1pSPc18. Optimized hermetic transform beam-forming of acoustic arrays via cascaded spatial filter arrangements derived using a chimerical evolutionary genetic algorithm

Harvey Woodsum* and Christopher Woodsum

*Corresponding author’s address: Nergetic System Dynamics, 70 Campbell Road, Bedford, NH 03110, hwhwoodsum@gmail.com

Hermetic Transforms are complex matrices, having particular mathematical properties, that have recently been introduced to the field of acoustic array signal processing. Cascade sequences of Hermetic Transform Matrices have been shown to have direct utility in accomplishing spatial filtering and beam-forming of data from oversampled arrays. The present work details the adaptation of techniques previously shown to be successful in the processing of radio-wave phased-array antenna systems, [Woodsum et al, 16th International Conference on Cognitive and Neural Systems, 2012] to the processing of sampled digital data from acoustic arrays. As in our earlier work, the use of a Chimerical, Evolutionary, Genetic Algorithm having a "feature seeking" fitness function, is retained, for deriving optimal multiplicative arrangements of non-commuting elemental transform matrices. Each elemental matrix represents a spatial "pole" or "zero", and cascaded arrangements of these are utilized to create a desired spatial pattern response for the array. In terms of acoustic reception, the technique is especially successful in dealing with null placement in order to mitigate large numbers of interfering signals, and in achieving super-resolution beams for arrays that are "acoustically small". Experimental results are compared to theoretical predictions of performance.
INTRODUCTION
Optimal Spatial filtering

The general concepts behind conventional approaches to optimal spatial filtering are founded upon statistical treatments of signal detection and estimation\(^1\). Our approach to this problem differs significantly from these canonical techniques, and so for completeness, it is necessary to review here, the basis and background of our methods. In terms of notation, we represent sampled outputs of \((M)\) acoustic sensor array elements in terms of the analytic signal, i.e the output of each of elements are complex time series,

\[
X(n) = \{ X_k(n) \}, k = 0, 1, 2, \ldots, (M-1)
\]

where \(X\) is the Vector Sensor time series, the index “\(k\)” refers to the specific individual sensor element(s), and \(n\) refers the \(n\)-th time snapshot from each sensor. Time-synchronous sampling is assumed. Each time snapshot generates a single vector of complex data, taken to be arranged as a column.

The set of vector responses of the \(M\) array elements to a complex sinusoidal signal arriving from a particular direction \(\Omega = \{ \theta, \phi \}\) comprises what is termed an “array manifold” or “calibration matrix”, \(\Sigma\). In our representation, the array manifold is a matrix \(\Sigma(\theta, \phi ; \omega)\) which has a set of \(N\) column vectors with \(M\) rows (the number of rows being equal to the number of array sensor elements). Here \(N\) is the number of discrete angles of arrival \(\{ \Omega_k \}\). The manifold matrix can be obtained either through modeling of the array in question, or through empirical calibration.

We assume the signal is narrow band at a particular frequency \((\omega)\), where the Array Manifold \(\Sigma(\theta, \phi ; \omega)\) is known. This constraint is not overly restrictive, as any signal can be transformed into the frequency domain and then dealt with as sum of narrow-band components. For the case of spatially uncorrelated noise interfering with our signal, it has been shown that the optimum signal to noise ratio for detection is achieved by applying a Matched Filter Beam,

\[
z(n) = \Sigma^H(\theta, \phi ; \omega) \cdot X(n)
\]

The Matched Filter is the Hermitian Conjugate of the Array Manifold vector corresponding to a particular search direction, as indicated by the angles \((\theta, \phi)\). This is a special case of applying a beam-weight vector \((\mathbf{w}^H)\) to form the detection channel

\[
z = \mathbf{w}^H \cdot \mathbf{X}
\]

The case of spatially uncorrelated noise roughly corresponds to a half wavelength \((\lambda/2)\) spacing between elements, for an isotropic noise field. For an array with more closely space elements than \(\lambda/2\), the element noises will have inter-element correlations that are non-zero. This is a critical factor in selecting the “optimal” approach for signal detection and spatial filtering. For the case where the noise does have spatial correlations, one formulation of an optimal algorithm applies first the inverse of the noise covariance matrix, \(\mathbf{R}_n\), and then the matched filter \(\Sigma^H\).

