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3pNSc6. Position optimization of Helmholtz resonator in ducts using a genetic algorithm
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The equipments layout, the maintenance and operation purposes in industries limit the installation of reactive mufflers in terms of space and location. As part of this device design it is necessary to consider these restrictions and an optimization process may necessary. Keep in mind that in real application the downstream radiating end of the duct must be modeled as an open end radiating into free space, the Insertion Loss (IL) parameter is more proper for evaluating the HR’s performance than the Transmission Loss. Using the IL to estimate the effectiveness of the acoustic filter, the main purpose of this paper is to numerically analyze and maximize this parameter in the maximum attenuation frequency considering position restrictions (bounds constraints) in a duct. An evolutionary search algorithm (GA) has been applied in order to solve the best position for a fixed shape HR in a duct. The finite element method was used to model the acoustic system HR/duct. The pressure data and the optimization step were processed in Matlab®. For optimal positions the results reveal an increase of 19 dB in the IL parameter at the desired frequency. To verify the sensibility of the methodology simulations were performed varying some GA parameters.

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INTRODUCTION

Industrial equipments like compressors, blowers and fans generate a combination of broadband noise and specifically low frequency tonal noise. In general these equipments are connected with ducts or exhaust systems, through the noise is transmitted to the external environmental. The fan, for example, generates strong tones at the fan’s blade passage frequency (BPF) and associated harmonics (Cazzolato et al, 1997).

Low frequency tonal noise transmission can be controlled in a number of ways, such as by the installation of reactive silencers, side branch elements and active noise. In particular, the Helmholtz Resonators (called simply HR, here after) are reactive devices with a resonance peak designed to control this kind of noise. They are useful against noise centralized in a narrow frequency band (Rayleigh, 1945).

The HR is usually composed of an acoustic cavity and a short tube. It is widely used as an acoustic absorption filter and noise control device because of the strong absorption in its resonant frequency (Singh, 2008). When the HR is mounted on a duct, it reduces the radiated acoustical power primarily that reflect the sound back toward the source (Beranek and Ver, 1992).

Although this filter be a classical device and its principle be widely known from many time ago, researches still are developed due to its high efficiency, applicability and low cost when compared with resistive filters. New configurations like HR array were recently developed and showed good results (Lu et al, 2012; Wang and Mak, 2012).

Reactive silencers provide a feasible alternative to the more conventional dissipative systems and do not suffer from a reduction in performance from buildup of particulates present in the acoustic fluid. In addition, reactive silencers offer better low frequency performance than dissipative silencers of similar dimensions (Cazzolato et al, 1997).

Before the HR design (shape and dimensions) it is necessary to have in hands the noise source characteristics. The noise source identification should be done by the acoustic designer as a first step in order to know which is the tonal frequency (and the harmonics) and its magnitude. Many experimental techniques can be used in this step like vibration measurement, intensity measurements and others (Bies and Hansen, 2003). This frequency characterization may be used in the HR modeling as the second step of the HR design.

Once the acoustic performance of the HR is direct related with its dimensions, like the cavity volume, neck’s radio and length, the physically space constraints should be treated with relevancy. Mainly in industries this fact is extremely relevant. In such places the equipment layout can be tight and the available space for a muffler is limited for maintenance and operation purposes (Chang, 2008). In this context, in the HR design should be considerate the need to maximize the acoustic performance of this device in a limited space for installation. So an optimizer becomes crucial in order to enhance a proficiently acoustic performance. Both cases should be considerate: muffler dimension (shape) and position (location) limitation.

Several researches about mufflers optimization have been developed (Airaksinen and Heikkola, 2011; Lee and Kim, 2009; Chang and Chiu, 2008) over the last few years. However, most of them deal with shape optimization maximizing the transmission loss and improvement of optimization techniques for this kind of problem. Yet, the investigation of optimal muffler design under position constraints is rarely tackled. In order to complement this design step and contribute to increase the acoustic performance of a designed HR this work show numerically that the location/position of the muffler mounted in a duct has its acoustic performance changed (maximum attenuation frequency and insertion loss) when this parameter is altered, so a position optimization methodology is presented.

Today, many optimization methods have already applied in acoustic problems, like Search methods and Heuristic optimization. One in special is larger used for this purpose: the Genetic Algorithmic. This method, based on Darwin’s concept of selection and natural evolution, has advantages which justify its utilization for obtaining optimum solutions for many engineering efforts (Key, 2009). The most advantage is that does not need derivatives during the optimization process and it tends to the global optimum even with initial bad data points (Goldberg, 1989). Many researchers used this algorithm with successful results applied to muffler optimization (Lima et al, 2011; Chiu, 2010, Yeh et al, 2006).

