2aMU5. Numerical analysis of the interaction between fluid flow and acoustic field at the mouth-opening of a flue instrument

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The fluid-sound interaction is the key to understanding the sounding mechanism of flue instruments. The formula introduced by Howe allows us to estimate the energy transfer between acoustic field and hydrodynamic field. For calculation of Howe's formula, it is necessary to divide acoustic field from fluid flow. Recently, several authors developed approximate methods to evaluate Howe's formula and applied to experiments of cavity noise and flue instruments. In this talk, we introduce a numerical method to calculate Howe's formula, which is similar to those introduced by the above authors. Our model is a small flue-organ like instrument with an end stop. We use compressible Large-eddy simulation (LES), which is able to reproduce the fluid flow and acoustic field, simultaneously. First, the pipe is driven by the jet and the fluid flow and acoustic oscillation excited in the pipe are reproduced by LES. Next, an acoustic field generated without the jet-injection but with driving at the far end is reproduced by LES. To excite the acoustic field, we can use several methods, pressure driving, particle velocity driving or oscillating wall driving (like a loudspeaker). Combining those results enables us to calculate Howe's formula and to estimate the fluid-sound interactions.

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INTRODUCTION

The fluid-sound interaction is the key to understanding the sounding mechanism of flue instruments[1, 2, 3, 4, 5, 6]. The formula introduced by Howe allows us to estimate the energy transfer between acoustic field and hydrodynamic field[5]. For calculation of Howe’s formula, it is necessary to divide acoustic field from fluid flow, but we do not have any established method to do it, yet. Recently, several authors developed approximate methods to evaluate Howe’s formula and applied to experiments of cavity noise, flue instruments and so on[7, 8, 9].

In this paper, we introduce a numerical method to calculate Howe’s formula, which is similar to those introduced in Refs.[7, 8, 9]. Our model is a small flue-organ like instrument with an end stop. We use compressible Large-eddy simulation (LES), which is able to reproduce the fluid flow and acoustic field, simultaneously[10, 11]. First, the pipe is driven by the jet and the fluid flow and acoustic oscillation excited in the pipe are reproduced by LES. Next, an acoustic field generated without the jet-injection but with driving at the far end is reproduced by LES. To excite the acoustic field, we can use several methods, pressure driving, particle velocity driving or oscillating wall driving (like a loudspeaker). Combining those results enables us to calculate Howe’s formula and to estimate the fluid-sound interactions.

MODEL AND SCHEME

In this section, we introduce a 2D model of a flue-organ pipe like instrument with an end stop. As shown later, the existence of the end stop makes the calculation of Howe’s formula easier. Fig.1 (a) shows the geometry of the 2D model, which is the same as that used in our recent paper[11]. As shown in the figure, the pipe length L is 90mm, pipe height h = 10mm, width of mouth aperture l = 5mm, flue height d = 1mm, flue length e = 3mm and the edge angle of the labium is 25°. Fig.1 (b) shows the numerical mesh of 2D model, 450 × 200mm² and Table 1 shows the parameters of the mesh. The upper, right and left walls are transparent and the transparent boundary condition is calculated by the Poinsot-Lele method[12]. The other boundaries are non-slip solid walls. The smallest grid size near the labium is around 0.1mm.

We use the compressible LES as the numerical scheme[10]. In LES a statistical model, so-called SGS(sub-grid-scale) model, is used to estimate the effect of eddies smaller than the grid size, which is included into the large scale dynamics of the fluid. LES has less reproducibility than the direct numerical simulation(DNS), but LES is more stable and saves much numerical costs. In this sense, using LES is reasonable. Actually, we adopt a compressible LES solver in the open software OpenFOAM[13] and the time step of the numerical integration is Δt = 10⁻⁷ sec.