\[
z = \Sigma^H(\theta, \phi ; \omega) \cdot \mathbf{R}_n^{-1} \cdot \mathbf{X}
\]

This form arises in several different ways, particularly as a “maximum signal-to-noise” filter, or a “minimum variance” filter. When normalized to have unity gain in the beam-steering direction, the beam is said to have a “distortionless response”. In this case, we start with weight vector above and apply normalization to achieve unity gain.

\[
\mathbf{w}^H = \left[ \Sigma^H \cdot \mathbf{R}_n^{-1} \cdot \Sigma \right]^{-1} \Sigma^H(\theta, \phi ; \omega) \cdot \mathbf{R}_n^{-1}
\]

This beam forming approach is referred to as MVDR – Minimum Variance Distortionless Response approach\(^2\). One of the problems with this approach is that the noise covariance must be estimated from data, which is difficult in practice for many real-world applications. One of us (HICW) has developed a new transform theory which bears
on the problem of optimal spatial filtering. The general term for these highly useful mathematical constructs is *Kything Transform*, the word deriving from *kythe*, meaning, “to make visible”. *Kything Transforms* are in turn based on the *Hermetic Transform*\(^{16}\) which is used in beam-forming of arrays that are spatially oversampled, i.e., where the inter-element spacings of the array are less than (in many cases much less than) one half the wavelength. Equivalently, the primary noise is assumed to be spatially correlated. In order to motivate the Hermetic Transform\(^{14,15}\), we consider a signal arriving from a particular direction \(\Omega_0 = (\theta_0, \phi_0)\) while the noise arrivals are assumed to be distributed, coming from various directions. It is reasonable to believe that the “ideal”, optimum, beam (spatial filter) in the case of omnidirectional noise has a delta function beam pattern, i.e., a beam which accepts the signal and rejects everything else. It turns out that such a beam can be constructed, but only in a minimum quadratic norm (least-squares) sense. We will briefly show how to derive such a beam. We start with the Matched Filter weight vector result as previously discussed:

\[
z(n) = \Sigma^H(\theta, \phi \mid \omega) X
\]

and construct a set of these for various angles \((\theta, \phi)\),

\[
z = \Sigma^H(\theta, \phi \mid \omega) X
\]

with \(z\) being construed as a sequence of column vectors generated by multiplying a matrix \(\Sigma^H\) times a sequence of column vectors (time snapshots) output from the various array elements of \(X\). The \(k\)th row of the matrix \(\Sigma^H\) is the complex conjugate transpose of the Array Manifold vector corresponding to arrival direction \(\Omega_k = (\theta_k, \phi_k)\). Equivalently, the matrix \(\Sigma\) consists of a set of column vectors, each column vector being the Array Manifold response corresponding to a particular direction of arrival. In matrix form then,

\[
z = \Sigma^H X
\]

The matrix \(\Sigma^H\) can be seen as a transformation that takes the signal from element space into beam space, where the beam directions correspond to the manifold directions. In the case of linear or planar arrays, the matrix \(\Sigma^H\) would correspond to a Discrete Fourier Transform matrix. It is often the case that each sensor element output component is weighted by a weighting or “apodizing factor” prior to matched filtering in order to control sidelobes. This process can be represented as a diagonal weight matrix \(W\) which contains these factors along the main diagonal of the matrix and zeros elsewhere. For example in the case of a linear or planar array, Hanning weights are often used (as in Fourier Transform spectrum analysis). The Matrix form of this equation is thus:

\[
z = \Sigma^H W X
\]

