2pNSa9. The impact of including diffraction when predicting the effect of listener environment on the perceived loudness of outdoor sonic booms

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The human impact of sonic booms varies with listening environment. Given the incident sonic boom waveform, the specular field around a realistic geometry has been predicted via a C++ implementation of image source method (ISM) tailored to this outdoor application. This work explores the necessity of including the diffracted field when predicting time series and perceived loudness (PLdB), both in and out of the shadow zone. The impulsive nature of the excitation, and the sensitivity of the PLdB to temporal details, constrains appropriate diffraction modeling techniques to those capable of time domain accuracy. Uniform Theory of Diffraction (UTD) and Biot Tolstoy Medwin (BTM) models are considered, exploring the benefits and challenges of each approach, particularly with regards to scalability and bandwidth. The importance of accurately predicting diffraction for this application is evaluated through comparison with booms recorded around a building corresponding to the simulated geometry.

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**CONTEXT**

Motivated by a need for more accurate simulated outdoor sonic booms, this work builds on an existing C++ specular reflection model, with the ultimate goal of best simulating the effects of the listening environment for a given incident sonic boom waveform. Waveforms output by a sufficiently accurate model would find application in listening tests and utility in calculating surface pressure loading for the indoor transmission problem. Such listening tests would provide useful information for the public and policy makers regarding boom perception and possible rule changes regarding overland supersonic flight, respectively. No such tests are described in the current work. Presented here is a progress report toward synthesizing more realistic waveforms that could be used in laboratory jury tests.

The implemented specular model is based on Image Source Theory, as presented by Mechel (1), but tailored to outdoor applications and planar incident wavefronts. This purely specular model erroneously predicts silence in the shadow zones and discontinuities across shadow boundaries. ISM, and other ray-based approaches are often said to be “high frequency models”, accurate down to a cutoff frequency, the wavelength of which is often defined by the smallest facet in the 3D geometry. Awareness of this, and other heuristic statements about diffraction, specifically that low frequencies propagate past acoustically small obstacles without reflection or occlusion, and that the width of the shadow boundary transition is proportional to wavelength, led to an initial pragmatic approach: each specular reflection was high pass filtered with a roll-off based upon the pressure reflected by a finite disk (2).

In order to assess this first approach (ISM + disk approximation), impulse responses were generated for a geometry approximating that of Edwards Airforce base residence instrumented in the 2007 experiment known as HouseVIBES (3).

![Figure 1: Simulated Geometry and Microphone Positions](image)

For each simulation, the input incident waveform was approximated by the sonic boom recorded on the desert surface, 300 m West of the structure, scaled by 1/2. This input waveform was then convolved with the impulse response (IR) predicted using the geometry shown Figure
1. Thirty four impulse responses were obtained, corresponding to the incident arrival angles of the recorded sonic booms comprising the dataset. It was found that the ISM+disk approach significantly under predicted peak pressures, and that the agreement between simulated and recorded waveforms was better without the disk approximation. Nonetheless, without accounting for diffraction using the finite disk model, the simulations consistently over predict the peak pressure of the waveform, although by a smaller margin. It is likely that a diffraction model more closely related to the actual geometry would offer greater agreement in the time domain waveforms.

A similar question was addressed by Albert in an exhaustive study in which he compared Pierce’s diffraction model with traditional BTM. Eight propagation paths around a simplified house were considered. Despite agreement at low frequencies, above 500 Hz the agreement was poor. Automatic calculation of diffracted paths in a more finely detailed geometry may clarify of the poor high frequency agreement could be attributed those simplifications. The question remains, however, as to whether the additional accuracy will result in a significant change in PLdB. This work serves as a preliminary look at two edge diffraction models which are widely used, the Uniform Theory of Diffraction, and the Biot-Tolstoy Medwin approach, in light of this application and the HouseVIBES dataset.

**APPROACH**

In order to better assess the necessity of the inclusion of diffraction, and evaluate two accepted methods of doing so, we begin by isolating a single edge and single microphone. We are fortunate that microphone 106 receives no specular reflections, as illustrated for a single boom event in Figure 2. Following a procedure similar to that used to quantify the accuracy of the finite disk model, we use the microphone located 300 m West to approximate the input waveform.

![Figure 2](image_url): Change in PLdB due to Predicted Specular Reflections for Boom Event 37 (boom events 2, 27 and 37 were discarded), illustrating the change in PLdB when considering specular reflections. There are no specular reflections at the position of microphone 106, for all boom events.

