5aNSa5. Outdoor sound propagation for high-speed moving sources
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This paper deals with modeling of sources in motion in time-domain solvers. In the context of transportation noise, acoustic sources are complex. Indeed, they are in motion, and they are generally not compact. Equivalent point sources are often used to simplify the problem. Heuristic methods are then applied to handle acoustic propagation over complex sites. Besides, time-domain solutions of the linearized Euler equations have proved to be an attractive approach to study outdoor sound propagation, and can then be used to validate these models. However point sources in arbitrary motion are difficult to account for in these approaches. Distributed volume sources can be used instead. First, influence of the spatial support of the source on the acoustic field is investigated. The case of a harmonic source moving at a constant speed is studied. Then, simulations of a broadband moving source above a rigid ground surface in a three-dimensional geometry are presented, and ground effect is highlighted.
**INTRODUCTION**

Time-domain methods are well suited for applications in transportation noise studies. Indeed, they can account for realistic meteorological conditions (see e.g. Van Renterghem and Botitteloooren (2003)), effects of surface impedances (Cotté et al., 2009; Dragna et al., 2011) and topography (Blumrich and Heimann, 2002; Dragna et al., 2010). Among the remaining issues is the modeling of complex sources. Besides, in existing propagation studies, complex sources are often described as a sum of more simple equivalent sources. For instance, in railway noise applications, existing models are based on description of the train by a line source or by a set of point sources. Heuristic propagation models based on ray-tracing or parabolic equation computations have been proposed to treat long range propagation (see e.g. Cotté et al. (2007)), and can then be validated against reference time-domain methods.

However, it is not straightforward to implement moving point sources in time-domain solvers, because, for a given trajectory, the source must be located on a grid node at each iteration. To avoid these issues, volume sources are used in this paper. For a fixed source, it is known (Crighton, 1975; Dowling and Ffowcs Williams, 1983) that the spatial support modifies the acoustic field by modulating the amplitude of the pressure by the Fourier transform of the volume source distribution. For a moving source, this Fourier transform is also expected to have an influence on the acoustic pressure field.

In a first section, the effects of the spatial support of a volume source moving at constant speed on the acoustic field are studied. Results obtained with a solver of the linearized Euler equations for a 2-D geometry are compared to that obtained with the derived analytical solution. In a second section, numerical simulations of a broadband source moving above a rigid ground surface are presented. Results are compared to those obtained with an analytical solution.

**EFFECT OF THE COMPACITY OF A MOVING SOURCE ON THE ACOUSTIC FIELD**

In this first section, the effect of the spatial support for a harmonic moving volume source is investigated, with both analytical and numerical solutions. The schematic of the problem is represented in Fig. 1.

![Figure 1: Volume source moving at constant speed $V_0$.](image)

**Analytical Solution in Geometrical Far-Field**

The wave equation for the velocity potential is given by:

$$\frac{1}{c_0^2} \frac{\partial^2 \phi}{\partial t^2} - \Delta \phi = S(\mathbf{x}, t),$$

(1)
where $t$ denotes the time and $c_0$ the sound speed. The source term $S(x,t)$ is written as:

$$S(x,t) = Q(x - V_0 t) \exp(-i\omega_0 t), \quad (2)$$

with the angular frequency $\omega_0$, the spatial support $Q$, and the constant source speed $V_0$.

Let us denote by $B$ a characteristic length scale of the source spatial support and by $\hat{Q}$ the spatial Fourier transform of $Q$. In the retarded time coordinate system $(\text{Re}, \cos \theta_e)$ whose origin is the center of the source spatial support at the retarded time, it is shown in Dragna et al. (2012), that the solution in geometrical far-field, i.e. $R = |x - V_0 t| \gg B$, is:

$$\phi(x,t) = \exp(-i\omega_0 t) \frac{\exp(ik\text{Re})}{4\pi\text{Re}(1 - M_0 \cos \theta_e)} \hat{Q}(k_D), \quad (3)$$

where $M_0 = |V_0|/c_0$ is the Mach number. The wavenumber $k_D$ is given by:

$$k_D = \frac{k}{1 - M_0 \cos \theta_e \text{Re}}. \quad (4)$$

It is directed from the source to the receiver at the emission time. The analytical solution in Eq. (3) is the product of two terms: the analytical solution for a point source (see e.g. Morse and Ingard (1968)) and the Fourier transform $\hat{Q}(k_D)$. It shows, that for a harmonic volume source moving at a constant speed, the spatial support of the source causes an additional amplification factor due to its spatial Fourier transform.