Now, the conditions for creating decomposable Hermetic Transform is created by allowing the weight matrix \(W\) to be an arbitrary matrix, (in general, non-diagonal, and complex) and by imposing an equation which states delta-function beam response criterion. If we assume the input matrix \(X\) is the Array Manifold Matrix itself, then we have a mathematical statement of the requirement for \(W\):

\[
\Sigma^H W \Sigma = I
\]

which is the discrete form of the delta function condition. This equation can be solved for \(W\) and hence for the Hermetic Transform \((H)\) defined to be

\[
H = \Sigma^H W
\]

and

\[
W = (\Sigma \Sigma^H)^+ \Sigma (\Sigma \Sigma^H)^{\dagger} \Sigma \Sigma^H \Sigma (\Sigma \Sigma^H)^{\dagger}\#
\]

where the identity matrix is shown explicitly, (other desired beam responses than \(I\) could be substituted), and the \# symbol indicates the pseudo-inverse (Gelb notation). If the self-noise (with diagonal covariance) at the individual sensor elements is sufficiently large, another term needs to be added, a conditioning matrix \(K\), where \(K\) is given by
\[ K = R_{\text{NN}} \{ R_{\text{NN}} + R_{\text{xx}} \}^\theta \]

Here \( R_{\text{NN}} \) is the self-noise covariance, and \( R_{\text{xx}} \) is the scaled manifold covariance; so that the conditioned Hermetic Transform is given by:

\[ \mathbf{H} = \Sigma^{-1} \mathbf{W} K \]

It should be noted that the Hermetic Transform weights (rows of the Hermetic Transform Matrix) are not dependent on the data (unlike MVDR or other data-adaptive beam-forming) but rather only on the manifold, and self-noise statistics, if conditioning is warranted. This is an essential difference between the Hermetic Transform Beam-Forming approach and classical Adaptive Beam-Forming approaches.

**Non-decomposable Hermetic Transform Weights**

For large manifolds, or large numbers of elements, or both, the matrix operations required to generate the full transform, as outlined above, can be prohibitively difficult to accomplish. Fortunately, there is an equivalent formulation, which is surprisingly simple. It can be shown, that a single beam weight, corresponding to one row of the Hermetic Transform, can be formed using the following expression, for a beam directed in the direction indicated by angles \((\theta, \phi)\)

\[ w^H(\theta, \phi) = \Sigma^{-H}(\theta, \phi; \omega) \{ [D_\theta \Sigma] [D_\phi \Sigma]^H \}^\theta \]

where \( D_\theta \) is an operator that zeros out the column in the matrix \( \Sigma \) corresponding to the direction \((\theta, \phi)\). Essentially, this procedure is a lot like MVDR, except that the interfering “noise” which is being nulled out consists of all angles of arrival, with exclusion of the “look” direction \((\theta, \phi)\). Of course the weight vector can be set up to normalize the gain of the beam to unity in the look direction, just like MVDR. The two equivalent procedures above produce the same identical Hermetic Transform when the weights for all of the directions are calculated. This is because each row of the Hermetic Transform corresponds to a complex weight vector which attempts to make a beam in a particular “look direction” \( \Omega_0 \) that is as close as possible, in a minimum-norm sense, to a delta function beam, \( \delta(\Omega - \Omega_0) \).

**Creation of Spatial Filters Using Hermetic Transforms**

General spatial filters can be created using Hermetic Transforms. The most elemental transform applies the following mathematical operations:

\[ F = \mathbf{H}^H \Delta \mathbf{H} \]

Here the complex filter matrix \( \mathbf{F} \) is of dimension \( M \times M \) (for an \( M \)-element array) and can be interpreted in terms of the beam-transform space operations. First, the Hermetic Transform \( \mathbf{H} \) is applied to an input signal vector (time snapshot from the array) in order to transform the signal into the wave-vector (or beam) domain. The result is then multiplied by a diagonal matrix \( \Delta \) which applies weights to each “beam”. Finally, the pseudo-inverse of the Hermetic Transform is applied to move back to the spatial domain. If we choose \( \Delta \) as the identity matrix, the signal would remain unchanged by applying the filter matrix \( \mathbf{F} \). If we instead choose \( \Delta \) as a matrix with all but one non-zero elements, having a “one” on the \( p \)th row diagonal element, the filter will project out of the signal all of the data except for that part of that signal that lies in the Hermetic Transform beam pointed at the \( pt \) look direction.