The success of an optimization technique depends of many factors (design parameters) but the choice of an appropriate objective function is essential. For muffler optimization two acoustic performance criteria can be used like objective function: the transmission loss (TL) and the insertion loss (IL). The sound transmission loss (TL) is defined as the ratio of the sound energy incident on the muffler inlet to that of the sound energy leaving the muffler at the outlet. This parameter depends only the muffler and not also on connected elements. The insertion loss (IL) is the difference between the acoustic power radiated without any filter and with the filter (Munjal, 1987).
The TL criteria is not representative of the practical point of view, because for calculate it is necessary consider an anechoic termination in the duct (opposite to the noise source). This condition is not true in an industrial environmental. In industry the noise commonly propagates in an exhaust duct which the downstream radiation end should be modeling as an open end radiating to the free space. In addition, the IL involves measuring the acoustic power of the system before and after the installation of the filter, that is more useful from a practical point of view (Singh, 2008). Once the IL considerate the whole system filter/duct, this one will be used to estimate the acoustic performance of the HR evaluating the position influence of the muffler. The Finite Method Elements (FEM) was used to modeling the acoustic system in order to estimate this parameter.

Using the IL to estimate the acoustic performance of the HR, the aim of this paper is to numerically analyze and maximize this parameter in the maximum attenuation frequency considering position restrictions (bounds constraints) in a duct. An evolutionary search algorithm (GA) is used to solve the best position for a fixed shape HR in a duct with defined length. The FEM was used to model the acoustic system HR/duct and to obtain the acoustic pressure in the points of interested. The IL estimation through the sound pressure data was processed in Matlab® as the GA algorithm. In order to evaluate the optimization technique applied to this specific device, several parameters like population size, generation and others were modified and tested.

THEORETICAL BACKGROUND

Insertion Loss

The sound power insertion loss is defined as the logarithmic ratio of the sound power transmitted by a system before the insertion of a noise-control device to that after insertion. This parameter not only accounts for the performance of the isolated attenuator (unlike TL) but also for any effects of insertion, such as alteration of source sound power. Insertion loss is therefore installation sensitive, and not unique to an attenuator. It provides a more realistic and reliable measure of attenuator performance (Fahy, 2001). This definition justifies its use in this paper like objective function for location optimization.

Modal decomposition technique can be used to determine the amplitude of the acoustic waves propagating each way inside the duct, that can be easily separate into incident and reflective parts. It is assumed a uniform straight duct with rigid walls that carries a uniform axial mean flow and the linear acoustic theory is valid. The acoustic power associated to the incident waves, $W_+$, and reflective waves $W_-$, are described as (Ábom, 1988):

$$W_+ = \frac{S|\hat{a}_+|^2}{2\rho c}$$

$$W_- = \frac{S|\hat{a}_-|^2}{2\rho c}$$

where $\hat{a}_+$ and $\hat{a}_-$ are the modal amplitudes of the acoustic pressure associated with the incident and reflective waves, respectively, $\rho$ is the density of the fluid medium, $c$ is the speed of the sound and $S$ is the cross-sectional area.

The net acoustic power transmitted along the duct is estimated by the difference between (1) and (2):

$$W_{net} = W_+ - W_- = \frac{S}{2\rho c} \left[|\hat{a}_+|^2 - |\hat{a}_-|^2\right]$$

The IL can be determined as:

$$IL = W_{net1} - W_{net2}$$

The subscripts 1 and 2 in equation (4) denote acoustic systems without filter and with filter, respectively. The Fig. 1 illustrates the IL principle.
Holland (1975) was the first to formalize the concept of Genetic Algorithm followed by DeJong (1975) who extended it to a functional optimization. This evolutionary algorithm involves the use of optimization search strategies patterned after Darwinian notion of natural selection and evolution (Chiu, 2010). These algorithms use simulated evolution to search for solutions to complex problems.

The GA, defined as stochastic search technique, differs from deterministic search methods by the way the search vector is calculated. From an initial population, the GA starts the optimal search. The most common search method is to randomly generate solutions for the entire population by the use of operators (Yeh et al., 2004). Each individual in the population represents a point in the search space for which the dimension is determined by the number of design variables. The typical genetic algorithm encodes solutions as bit strings, enabling the use of standard crossover operators such as one-point and two-point crossover. For each generation, the genetic search calculates the fitness of every individual in the population. The reproductive opportunities of the evaluated structures are allocated in such a way that those chromosomes which represent a better solution to the target problem are given more chances to reproduce than those chromosomes which are poorer solutions (DeJong, 1975).