**Comparison with a Numerical Solution**

An illustration of the effect of the source compactness is proposed in this section. To do so, a 2-D numerical simulation of a harmonic source moving at a constant speed is performed. The linearized Euler equations are solved using finite-difference schemes developed in the computational aeroacoustics community. Details on the solver can be found in Dragna et al. (2011). The source is moving along the $x$-axis at speed $V_0 = 100$ m.s$^{-1}$. Therefore, the Mach number is equal to 0.3. At $t = 0$ s, the source is located at $x = 0$ m and $z = 0$ m. The source spatial support is a Gaussian distribution:

$$Q(x) = \frac{S_0}{(\sqrt{\pi}B)^n} \exp\left(-\frac{x^2}{B^2}\right), \quad (5)$$

where $n$ is the dimensionality of the problem and $S_0$ is a normalization parameter set to 1 m$^n$.s$^{-1}$. The parameter $B$ is chosen as 0.36 m. Two frequencies are investigated. In the first case, $f_0$ is equal to 50 Hz. The parameter $kB = 0.33$ is small compared to 1. Hence, the emission time is assumed to be the same for all elementary volume sources, and the source is called compact (see e.g. Crighton (1975)). In that case, it is expected that the source behaves like a point source. In the second case, the source frequency is set to $f_0 = 300$ Hz, and the source is not compact anymore.

Pressure waveforms obtained at the receiver at $x = 0$ m and $y = 15$ m are plotted in Fig. 2. For the frequency $f_0 = 50$ Hz, it is observed in Fig. 2 (a) that the absolute value of the acoustic pressure is greater when the source approaches the receiver than when it recedes from the receiver. This classical behavior is due to the convective amplification. In that case, the envelopes calculated with the analytical solutions for a point source and for the volume source using the geometrical far-field approximation are almost indistinguishable. For the frequency $f_0 = 300$ Hz, the time behavior of the acoustic pressure is reversed compared to the previous case. Indeed, it is seen in Fig. 2 (b) that the absolute value of the acoustic pressure is greater when the source recedes from the receiver. In the previous section, it has been noted that the source spatial support induces an other amplification factor. In that particular case, it compensates the convective...
amplification factor. This is confirmed by the envelopes of the analytical solutions. The envelope using Eq. (3) coincides with the numerical solution, while the envelope calculated with the analytical solution for a point source does not correspond at all with the numerical solution.

Besides, it can be noted from Eq. (3) that in geometrical far-field, the source spatial support plays only a role on the amplitude of the solution. The wave phase is the same than that of the solution for a point source. It is shown in Dragna et al. (2012) that the Doppler shift is retrieved in the results of the numerical simulation for both cases.

**TIME-DOMAIN SIMULATIONS OF MOVING SOURCES ABOVE A GROUND SURFACE**

This section deals with numerical simulation of a broadband source moving above a flat rigid ground. Few studies have been published investigating moving sources above a ground surface. Among them, one can cite the studies of Norum and Liu (1978) and Li et al. (1998), that have proposed an analytical solution for a point source moving at a constant speed and a constant height above a ground of finite impedance in acoustic far-field, based on a Lorentz transformation. For more general configurations, analytical solutions based on a heuristic approach (Attenborough et al., 2007) have been proposed. In this section, acoustic pressure field obtained above a rigid ground is studied. Similar simulations have been performed for an impedance ground surface in Dragna et al. (2012). A schematic of the problem is depicted in Fig. 3.

