4pMU3. A trombone model emphasizing acoustic accuracy and playability
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This work contributes a physical synthesis model of the trombone, a virtual musical instrument emphasizing quality sound production, and interactivity. The focus is on modelling and coupling four parts of the trombone: the instrument bore, the bell, the vibrating lips and the mouthpiece. The model of the instrument is made parametrically flexible by using a combination of filter elements modeled either using known theory or, for elements not well described theoretically, from acoustic measurement. In particular, acoustic accuracy of the bell reflection and transmission is explored by comparing results obtained from measurement, to those obtained from a piecewise conical model. In addition, the playability and performance characteristics of the complete sounding model---when coupled to a mouthpiece and a configurable generalized reed model---is discussed with reference to expected acoustic behaviour.
INTRODUCTION

The sound produced by the trombone may be seen as the coupling of the input pressure from the lips (the product of the volume velocity output from a pressure-controlled valve and the bore’s characteristic impedance) with the instrument mouthpiece, bore and bell. In this work, the current status of trombone physical modelling, and the synthesis and/or measurement of these components, is presented.

TROMBONE BORE AND BELL

Though wave propagation in many wind instruments bores may be modeled as a one-dimensional waveguide, more attention is required when the bore departs from a purely cylindrical or conical contour—such as at the mouthpiece (discussed in Section 4) and at the bell (discussed herein).

The model depicted in Figure 1 has several filter elements describing the acoustic characteristics of the system that may, or may not, change over time:

- **The delay of \( M \) samples** accounts for the acoustic propagation delay in the bore, the value typically being set according to the bore’s effected length or the desired sounding pitch.

- **Propagation/wall losses** \( \lambda(z) \) are well described theoretically [1, pp. 193-196], with a parametric filter described in [2], allowing for real-time changes according to tube size and length.

- **The reflection and transmission at the bell**, \( R_L(z) \) and \( T_L(z) \), respectively, may be derived either from a computational model or from measurement (see Figure 2), with the former emphasizing parametrization and ability to change the bell contour during performance, and with the latter offering assumed greater accuracy. Because the trombone bell is not expected to change during performance, and because it disassembles easily from the trombone bore, its reflection and transmission functions may be estimated using the measurement technique described in [3].

- **The reflection at the mouthpiece position** \( R_0(z) \). As this is expected to change during performance with the vibrating lips changing both the mouthpiece volume and the opening to the bore, this waveguide element is not easily measured but better developed within the context of coupling with the dynamic lip reed model and mouthpiece, discussed in Section 3 and 4.
LIP VALVE MODEL

When blowing into a trombone, air pressure from the lungs/mouth creates a pressure difference across the surface of the lips, causing them to vibrate. The oscillation of vibrating lips in a brass instrument is typically characterized as being “blown open” [4], and strongly coupled to the bore, making playability (regular non-chaotic oscillation of lips), highly dependent on the resonances of the bore and bell.

Here, the generalized pressure-controlled valve model, first introduced in [5], is used in its “blown-open” configuration as illustrated in Figure 3. The displacement of the valve is given by its angle $\theta$ from the vertical axis and the valve classification is determined by 1) its initial position $\theta_0$ (its equilibrium position in the absence of flow), and 2) by the use of an optional stop—a numerical limit placed to constrain the range of $\theta$. Since the stop is placed at the center vertical axis, $\theta = 0$, and the initial equilibrium position of the valve $\theta_0$ is to the right of the stop, $\theta_0 > 0$, an increased blowing pressure will cause the reed to blow open.

As the reed angle $\theta$ changes during oscillation, the valve opening area $A$ changes according to

$$ A(\theta) = wH_c(\theta), $$

where $H_c(\theta)$ is the characteristic function of the valve.
where \( w \) is the width of the channel and \( H_c(\theta) \) is the channel height, which may be specified by a number of possible functions, such as

\[
H_c(\theta) = 1 - \cos \theta, \quad (2)
\]

suitable for lip reeds.

The geometry of the valve may be further specified by setting the effective length of the reed that sees the mouth pressure \( \lambda_m \), the reed length that sees the bore pressure \( \lambda_b \), and the reed length that sees the flow, given by \( \mu \) (see Figure 3). These variables have an audible effect on the overall driving force acting on the reed, given by \( F \) in (3), and can be seen as offering finer control of embouchure.