The operators, which provide the search mechanism of the GA, are used to create the new solutions based on existing solutions in the current population. There are two basic types of operators, crossover and mutation. Crossover takes two individuals and produces two new individuals while mutation alters one individual to produce a single new solution. The GA function calls each of the operators to produce new solutions (Houk et al., 1995).

The GA moves from generation to generation selecting and reproducing parents until a termination criterion is met which can be the maximum number of generations or population convergence criteria. In general, GAs will force much of the entire population to converge to a single solution (Houk et al., 1995). Figure 2 shows a block diagram with the operations in the GA method.

In this work the GA was implemented as Matlab® toolbox by Houk et al (1995), named GAOT (Genetic Algorithms for Optimization Toolbox). This algorithm uses a real-valued alphabet (float genetic algorithm) which was employed in conjunction with the selection, mutation and crossover operators. Each module of the algorithm is implemented using a Matlab function in order to provide easy extensibility, as well as modularity.
As input parameters are necessary to define the bounds which is a matrix of upper and lower bounds on the variables, the evaluation function or objective function and the change required to consider two solutions different \( (\epsilon = 1 \times 10^{-6}) \). The starting population defaults to a randomly created population created with an initialize function.

As output parameters the algorithm gives the best solution string, the final population, the matrix of the best individuals and the corresponding generation they were found and a matrix of maximum and mean functional value of the population for each generation.

**METHODOLOGY**

Before performing the GA simulation, a sensibility analysis was performed in order to check the dependency between both parameters: HR acoustic performance (IL) and its position along the duct, considering a predefined HR shape.

The first step consists in to estimate the net acoustic power of the system without a muffler. A three-dimensional model of the considered duct was built using the finite element software Ansys®. The duct geometry is: 3 meters of length and 0.155 meters of internal diameter. It was considered a duct with rigid-walled, so only the fluid inside it was modeled like finite elements. The left end of the duct had being driven by a piston and the right end is open and radiating sound into free space.

The open end of the duct was modeled using the radiation impedance condition (Eq. 5) for an unflanged open end (Kinsler, 1982):

\[
Z_i = \frac{\rho c}{S} \left[ \frac{(ka)^2}{4} + j(0.6)ka \right]
\]

where \( \rho \) is the density of the fluid medium, \( c \) is the speed of the sound and \( S \) is the cross-sectional area, \( k \) is the wave number, \( a \) is the duct’s radius and \( j \) is the imaginary number.

As the radiation impedance is a complex value and it is a frequency dependent, it is necessary to consider both parts (real and imaginary) as input parameters in the FE model. The surface element SURF153 was used to model this radiation impedance. The impedance real part is modeled using the parameter VISC, which is a material property of this surface effect element. The imaginary part can be modeling using ADMSUA which is a real constant of this element (Imaoka, 2004).

The medium fluid inside the duct was meshed with FLUID30 elements. This kind of element has eight corner nodes with four degrees of freedom per node: translations in the nodal x, y and z directions and pressure. The elements without structural interface has only pressure degree-freedom at each node (defined by the input parameter KEYOPT(2) = 1), which are represented by blue color in the Fig. 3. So, the fluid elements placed at the right end were modified having both pressure and displacement degree-of-freedom at each node (KEYOPT(2) = 0), as indicate Fig. 3 by purple color elements. It is possible to note in Fig. 4 the presence of the SURF153 elements like a layer (red). The global element size was 30 mm, resulting in a model with 21770 elements.

The rigid-walled is modeled if no boundary condition is applied on the boundary of the pure acoustic element mesh. In order to reproduce the duct being drive by a piston at the left end the unit volume acceleration (denoted by FLOW load parameter) was applied in the nodes located in that termination.
A harmonic analysis in the interesting frequency range (0 - 400 Hz) was performed in order to obtain the acoustic pressure in points of interest along the duct model. The sound pressure data from the FE model was processed in Matlab® to estimate the net acoustic power ($W_{net1}$) of the duct. After that, all the previous methodology should be repeated in order to estimate the net acoustic power of the system with the HR. So, in this step it is necessary to include the HR geometry in the duct model according to the dimensions and boundary conditions illustrated at Fig. 5. All the dimensions are in meters (SI units).

In the case of the HR/duct system, the damping of the HR neck needs to be considered. It is included by using the boundary admittance coefficient. In Ansys®, it can be add by using the material property parameter (MU), which range from 0 to 1. If MU=0 no sound absorption exists and the maximum value considers total sound absorption like an anechoic termination. Once the HR neck is a region with non-planar sound waves the mesh in this subsystem needs to be refined in order to model the sound field variations with accuracy. The size mesh in the HR volume was reduced in a proportion of three when compared with the duct mesh. The complete model duct/HR has 42184 elements (approximately two times more when compared with only the duct model).