We refer to this type of filter transform as a elemental spatial “pole” analogous to a pole in the frequency domain response of a time-series filter. Similarly, if we choose for \( \Delta \) a modified identity matrix which has one diagonal element in the \( p \)th row zeroed out, the transform \( \mathbf{F} \) will project data out the data from one beam (look) direction, making a null in that direction. We refer to this type of transform as an elemental spatial “zero”. By cascading groups of elemental transforms of this type together, one can reasonable construct nearly arbitrary spatial filters which can be optimized to approach a desired spatial response.

The generic type of transform resulting has been termed a *Kything Transform*. It is a general type of spatial filter...
which is constructed by cascading a series of elemental spatial pole-filters and zero-filters that are, in turn, constructed using the Hermetic Transform.

Previously, we have applied this concept to the creation of spatial filters for small aperture antennas\(^5\). Key to this work was the development of an evolutionary Genetic Algorithm, including a Feature-Seeking Fitness Function, developed by one of us (CMW) for achieving optimization of the spatial response. The Genetic Algorithm is used as a means of guided, intelligent search, in order to produce a cascaded arrangement of spatial poles and zeros that achieves the maximum correspondence to a desired spatial response in terms of matching an Objective Function. In this paper we have successfully applied this concept to the spatial filtering of acoustic arrays.

**EVOLUTIONARY GENETIC ALGORITHMS**

**Review of Evolutionary Genetic Algorithms**

Although elemental beam and null matrices can be cascaded to form transform matrices, the fact that these matrices do not commute makes the number of permutations of elemental matrices too large to optimize by manual selection alone. This problem was overcome through the use of evolutionary genetic algorithms\(^5\). Genetic algorithms perform iterative optimization using functionally specified fitness criteria to evaluate the desirability of prospective solutions. Problem-specific variables are organized as “genes” or “chromosomes” within “genomes” that represent particular solutions to the specified problem. This general process performed by the genetic algorithm can be summarized by a series of steps shown in Figure 1. Starting from a predefined starting population (1), the set of genomes are selected for sexual reproduction based on compatibility of genes between prospective genome pairs (2). Compatible pairs of genome are mated (3), and mutations are induced through a random or intelligent process during the reproductive portion of the cycle (4). The fitness of the resultant population of genomes is measured by a carefully specified fitness function (5), after which a specified number of the members of the population are selected for survival and the least desirable genomes are eliminated from the population.

**FIGURE 1.** This figure shows a conceptual representation of an evolutionary genetic algorithm. Predefined genomes are selected for sexual reproduction based on genetic compatibility, are mated and are mutated. The fitness of the resultant population is measured and the least desirable genomes are eliminated from the population.
Implementation of the Genetic Algorithm

We represented beam and null matrices as genes in a “male” and “female” chromosome, respectively, within genomes consisting of a consistent number of both beam and null matrices. Each of the beam matrices would individually select signals from a different specific direction. Similarly, each of the null matrices would individually reject signals from a particular unique direction. Sexual reproduction between random pairs of genomes was accomplished by selecting the best genes within each chromosome and maintaining the total number of genes in each chromosome. In order to improve the quality of gene selection, a probability density function (PDF) was created for the fitness of gene pairs by taking an average fitness measurement of each gene and its possible adjacent genes and normalizing each of these PDFs. This allowed improved selection for an optimal gene sequence. It was quickly discovered that the factor most limiting the value of solutions generated with the genetic algorithm was the specification of desired results through the fitness function. Therefore, a unique form of fitness function, which is referred to as a feature-seeking fitness function, was created to solve this problem.

Introduction of a Feature-Seeking Fitness Function

The problem which was specifically explored required highly effective signal nulling in only one specific direction. However, conventional fitness functions based on the angle-dependent standard deviation of the solution’s beam pattern from the specified ideal beam pattern produced results with only partially effective nulls since the importance of this feature was not adequately specified. A fitness function, referred to as a feature-seeking fitness function was created to address this problem. This fitness function fit beam patterns to an angle dependent polynomial in Cartesian coordinates (for simplicity). The function was then differentiated several times and a vector was created for each point to represent the value and successive derivatives of the function at each point. Each beam pattern from the population of prospective solutions was compared to the specified beam pattern by taking the squared error of the derivative vectors for each angle and creating a deviation matrix. Deviation values were normalized within each row corresponding to a unique derivative order to give all derivatives equal weight, and each column corresponding to an angle in space was subsequently normalized to give each angle equal weighting. Although this is somewhat arbitrary, it is difficult to optimally weight the angles and beam pattern derivatives to gain the desired characteristics of solutions without knowing the specific application of the genetic algorithm, and it was desired to have an algorithm that was robust to changes in application. The results produced by application of the feature-seeking fitness function were greatly improved in comparison to the conventional fitness function.