Once the valve is set into motion, the value for \( \theta \) is determined by the second order differential equation

\[
m \frac{d^2 \theta(t)}{dt^2} + m2\gamma \frac{d\theta(t)}{dt} + k(\theta(t) - \theta_0) = F,
\]

where \( m \) is the effective mass of the reed, \( \gamma \) is the damping coefficient, \( k \) is the stiffness of the reed, and \( F \) is the overall driving force acting on the reed, a function of the mouth and bore pressure, and flow in contact with the reed. The frequency of vibration for this mode is given by

\[
\omega_v = \sqrt{k/m - \gamma^2}.
\]

Discretization, equivalent to applying a bilinear transform, yields the transfer function in the \( z \) domain

\[
\frac{\theta(z)}{F(z) + k\theta_0} = \frac{1 + 2z^{-1} + z^{-2}}{a_0 + a_1z^{-1} + a_2z^{-2}},
\]

and the corresponding difference equation

\[
\theta(n) = [F_k(n) + 2F_k(n-1) + F_k(n-2) - a_1\theta(n-1) - a_2\theta(n-2)]/a_0,
\]

where \( F_k(n) = F(n) + k\theta_0 \), and

\[
\begin{align*}
a_0 &= ma^2 + mg\alpha + k, \\
a_1 &= -2(ma^2 - k), \\
a_2 &= ma^2 - mg\alpha + k,
\end{align*}
\]

and \( \alpha = 2/T \), where \( T \) is the sampling period, and \( g = 2\lambda \). Since pole frequencies are well below half the sampling rate, there is no need for pre-warping.

The force driving the reed \( F \) is equal to the sum of the forces acting on the reed,

\[ F = F_m + F_b + F_U, \]

where \( F_m = w\lambda_m p_m \) is the force acting (in the positive \( \theta \) direction) on the surface area \( \lambda_m w \), \( F_b = -w\lambda_b b \), is the force acting (in the negative \( \theta \) direction) on the surface area \( \lambda_b w \), and \( F_U \) is the force applied by the flow (which forces the reed open) given by

\[
F_U = \text{sign}(\theta)w\mu \left( p_m - \frac{\rho}{2} \left( \frac{U(t)}{A(t)} \right)^2 \right).
\]

As can be seen by (6), the total force driving the reed is dependent on the valve classification, since the sign of \( \theta \) is determined by its limits.

The differential equation governing air flow through the valve, fully derived in [6], is given by

\[
\frac{d U(t)}{dt} = (p_m - p_b) \frac{A(t)}{\mu \rho} - \frac{U(t)^2}{2\mu A(t) + U(t) T},
\]

where \( p_m \) is mouth pressure, \( p_b \) is the bore pressure (see discussion in the following section), \( A(t) \) is the cross sectional area of the valve channel, and \( \mu \) is the length of reed that sees the
flow. Equation (7) is used to update the flow $U$ every sample period (given by the inverse of the sampling rate).

There are, therefore, three variables that evolve over time in response to an applied mouth pressure $p_m$: the displacement of the reed $\theta$ (determined using 5), the flow $U$, determined using the update given by (7), and the pressure at the base of the bore $p_b$, obtained from the waveguide model as shown in Figure 1.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of exhaust</td>
<td>$a$</td>
<td>8 mm</td>
</tr>
<tr>
<td>Valve width</td>
<td>$w$</td>
<td>2.3 mm</td>
</tr>
<tr>
<td>Valve length</td>
<td>$\lambda_m = \lambda_b$</td>
<td>23.2 mm</td>
</tr>
<tr>
<td>Valve mass</td>
<td>$m$</td>
<td>.3 g</td>
</tr>
<tr>
<td>Valve thickness</td>
<td>$v$</td>
<td>6 mm</td>
</tr>
<tr>
<td>Initial displacement</td>
<td>$\theta_0$</td>
<td>0.01 mm</td>
</tr>
<tr>
<td>Mouthpiece volume</td>
<td>V</td>
<td>$5 \times 10^{-6}$ m$^3$</td>
</tr>
<tr>
<td>Mouthpiece choke length</td>
<td>$l_c$</td>
<td>48 cm</td>
</tr>
<tr>
<td>Mouthpiece choke radius</td>
<td>$a_c$</td>
<td>4.5 mm</td>
</tr>
</tbody>
</table>

**MOUTHPIECE**

Connecting the lip model to the instrument is done via the mouthpiece to account for the resonance created by the mouthpiece’s cup volume and its backbore constriction.