**THEORY**

Uniform Theory of Diffraction

Finding it’s foundation in Keller’s Geometrical Theory of Diffraction (GTD), the Uniform Theory of Diffraction (UTD) is a widely accepted 'high frequency' diffraction model for
diffraction by hard edges. “By high-frequency phenomena, we mean that fields are being considered in a system where the properties of the medium and scatterer size parameters vary little over an interval on the order of a wavelength” (4). For a building on the order of 20’, 54 Hz corresponds to a wavelength on the order of the scatterer size. With major features of the home being about a fifth of the size of the entire structure, the frequency range of theoretical validity is very restricted.

Considering an infinitely thin barrier, away from the shadow boundaries the edge diffracted contribution is proportional to $O(k^{-1/2})$. If that barrier is finite, the vertex diffracted field is proportional to $O(\frac{1}{k})$ (5). The termination of each edge gives rise to its own vertex diffracted field; each vertex of a cube will give rise to 3 co-located vertex diffracted contributions. While UTD is only theoretically accurate for higher frequencies, this work aims to quantify how significant this error is for this application, for both the prediction of time series and PLdB. Perhaps the error will not be significant considering the approximation used to obtain our input waveform.

GTD and UTD resemble a generalization of Snell’s law. In these ray based models, every ray which strikes an edge gives rise to a cone of diffracted rays, each ray sharing the same angular relationship with the edge as the incident ray. This theory provides an analytical expression for the diffracted contribution by means of a complex diffraction coefficient and ray based arrival time from Fermat’s principle. UTD is distinguished from GTD by fact that the diffraction coefficients are scaled by a transition function, making finite the singular solutions of the GTD diffraction coefficient that arises at shadow zone boundaries.

The equation for the diffraction coefficient is greatly simplified by the details of the particular application, specifically the geometry composed of planar facets and the planar incident wavefront. For this circumstance, the ‘Distance Parameters’, $L_i$, $L_{ro}$ and $L_{rn}$, substantially simplify to:

$$n = \frac{2\pi - \alpha}{\pi}$$

(1)

Where $\alpha$ is the solid angle of the wedge. The Associated Function is given by,

$$a^{\pm}(x) = 2\cos^2\left(\frac{2\pi N^\pm - x}{2}\right),$$

(2)

where $N^\pm$ are integers which most closely satisfy,

$$2\pi n N^+ - x = \pi$$

(3)

or

$$2\pi n N^- - x = -\pi.$$  

(4)

For this application, all distance parameters reduce to

$$L_i, L_{ro}, L_{rn} = s \cdot (\sin[\beta_o])^2,$$

(5)

where $\beta_o$ is the acute angle subtended by the incident ray and the edge, as well as the reflected ray and the edge and $s$ is the distance from $Qe$, or the edge diffraction point, to the receiver. $Qe$ is the point on the edge defined by the shortest distance from the source, to the infinitely extended edge, to the receiver.

The Transition Function, which compensates for the discontinuity in GTD around the shadow boundary, making the field uniform, is given by

$$F(x) = 2j \sqrt{\pi x} j^x \int_{\sqrt{\pi}}^{\infty} e^{ju^2} du = \frac{1}{2} \sqrt{\frac{\pi}{2}} \left[ 1 - 2C\left(\sqrt{\frac{2x}{\pi}}\right) \right] + j \frac{1}{2} \sqrt{\frac{\pi}{2}} \left[ -1 + 2S\left(\sqrt{\frac{2x}{\pi}}\right) \right],$$

(6)
which maybe expressed in Fresnel integrals and approximated using Chebyshev polynomials. Using these parameters, the diffraction coefficients for the incident and reflected fields are given by:

\[
D_1 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k \sin \beta_o}} \cot \left( \frac{\pi + (\Phi - \Phi')}{2n} \right) F[kL^i a^+(\Phi - \Phi')],
\]

(7)

\[
D_2 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k \sin \beta_o}} \cot \left( \frac{\pi - (\Phi - \Phi')}{2n} \right) F[kL^i a^-(\Phi - \Phi')],
\]

(8)

\[
D_3 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k \sin \beta_o}} \cot \left( \frac{\pi + (\Phi + \Phi')}{2n} \right) F[kL^r a^+(\Phi + \Phi')],
\]

(9)

\[
D_4 = \frac{-e^{-j\pi/4}}{2n\sqrt{2\pi k \sin \beta_o}} \cot \left( \frac{\pi - (\Phi + \Phi')}{2n} \right) F[kL^r a^-(\Phi + \Phi')]
\]

(10)

For the case of a scalar field and infinitely rigid surfaces, their contributions sum simply:

\[
D = D_1 + D_2 + D_3 + D_4
\]

(11)

For an incident plane wave, the spreading term reduces to:

\[
\frac{1}{\sqrt{s}}
\]

(12)

Assembling the terms, we obtain the complex amplitude of the edge diffracted contribution at the field point, \(E^d\), and it’s relation to the complex amplitude of the pressure incident at the point of diffraction on the edge, \(E^i\).

\[
E^d = E^i \cdot D \frac{1}{\sqrt{s}} e^{jks}
\]

(13)

While this approach is a frequency domain model, that description is only accurate for an isolated edge. The formulation of the theory in terms of rays allows diffraction contributions from various edges to be distinguished temporally. The frequency domain nature of the model, and the limited bandwidth of theoretical validity, are significant detractors from the appeal of the approach for this specific application. The present work, however, explores how detrimental these limitations are to agreement with experiment.