![Figure 1: 2D model and mesh. (a) Dimensions of the model, (b) Mesh.](image)
HOWE’S ENERGY COROLLARY AND A STRATEGY FOR NUMERICAL CALCULATION

In order to pursue the acoustic mechanism of flue instruments in terms of aerodynamic sound theory, we have to study the interaction between acoustic and hydrodynamic fields. Howe introduced a very interesting formula, so called Howe’s energy corollary, which enables us to estimate the energy transfer between the acoustic and hydrodynamic fields[5]. It is given by

$$\Pi = -\rho_0 \int (\vec{\omega} \times \vec{U}) \cdot \vec{u} \, d\vec{r},$$

(1)

where $\vec{U}$, $\vec{\omega}$ and $\vec{u}$ denotes the fluid velocity, its vorticity and acoustic particle velocity, respectively. Even though Howe’s integral formula was introduced for a hydrodynamically and acoustically free field, it may be applicable for the space containing solid bodies. Further one may assume that the integrand in eq.(1) predicts local creation and absorption of the acoustic energy: positive and negative values of $(\vec{\omega} \wedge \vec{U}) \cdot \vec{u}$ indicate absorption and creation, respectively.

In order to calculate Howe’s formula, it is necessary to separate the acoustic field from the hydrodynamic field with sufficient accuracy. Now we attempt to calculate it by using a method recently developed by experimentalists[7, 8, 9], which allows us to approximately obtain the acoustic field separated from the hydrodynamic field. The procedure for the numerical calculation is sketched in Fig.2.

First by using the compressible LES we reproduce oscillations excited in the instrument driven by the jet and obtain the flow field together with the acoustic field(see the lower right box). Then we obtain velocities $\vec{U}$ and velocities $\vec{\omega}$, especially those near the mouth opening and calculate the cross product $\vec{\omega} \times \vec{U}$. Further, we get the data of pressure fluctuation at the center of the end stop, the right end of the instrument, at which it is regarded as almost acoustic pressure, because the pressure fluctuation of the fluid becomes extremely small.

Next, comparing with the data of the pressure fluctuations at the end stop, we calculate oscillations of the acoustic field in the instrument without the jet injection but with driving at the end stop in some way, e.g., pressure driving, particle velocity driving or oscillating wall driving like a loudspeaker (see the lower left box in Fig.2). In this work, we choose the particle velocity driving method, namely alternate injection and outjection of small amounts of fluid volume with the same period as the pressure fluctuation observed at the end stop. Since we use the same compressible LES scheme for the calculation, then it is easy to handle the boundary condition at end stop, i.e., driving portion. As a result, acoustic fields obtained are very stable and we get the acoustic particle velocities passing through the mouth opening. Finally we can calculate the integrand $(\vec{\omega} \times \vec{U}) \cdot \vec{u}$ and Howe’s integral formula to estimate energy transfer between the acoustic and fluid fields.

NUMERICAL RESULTS

Fluid field and acoustic fields

First, we numerically reproduce oscillations excited in the instrument driven by the jet at $V = 12m/s$. Spatial distributions of pressure fluctuation, magnitude of velocity and vorticity at a
certain time are given in Fig.3 (a),(c) and (e), respectively. Fig.4 (a) and (b) show the pressure fluctuation observed at the center of the end stop and its power spectrum, respectively.

As shown in Fig.3 (a), there exists a very strong field of pressure fluctuation in the instrument compared with that of the outside. It means that the oscillations are in resonance. Actually as shown in Fig.4, the pressure fluctuation at the end stop oscillates periodically at the frequency $f = 830$Hz.

As shown in Fig.3 (c), the jets oscillate periodically with the same pitches as the acoustic pressure and the eddies created by the collision of the jet with the labium. The eddies created by the collision soon roll up and form clear vortex tubes, which have considerably long lives. Eddies outside the instrument gradually separate from the wall of the pipe and stagnate on an upper side. On the other hand, those inside the instrument are reduced into a large rotor or a few rotors near the open mouth, which never spread further into the right hand side.

As shown in Fig.3 (e), the vorticity takes a positive or negative value at the center of a rolled up eddy depending on its rotational direction. On the other hand, the vorticity along the jet takes positive and negative values along its upper and lower parts, respectively. According to eq.(1), the aerodynamic sound power should be generated in the area of strong vorticity, which appears near the mouth opening. Hereafter, we concentrate our attention to that area.

