therefore, the thickness of back and top plate, the fillet style, the bracing, or the fan-bracing amongst others are used. The choosen collection is a mixture from high to less grade instruments. Also different designs in bracing or shape of body are covered with the sample.

All possible tones on the instrument up the 12th fret for all six strings are played and recorded with a microphone array in front of the top plate. The 121 microphones of the array enables to measure the pressurefield in front of the guitar, 3 cm distance was choosen. To measure all natural playable frequencies the strings are plucked for each fret. While doing so the vibrational behaviour of the body is not disturbed by any kinds of device, like e.g. a piezo detector. Thus the measured radiation of the pressure from the soundboard and air cavity is the pure vibrational behavior of the instrument. After performing a FFT over all sounds, the complex amplitudes are back-propagated for 20 partials of all 78 recorded notes. The result of the back-propagation are the vibrational patterns directly at the surface of the soundboard.

The findings presented here are the forward propagated forced-oscillation patterns from the radiation for each top plate for the existing sample of guitars. The computed data of the pressure enables to calculate the radiated pattern in front of the instrument. These radiation field enables an insight into influences of different body shapes and bracing designs on the guitars sound characteristics. Indeed useful information for luthiers, too.

METHOD

The back-propagated sound field is a vector of 121 complex pressure values, for each entry of a 11 x 11 grid, denoted below by \( p_{bp} \). The grid is arranged rectangularly with a grid constance of \( dx = 4.75 \) cm and \( dy = 4.75 \) cm. To ensure that no cross-feeds affect the back-propagation, the Minimum Energy Method (MEM) is used (Bader et al., 2009). The radiation pattern computed by MEM can then be used to compute the sound-field in front of the guitar in its forward direction. Therefore, a forward-propagation matrix \( M_{nm} \) is used connecting the radiation and sound field points, with \( n=121 \) and \( m \) according to the amount of points of interest in the sound field, including all distances between the top plate and the sound field points and their phase changes. The radiation is taken as a sum of monopole sources. Obviously, each distance \( r_{x,y,z} \) between each of the 121 sources at positions \( l^G \) and all sound field points at positions \( l^S \) can be computed in a three dimensional coordinate system like

\[
 r = \sqrt{(l^G_x-l^S_x)^2 + (l^G_y-l^S_y)^2 + (l^G_z-l^S_z)^2}. \tag{1}
\]

The pressure \( p^R \) at a position in the sound field can be calculated using one of the known pressure \( p_{bp}^{i} \) for the \( i \)th position in \( P_{bp} \) as a monopole source like (Möser, 2009)

\[
p^R = p_{bp}^{i} \frac{1}{r} e^{ikr}, \tag{2}
\]

with wave vector \( k \) and distance \( r \) between the two points. Taking into account the direction of the radiation from the surface, \( r \) is substituted by the variable \( \Gamma \) like

\[
 \Gamma = r(1 + \alpha(1 + \beta)). \tag{3}
\]
This enables a narrowing of the directional pattern in the normal direction of radiation, where $\beta$ is the angle away from the normal direction, and $\alpha$ is the strength of the directionality. $\alpha$ has previously been calculated in the MEM to ensure stability of the back-propagation and therefore need to be used in the forward-propagation, too.

The pressures $p^{bp}$ at all 121 positions on the guitar are known, therefore the radiated pressure $p^R$ in the positive $z$ direction is given as the sum of all radiating monopoles like

$$p^R_m = \sum_{n=1}^{121} p^{bp}_n M_{nm}$$

with

$$M_{nm} = \frac{1}{\Gamma_{nm}} e^{ikr_{nm}}.$$  

This is performed for all positions in $M_{nm}$. The absolute value pressure $|p^R|$ is used throughout this paper omitting the phase information.

**Array Arrangements and Handling of Data**

Two sound field arrangements are used. One to represent the three dimensional space right in front of the player as shown in figure 2. The other assumes an audience in some distance from the guitar. The latter uncovers how a guitar design affects the sound experience of an audience in the case of a concert situation. Five listeners in a radius of $r = 250$ cm in front of the player are assumed at different angles, arranged as a semi circle as displayed in figure 1. The numbers $-2, -1, 0, 1, 2$ in the figures are referring to the positions of five assumed listeners, see below.

**FIGURE 1:** Arrangement for audience case. The five thick black points are the assumed listeners of the audience matrix for $r = 100$ cm. In the background the source matrix $p^{bp}$ with the position of the guitar is displayed.

**FIGURE 2:** Arrangement for entire space solution. Black thick points are similar to the source matrix $p^{bp}$, small gray points representing $M_{nm}$ for several distances $r$.

**RESULTS**

The sample of classical guitars investigated represent a wide range of different top plate designs. To cover the entire diversity and their affects on the radiation characteristics, the 32 guitars used had to be classified into several groups. A first group consists of six instruments all with a standard fan-bracing. A second group includes asymmetric fan-braced top plate guitars, again six guitars. All other instruments were either low-class or had an even more complex top plate design and were therefore excluded from this investigation. The two basic fan-bracing are shown in figure 3.
Comparison of Standard and Asymmetric Fanbracing

The measurements performed with the Microphone Array and the computation of the forward-propagation of the radiation patterns are quite large. With 32 instruments, 78 plucked tones for top and the back plate, where again from each tone 20 partials were back-propagated, a total of 99840 radiation patterns were calculated to estimate the characteristics of the guitars. Here, to make the results not too complex, only the open string notes were used with only their first partial.