The range of validity of UTD is in fact quantified by the Largeness Parameter, \(\kappa\), which is, in this case, defined in terms of distance parameter \(L^i\) and frequency:

\[
\kappa = kL^i \sin^2 \beta_o.
\]

(14)

This equation communicates that UTD is technically not valid at low frequencies, or when the source or receiver is very close to the edge. However, perhaps the model is sufficiently accurate for the purposes of PLdB and jury tests, and perhaps after validation with another experiment, the transmission problem.
Biot Tolstoy Medwin

Based on Biot and Tolstoy’s closed-form solution for diffraction from a pulse by an infinite wedge (6), Medwin presented expressions with which one can computationally obtain the pressure diffracted by a finite edge (7). Contrasting with UTD, where a single ray makes its way from the source to edge to receiver, that ray being associated with a complex diffraction coefficient for each frequency, BTM models the diffracted contribution as a line source on the edge. BTM provides a time domain expression of the pressure at the field point in terms of the contribution from spherically spreading sources along the edge. The BTM model has many theoretical advantages; it is free of high frequency asymptotic assumptions, and avoids the Kirchhoff approximation, offering better agreement at high frequencies and at grazing incidence. The expressions given by Medwin are also appropriate for evaluation on the faces of the diffracting edge with after simple scaling by a factor of 2 (8).

Another appealing feature of the BTM model is that higher order diffraction may be calculated by representing the first order diffracted field as Huygens wavelets, or secondary sources (8). These secondary sources then radiate on to other edges, providing a means of calculating higher order diffraction. An approach pursued at length and further developed by Svensson(9; 10).

First order BTM edge diffraction is given by

\[ p(t) = \left( -\frac{S \rho c}{4\pi \theta_w} \right) \beta_{\pm\pm} \left( \frac{1}{rr_o \sin hY} \right) e^{-2\pi y/\theta_w}, \]  

(15)

S is the volume velocity of the original spherical source, and \( \theta_w \) is the fluid angle of the edge, 2\( \pi \) corresponding to an infinitely thin barrier. The source position is given by \([r_o, \theta_o, 0]\), and the receiver is located at \([r, \theta, Z]\).

The other terms are given by,

\[ y = \arccosh \left( \frac{c^2 t^2 - (r^2 + r_o^2 + Z^2)}{2rr_o} \right). \]  

(16)

and

\[ \beta = \sin \left( \frac{\pi}{\theta_w} (\pi \pm \theta \pm \theta_o) \right) \left[ 1 - 2e^{-\pi y/\theta_w} \cos \left( \frac{\pi}{\theta_w} (\pi \pm \theta \pm \theta_o) \right) + e^{-2\pi y/\theta_w} \right]^{-1}, \]  

(17)

where \( \beta_{\pm\pm} \) in Eq. (15) is the sum of four terms, each containing one of the four possible combinations in \((\pi \pm \theta \pm \theta_o)\).

In Medwin’s formulation (8), it is also observed that the spectrum of the diffracted contribution away from the shadow zone is proportional to \(1/f\). However, due to the time domain approach, that trend is only clear after assuming that \(\tau/\tau_o << 1\). Where \(\tau_o\) is the time of arrival for the first diffracted contribution, and \(\tau\) is time measured after \(\tau_o\). Therefore, this assumption only holds for the entire diffracted IR when neither source nor receiver are very close to a very long edge.

RESULTS AND DISCUSSION

Because of the very early deadline for the written papers for this ICA/ASA meeting, no comparison results between the UTD and/or BTM methods with experimental data were available at the time of the writing. Those comparisons will be available at the time of the conference in June 2013.
CONCLUSION

This paper has described two methods that show promise for including the effects of diffraction in sonic booms scattering off of structures such as buildings. Both the Uniform Theory of Diffraction (UTD) and the Biot Tolstoy Medwin (BTM) approaches can add diffracted fields to otherwise purely-specular fields generated with images. It is postulated that the resulting synthesized fields will result in sonic boom waveforms including building reflections that are more realistic than those available currently. By future comparison to the NASA 2007 HouseVIBES datasets or other available datasets, assessment of the contributions of diffraction to human perception of sonic booms will be possible.

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REFERENCES