Creation of Chimeric Hybrids

In addition to creating a novel form of fitness function, a second unique approach was taken to optimization. Once the genetic algorithm had generated a set of prospective solutions, these genomes were converted into final transform matrices by multiplying the genes in each genome together into a single transform matrix. A novel approach referred to as chimeric optimization was used to find an optimal weighted linear combination of these resultant matrices. The fitness of each transform matrix was evaluated and a probability density function was created for selecting the genomes for use in hybridization.

EXPERIMENTAL RESULTS

A small inexpensive air-acoustic array was utilized to collect signal data for analysis. The data collection (photo, below) consisted of a five-element array of boosted capacitive microphones; a multi-channel synchronous data acquisition system derived from a Data Translations 94816UM six-channel capacity A/D converter, and a Dell laptop computer with data analysis software running under MATLAB™.
FIGURE 2. Air-acoustic array used for collection of the data analyzed in this paper. Note dollar bill for size comparison.

The A/D converter rate was set at 10 KHz, and the data was collected from a nearly omnidirectional 500 Hz source (CW) which was positioned at a variety of angles at constant distance from the receiving array. The array, consisting of four elements located around an approximately 7.5 cm diameter circle, plus one center element, was clearly much much smaller than the acoustic wavelength, thus satisfying the spatial oversampling requirement needed to gain performance from the Hermetic The data was converted to the frequency domain via FFT, with phase and amplitude at each element being measured. This set of 5 x N data points at the single frequency of 500 Hz comprised the Array Manifold for this array. A specification was made for a very deep null sector (> 80 dB below peak) over a 20 degree arc, with uniform spatial response (normalized to unity) desired elsewhere. The manifold was input to the Genetic Algorithm in order to generate a Kything Transform Spatial Filter with approximately the desired response. A model order of 8 (2 spatial poles and 6 spatial zeros) was chosen by trial and error. The Genetic Algorithm produced a final population after many iteration cycles of reproduction, mating, mutation, and down selection via the fitness function matching, that had eight surviving 5x5 transforms for null sector generation. These were hybridized using the Chimeric Algorithm and applied to real data at 500 Hz in order to measure the spatial response in relation to the desired response. The results are shown in the figure immediately below.

FIGURE 3. Spatial filter response shown in blue (data points) as compared to the desired response shown in red.

The relative response in dB is shown over an arc of 110 degrees where the spatial response was measured. The large depth and shape of the null can be clearly seen, and the response generated by filtering generally followed the desired response. The same technique can be used to generate beams with a specified response.
The figure below shows a measured beam pattern in dB which was generated with the same genetic algorithm.

Figure 4: Measured Spatial Beam Pattern corresponding to an “ideal” beam specification at 50 degrees bearing

The desired response, specified to the Genetic Algorithm in this case, was an “ideal beam” (unit response in the beam, zero response elsewhere) aimed at 50 degrees, and having a 20-degree beam width.

CONCLUSIONS

Our preliminary work with air-acoustic arrays illustrates the potential utility of the Kything Transform approach as well as advantages offered by the Hermetic Transforms. Because the Hermetic Transform offers the potential of generating sharp beam responses with small physical arrays, and which have high gain against ambient noise, it is anticipated that exploitation of these technique will gain popularity.

When combined with powerful computational tools such as the Chimerical Hybrid Evolutionary Genetic Algorithms described here, the general field of array spatial filter design optimization is expected to receive more attention.

Finally, it is worth noting that Kything Transforms for null generation with an N-element array have N*(N-1) degrees of freedom rather than (N-1) in traditional array processing, thus offering the potential of generating more spatial nulls than the number of sensors in the array. This topic is expected to be the subject of a future paper.
REFERENCES


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