For the sensibility analysis the net acoustic power ($W_{net2}$) of the system HR/duct was calculated considering five different localizations of the muffler mounted in the duct, keeping the same system geometry and boundary conditions. The first place was 0.5 m from the left end and the others positions was incremented of 0.5 m from this first. The IL was estimated for these five positions of the HR. The obtained results led to the GA optimization step.

Figure 7 illustrates the optimization methodology of the HR position along the duct. As indicated by the flow chart the GA algorithmic starts the optimization with a random population which represents the feasible positions of the HR along the duct bounded by the constraints defined by the user. In order to obtain the objective function value (IL) the GA implemented in Matlab® communicates with the software Ansys® which provides the sound pressure in the interest points of the duct system. After the fitness evaluation the algorithm computes the crossover and mutation parameters and a new loop is performed until a number of generations to exceed a pre-selected value.

It is easy to conclude that the FEM is used only to model the acoustic system and to supply the optimization algorithm with the pressure data values. In Fig. 7 the gray charts represents activities executed by Ansys®.

The objective function evaluates the IL in the frequency of 219 Hz which is the resonance frequency of HR considered obtained from the classical formula (Kinsler, 1982).

In order to evaluate the optimization technique used in this methodology, four GA input parameters, including population size ($pops$), maximum generation ($genno$), crossover ratio ($cr$) and mutation ratio ($mr$) were varied step by step during optimization. The combination between them resulted in twenty five sets of GA parameters which...
were simulated in a notebook ASUS Intel Core i7 using 4 processors in parallel. The simulation time varies about 15 to 24 hours, according to the defined number of generations.

Finally after the optimization step, evaluations of the IL in points closed to the optimal positions were performed in order to estimate the IL variations from deviations caused for practical applications like mounting tolerance. In these cases, the IL was calculated at 0.05m and 0.03m before and after the optimal position given by the GA.

![Flow chart of the optimization methodology.](image)

**FIGURE 7.** Flow chart of the optimization methodology.

**RESULTS AND DISCUSSION**

The results of the sensibility analysis are showed in Fig. 8 and 9. Each figure depicts the acoustic performance (IL) of the system duct/RH for some positions of the HR mounted in the duct wall. The legend shows the position of the HR center from the left end in the z coordinate. As noted, the IL curve and its maximum value modify according to HR localization, showing the importance of an optimization procedure in order to achieve the best position in order to obtain the maximum acoustic attenuation.

![IL for the HR positions: 0.5 m and 1 m (Z coordinate).](image)

**FIGURE 8.** IL for the HR positions: 0.5 m and 1 m (Z coordinate).
FIGURE 9. IL for the HR positions: 1.5 m and 2.5 m (z coordinate).

The HR has its geometry defined according to a frequency of interest which is the HR resonance frequency (for this study 219 Hz). The GA evaluated the IL maximizing it at 219 Hz. The obtained results are summarized in Tab. 1. As indicated in Tab. 1, seven configuration of GA parameters converge for optimal positions (grey lines), getting an IL of 52.9 dB at 219 Hz corresponding to the optimal position z=1.59m. Note that the optimal position cannot be achieved with a population size (pops) lower than 80 or when the mutation ratio (mr) is defined as 1.

TABLE 1. GA results for each set of GA parameters

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<th>cr</th>
<th>Position [m]</th>
<th>IL [dB]</th>
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<td>52.99</td>
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The estimate of the IL in positions which considers mounting errors (0.05m and 0.03m in z coordinate before and after the optimal locations) is presented in Tab. 2. The results show that the acoustic performance of the muffler is slightly changed due to deviations mounting which exist in real/practical applications. These results were already expected in view of the sensitivity analysis presented above. They reinforce the strongly dependency between IL and duct position.
### TABLE 2. Evaluation of IL in the optimal positions proximities.

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<th>(+) 0.03</th>
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### CONCLUSION

The present paper shows that the muffler position in a duct can change its acoustic performance (insertion loss). This fact justifies the using of the GA technique for noise control using Helmholtz resonator in a duct. Good results were obtained with this optimization technique implemented in Matlab® associated with the FEM (Ansys®). Twenty five sets of GA parameters are tested varying them step by step during. The HR optimal positions were reached with the appropriate set of GA parameters. Mounting errors in the assembly were considered and the results show that the IL can slightly change for small position deviation. This work contributes as an important and necessary step in the methodology for passive silencers design.

### ACKNOWLEDGMENTS

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### REFERENCES