As shown in [1, 7] and others, the mouthpiece may be modeled by the equivalent electrical circuit shown in Figure 4. The mouthpiece consists of a cup having a volume $V$, and presents an acoustic compliance $C$ given by

$$C = \frac{V}{\rho c^2},$$

where $\rho$ is the air density and $c$ is the velocity of sound in air. The cup is followed by a constricted passage before entering into the wider trombone bore, with the constriction behaving as a series inductance (inductance in electrical terms) given by

$$L = \frac{\rho l_c}{S_c},$$

where $l_c$ is the length and $S_c$ is the cross-sectional area of the constriction. The dissipative element $R$ in series with this inductance represents viscous and thermal losses. Its value for the mouthpiece has been obtained through experiment in [8].

Inserting a mouthpiece between the reed and bore models requires a new expression for the volume flow entering the bore (it is no longer that coming directly from the lips), as well as a new
expression for the downstream pressure used when in the dynamic lip reed model (it is no longer
the bore base pressure). These quantities are termed $U_2(t)$ and $p_1(t)$, respectively, in Figure 4.

The mouthpiece model provides a volume flow $U_2(t)$ into the bore and a pressure $p_1(t)$ in the
mouthpiece, in response to a volume flow $U_1(t)$ entered the mouthpiece (generated by the lip
reed model) affixed to the instrument having a pressure of $p_2(t)$ at the bore base.

Taking the Laplace transform of the differential equations describing the mouthpiece model
in Figure 4 leads to the system's frequency domain input-output matrix

$$
\begin{bmatrix}
U_1(s) \\
p_1(s)
\end{bmatrix} =
\begin{bmatrix}
\frac{s^2LC + sRC + 1}{sL + R} & sC \\
sL + R & 1
\end{bmatrix}
\begin{bmatrix}
U_2(s) \\
p_2(s)
\end{bmatrix},
$$

which may be rearranged and discretized to yield expressions for $U_2$ and $p_1$ in response to
$U_1$ and $p_2$, given in the $z$-domain as

$$
U_2(z) = \frac{U_1(z)(1 + 2z^{-1} + z^{-2}) - C a p_2(z)(1 - z^{-2})}{a_{m0} + a_{m1}z^{-1} + a_{m2}z^{-2}},
$$

where

$$
a_{m0} = LCa^2 + RCa + 1 \\
a_{m1} = -2(LCa^2 - 1) \\
a_{m2} = LCa^2 - RCa + 1
$$

and

$$
p_1(z) = \frac{U_2(z)(b_0 + b_1z^{-1}) + p_2(z)(1 + z^{-1})}{1 + z^{-1}},
$$

where

$$
b_0 = La + R \quad \text{and} \quad b_1 = -La + R,
$$

and $a = 2/T$, where $T$ is the sampling period. The corresponding difference equations are given
by

$$
U_2(n) = [U_1(n) + 2U_1(n - 1) + U_1(n - 2) - C a (p_2(n) - p_2(n - 2)) - a_1U_2(n - 1) - a_2U_2(n - 2)]/a_0,
$$

and

$$
p_1(n) = b_0U_2(n) + b_1U_2(n - 1) + p_2(n) + p_2(n - 1) - p_1(n - 1).
$$

Again, as for the case of discretizing the valve displacement, no pre-warping is required. The
effects of the mouthpiece can be viewed by comparing the model's output with and without a
mouthpiece, with all other parameters set as in Table 1.

**Conclusions**

This work assembles recent contributions to trombone synthesis. A measurement technique,
shown to produce accurate data for wind instrument bells, is combined with waveguide
synthesis, a generalized pressure-controlled valve model configured to act as a lip reed, and an
existing mouthpiece model to produce a quality real-time parametric trombone model.
FIGURE 5: The magnitude spectrum of the model output showing the resonant effects of the mouthpiece.

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REFERENCES