Fig.3 (b),(d) and (f) show the spatial distributions of pressure fluctuation, magnitude of particle velocity and vorticity of the reproduced acoustic field. Fig.5 (a) and (b) show the pressure fluctuation of the reproduced acoustic field at the end stop and its power spectrum, respectively. As shown in Fig.3 (b), a strong pressure fluctuation, namely acoustic field, is excited in the instrument like that of the instrument driven by the jet in Fig.3 (a). The pressure fluctuation at the end stop in Fig.5 (a) oscillates periodically at the same pitch as that of the pressure oscillation in Fig.4.

As shown in Fig.3 (d), large values of the velocity are observed near the mouth opening, although it also takes large values near the end stop due to the particle velocity driving. Therefore, a strong acoustic flow passes through the mouth opening. However, rolled up vortices, which are created by the collision of the acoustic flow with the labium, also appear near the mouth opening. This is confirmed by the vorticity distribution of the acoustic field, which takes non-zero values near the mouth opening as shown in Fig.3 (f). The existence of the vortices may affect the accuracy in calculation of Howe’s formula, but we connive at this problem in this paper.
**Distribution of \((\omega \times \vec{U}) \cdot \vec{u}\)**

Fig.6 (a) and (b) show distributions of Howe's integrand \((\omega \times \vec{U}) \cdot \vec{u}\) near the mouthpiece at representative phases, \(\phi = -\pi (\text{mod} 2\pi)\) and \(\phi = 0 (\text{mod} 2\pi)\), where \(\phi\) is the phase of the acoustic fluctuation at the end stop, when it is regarded as a sinusoidal function. The vector representation of the acoustic particle velocity field is superposed on it in each figure. In Fig.6 (a), the acoustic flow goes up through the opening, while it goes down in Fig.6 (b). Since the vectors of \(\omega \times \vec{U}\) take opposite directions at the upper and lower sides of the jet as shown in Fig.7, Howe’s integrand takes positive and negative values along the upper and lower edges of the jet in Fig.6 (a), respectively and it has an opposite pattern, i.e., negative at the upper and positive at the lower, in Fig.6 (b). Therefore, the spatial distributions of Howe’s integrand qualitatively coincide with those obtained in experiments by other authors[7, 9].

**FIGURE 3:** Spatial distributions of representative dynamical variables of the fluid and acoustic fields. a) Pressure fluctuation of the fluid field, b) Pressure fluctuation of the acoustic field, c) Magnitude of flow velocity, d) Magnitude of acoustic particle velocity, e) Vorticity of the fluid field, f) Vorticity of the acoustic field.

**FIGURE 4:** Pressure fluctuation at the end stop and its power spectrum. (a) Pressure fluctuation, (b) Power spectrum.
SUMMARY, CURRENT WORKS AND FUTURE DIRECTIONS

In this paper, we have introduced the method to numerically calculate Howe’s integral formula and we have successfully calculated the integrand of Howe’s integral formula, although we have not calculated Howe’s integral formula to the last. To calculate Howe’s formula, one needs to calculate the acoustic field separated from the hydrodynamic field. We have done it with particle velocity driving at the stop end, namely alternate injection and outjection of particle velocities at the end stop, which excites an acoustic field in the instrument. We use the compressible LES solver for the reproduction of the acoustic field instead of the acoustic solver. This is because we can use the same solver for both simulations, the instrument driven by the jet and the reproduction of acoustic resonance, then the data handling becomes much easier in aftertreatment.

However, this method has a drawback that the compressible fluid solver reproduces not only the acoustic field but also the vortices created by the collision of the acoustic flows with the
labium. Therefore, we can not obtain a purely acoustic field, which affects the accuracy of Howe’s formula obtained numerically. We are planning to calculate the acoustic field by using an acoustic solver and to compare the acoustic field obtained by LES with that by the acoustic solver to check the accuracy of the method.

In general, a vortex tube in 2D fluid is more robust than that in 3D fluid, due to the inverse energy cascade observed in 2D fluid[14, 15, 16]. This is the case of the 2D model and it is plausible that the well rolled up eddies considerably affect the energy transfer between the jet motion and acoustic field. Therefore, we need 3D simulations to pursue the sound generation of flue instruments in terms of aerodynamics sound theory.

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