**FIGURE 3:** Source moving above a flat impedance surface along x-axis at constant speed $V_0$ and at a constant height in a 3-D geometry.
A broadband source is considered. The source term is:

\[ S(\mathbf{x}, t) = Q(\mathbf{x} - \mathbf{V}_0 t) s(t). \] (6)

The source signal \( s(t) \) is constructed so that its power spectral density is Gaussian:

\[ \text{PSD}[s](f) = s_0 \exp \left[ -2 \frac{(f - f_c)^2}{f_b^2} \right], \] (7)

where \( s_0 \) is a normalization parameter set to 1 s. The central frequency \( f_c \) is chosen as 300 Hz. The parameter \( f_b \) controls the decrease of the Gaussian function and is set to 100 Hz. The frequency content of the source is significant for frequencies between 200 Hz and 400 Hz. The spatial support \( Q \) is Gaussian (see Eq. (5)). To avoid non-compacity effects, the parameter \( B \) is set to 0.1 m. The maximal value of \( kB \) obtained for \( f = 400 \) Hz is 0.7. Therefore, it is expected that the source behaves like a point source. Five simulations with different white noise realizations are performed. The time-frequency maps presented below are obtained by averaging the results.

![Figure 4](image)

**Figure 4:** Instantaneous power spectral density of the acoustic pressure in dB/Hz versus time and frequency at a receiver located (a) at \( x = 0 \) m, \( y = 5 \) m and \( z = 3 \) m and (b) at \( x = 0 \) m, \( y = 25 \) m and \( z = 3.5 \) m. Results are obtained from the numerical solution.

The instantaneous power spectral densities obtained at two receivers located at \( x = 0 \) m, \( y = 5 \) m and \( z = 3 \) m and at \( x = 0 \) m, \( y = 25 \) m and \( z = 3.5 \) m are plotted versus time and frequency in Fig. 4. The reference pressure is \( p_{ref} = 2.10^{-5} \) Pa. The Doppler shift is clear. Note that, as for a fixed source, strong destructive and constructive interferences appear. The line corresponding to the first destructive interference is represented in Fig. 4.

Results from the numerical solution are compared to that of an analytical solution. For a rigid ground, the acoustic pressure is the sum of the contributions from the source and from the image source. Assuming that the source is compact, the instantaneous power spectral density of the acoustic pressure is given by:

\[ \text{PSD}[p](\mathbf{x}, \omega, t) = \rho_0 \omega^2 \text{PSD}[s](\omega) \left| \frac{\exp(i k R_{e,1})}{4\pi R_{e,1}(1 - M_0 \cos \theta_{e,1})^2} + \frac{\exp(i k R_{e,2})}{4\pi R_{e,2}(1 - M_0 \cos \theta_{e,2})^2} \right|^2, \] (8)

where \((R_{e,1}, \cos \theta_{e,1})\) and \((R_{e,2}, \cos \theta_{e,2})\) are the retarded time coordinates with the origin set respectively to the position of the source and of the image source at the emission times. Time-frequency maps obtained with this analytical solution are shown versus time and versus the Doppler frequency \( f_D = f/(1 - M_0 \cos \theta_{e,1}) \) in Fig. 4. It can be seen that they are very close to the time-frequency maps obtained with the numerical solution. This confirms the feasibility of time-domain simulations of moving sources.
CONCLUSION

In this paper, modeling of moving sources in time-domain solver is examined. To do so, volume sources are considered. First, their effect on the acoustic field is studied. In geometrical far-field, it is seen that, compared to the analytical solution for a point source, an additional amplification factor linked to the Fourier transform of the source spatial support is introduced. The Fourier transform is evaluated at a wavenumber that depends on the time and on the source speed. For a compact source, the behavior is similar to that of a point source. For a non-compact source, it is dramatically modified. More energy can be radiated in the backward direction than in the forward direction. This behavior is retrieved with results of numerical simulations performed with a solver of the linearized Euler equations. At last, simulations of a broadband moving source above a rigid ground surface in a three-dimensional geometry are performed. As for a fixed source, strong destructive interferences are observed. Results are compared favorably with an analytical solution.

The feasibility of numerical time-domain simulations of moving sources in a three-dimensional geometry is demonstrated. For transportation noise applications, engineering models can then be validated against results obtained with time-domain solvers. These solvers can also be used to consider several scenarios, such as a train pass-by in a complex railway site. Moreover, to account for more realistic aerodynamic sources (such as those linked to the pantograph), coupling with LES calculations can be performed. This approach will be followed in future work.

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REFERENCES