As mentioned above, 2.1 two different arrangements of sound fields were used. We want to start with the positions in an audience. For both groups of guitars the mean and standard deviation of the radiated sound pressure at the audience positions were computed. All values are normalized to the maximum level at the source. The result is displayed in figures 4 to 15. Plotted are the sound pressure levels at a distance $r = 250$ cm for five assumed listener positions. Furthermore, the standard deviations corresponding to the six different strings are displayed on the right. The standard deviation of the entire sample is displayed and denoted as $\text{All}$.

Obviously, the design of the bracing of top plates influences the radiated sound pressure level for the $b$- and the high $e$-string, so for higher frequencies. There the asymmetric bracing results in a stronger SPL at 330 Hz, while the symmetric bracing enhances radiation at 246 Hz. Still this is not considerably the case for the lower strings. It is also interesting to see that the standard deviations of the radiated sound for the asymmetric braced top-plates are nearly always smaller than those for the symmetric ones.

Clearly, the influence of the bracing mainly affects the radiation of higher frequencies played on the guitar. At the chosen distance of 250 cm, the differences in sound pressure level are only about 1 dB or less at this frequency range. Indeed, 1 dB difference can no longer be perceived by listener due to the nature of the human ear (Möser, 2009). Still this indifference at low frequencies is expected, as the radiation of lower frequencies, like those of the fundamentals of the $1^{st}$, $2^{nd}$, and $3^{rd}$ string, are basically generated by the Helmholtz resonance of the air cavity and not by the top plate. Though this does not mean that this holds for the whole tone of the open strings, as here only the fundamental frequency is considered. Higher partials are beyond the low-frequency range and therefore are expected to again show differences in terms of symmetrical/asymmetrical fan-bracing. So the asymmetry of the top plate is expected to effect all notes played on the instrument.

Figures 16 and 17 display the overall mean for all strings and guitars of the investigated sample. It appears that the direction of radiation for guitars with standard fan-bracing is more narrow in comparison to the asymmetric design. The asymmetric design show slightly less radiated sound to the front but instead spreading in a more consistent manner to all off-normal listener positions. All in all the difference between the two groups is again about 1 dB or less, but referring to the direction of the radiated sound remarkable values can be observed. Clearly
**Figure 4:** Keynote E at 82 Hz. Mean for both groups.

**Figure 5:** Keynote E at 82 Hz. Standard Deviation for both groups and entire sample.

**Figure 6:** Keynote A at 110 Hz.

**Figure 7:** Keynote A at 110 Hz.

**Figure 8:** Keynote d at 146 Hz.

**Figure 9:** Keynote d at 146 Hz.

**Figure 10:** Keynote g at 195 Hz. Mean for both groups.

**Figure 11:** Keynote g at 195 Hz. Standard Deviation for both groups and entire sample.
a listener at position "0" achieves more volume than one at position ",225" or "225". This is of interest if one might think about a new design of top plate bracing, looking for a loud instrument with a broad radiation character. Contrary, an instrument which radiates sound evenly will show less loudness. Observing the mean of the sound pressure level, one can find that the asymmetric bracing results in a nearly similar radiation to all directions in front of the guitar, as shown in figure 16.

Figure 17 shows that guitars in the group with standard fan-bracing have a higher standard deviation than the ones with an asymmetric design. This means that the standard bracing has a larger amount of deviations.
Solution for the Near-Field

To get an impression of how the sound of the top plate scatters into space in front of the guitar, here the solution for the near-field arrangement of the array (see fig. 2) is shown. Due to lack of space, only one example is displayed, a dipole radiation from the top plate. Furthermore this pattern is assisted by a third area of radiated sound pressure of the Helmholtz resonance. The pattern radiated is generated by the played note cis' which has a frequency at 553 Hz; (6th string, 9th fret). The computed sound pressure level at a space from the top plate up to 100 cm

in front of the instrument is displayed in figure 18 and 19. Two slices of this 3D space are shown, one in the neck-direction of the instrument and one orthonormal to it. Both slices correspond to a row of the microphone array. Figure 18 shows a slice along the fourth row in neck direction and figure 19 is corresponding to the sixth row in orthogonal neck-direction. Clearly, at higher distances the radiated dipole pattern blurs more and more into a monopole. This appears from about 25 cm in front of the top-plate. A three dimensional isobar plot displaying a surface of a chosen sound pressure levels is displayed in figure 20. For orientation, the guitar top plate is shown, too.

**Figure 18:** Sliceplot of sound pressure level in front of the guitar alongside to the x axes. Dark areas illustrate regions of high sound pressure level.

**Figure 19:** Sliceplot of sound pressure level alongside the y axes. Dark areas illustrate regions of high sound pressure level.

**Figure 20:** Three dimensional plot of the radiation pattern for the space in front of the guitar.
REFERENCES


