4aSCb22. Flow development in the uniform glottis and viscosity effects

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Thirty-two pressure distributions at minimal diameters of \(d = 0.005, 0.0075, 0.01, 0.02, 0.04, 0.08, \) and \(0.16 \text{ cm} \) have been measured at a number of transglottal pressures of interest for phonation. Care is taken to identify those portions of the pressure distributions within the glottis that include substantial regions of uniform decrease with axial distance. These portions are further examined to identify their components that have a linear dependence on the volume velocity and those that have a quadratic dependence on the volume velocity. An analysis based on the Navier Stokes equation creates a natural framework for investigating corrections to the parabolic profile of fully developed flow, which leads to the Poiseuille formula. For glottal diameters between \(0.0075 \text{ cm} \) and \(0.02 \text{ cm} \) the Poiseuille formula is a good approximation. Overall, an inverse \(2.59\) power law to describe the diameter dependence of the linear coefficients is found to be superior to the inverse cube dependence of the Poiseuille formula. Glottal flow resistance is used as a means of comparing the accuracy of the two power laws.
INTRODUCTION

One of the early experiments that studied pressure-flow relationships in a context of relevance for phonation was carried out by van den Berg, Zantema, and Doornenbal (BZD), who constructed a physical model of the human larynx with a uniform glottis made to scale from dental material. BZD summarized the results of their measurements in a formula whose essential features may be conveyed in a general form \( \Delta P_{tr} \), that is,

\[
\Delta P_{tr} = \frac{(k_{ent} - k_{ex}) \rho U_{g}^2}{2 l_{g} d^2} + 12 \frac{\mu \Delta x}{l_{g} d^3} U_{g},
\]

where \( \rho \) and \( \mu \) are the density and viscosity of the air, respectively, \( U_{g} \) is the glottal volume flow rate, and \( l_{g}, d, \) and \( \Delta x \) are the glottal anterior-posterior length, diameter (often referred to as the width), and thickness, respectively. The pressure coefficient \( k_{ent} \) describes the effects responsible for the loss of energy at the glottal entrance, and the exit coefficient \( k_{ex} \) describes the effects of pressure recovery. The last term in Eq. (1) describes the Poiseuille effect for viscous flow. Entrance losses were summarized by van den Berg et al. with a coefficient of 1.375, and the value \( k_{ex} = 0.5 \) was chosen to describe the effects of pressure recovery.

In their seminal work on the two-mass model of the vocal folds, Ishizaka and Flanagan adopted the value \( k_{ent} = 1.375 \), but replaced the generic value of \( k_{ex} \) by a formula derived from momentum conservation at the glottal exit, which gave values for the exit coefficient considerably smaller than 0.5. Ishizaka and Flanagan also chose the inverse cube dependence of the second term of Eq. (1) to represent viscous effects within the glottis. However, Ishizaka and Matsudaira had realized that the Poiseuille term in Eq. (1) may not be an adequate approximation in many circumstances, since they had studied the development of the cross-channel velocity profile, as the flow moved along the glottal channel. A schematic of their results is shown in Fig. 1.

**FIGURE 1.** Schematic diagram of the velocity profile and growth of the boundary layer (dotted lines) along the direction of flow in the uniform glottis. A uniform velocity field enters from the left and develops into a parabolic profile, provided that the channel is long enough. Ishizaka and Matsudaira’s work was based on the solution of the Navier-Stokes equation.

M5 PRESSURE DISTRIBUTIONS

Sufficient pressure data within and near the glottis were not available to examine the implications of Ishizaka and Matsudaira’s calculation until the measurements obtained with the scaled-up (by a factor of 7.5) laryngeal model M5 became available. Two of the sets of pressure distributions measured with model M5 are shown in Fig. 2. Pressure distributions for the uniform glottis were also taken at minimal glottal diameters of \( d = 0.005, 0.02, 0.04, 0.08, 0.16, \) and 0.32 cm for a range of pressures from 1 cm H\(_2\)O to 25 cm H\(_2\)O, for a total of 35 pressure distributions.
FIGURE 2. Pressure distributions measured with static model M5 within and near the glottis. The first 5 taps are located on the inferior vocal fold surface, tap 6 is at the glottal entrance, and tap 13 is at the glottal exit. The transglottal pressures in cm H\textsubscript{2}O are given for each distribution (1 cm H\textsubscript{2}O = 98 Pa). The data of part (A) were taken with a uniform glottis with a minimal diameter of 0.0075 cm, and the data of part (B) were taken with a uniform glottis with minimal diameter of 0.01 cm.

The two pressure distributions of Fig. 2 show clearly that the M5 pressure distributions have a sufficient number of taps to separate the local behavior of the pressures in the subglottal region (taps 1 to 5) from that in the intraglottal region (taps 6 to 13). The pressure drop between the upstream tracheal reference (tap 0, not shown) and tap 6 at the glottal entrance may be used to define the entrance loss coefficient, and Fig. 3 is based on the table of values of these coefficients reported in Reference 6. From Fig. 3 it is clear that the entrance loss coefficients become quite large at small diameters and that the coefficients at small diameters vary rapidly with changes of these variables.

FIGURE 3. Entrance loss coefficients as a function of the minimal glottal diameter and the transglottal pressure.
Each of the distributions in Fig. 2 has a well-defined region (taps 6 to 11) where the pressure drop increases almost uniformly (because of the nearly uniform decrease in the pressure there). The slopes of the pressure drops for a given transglottal pressure are steeper for \( d = 0.0075 \) cm than for \( d = 0.01 \) cm. The M5 pressure distributions for minimal diameters \( d = 0.005, 0.02, 0.04 \) cm exhibit the same behavior. The M5 pressure distributions for \( d = 0.08 \) and 0.16 cm exhibit similar behavior, but over a smaller interval of the axial distance. These properties are what one would expect when viscous effects dominate the behavior of the pressure changes within a narrow channel. Only the M5 pressure distributions at \( d = 0.32 \) cm have no interval along the axis with linear behavior comparable to that of Fig. 2.

Further elucidation of the viscous effects within the glottis requires the help of the Navier-Stokes equation for one-dimensional flow \( v_x \) along the glottal axis shown in Fig. 1,

\[
\rho v_x \frac{\partial v_x}{\partial x} = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right). \tag{2}
\]

The second-derivative term is the source of possible corrections to the Poiseuille formula that are linear in the volume velocity \( U_g \). Thus, the pressure gradients calculated from the linear parts of the intraglottal pressures in the M5 distributions described above were analyzed for linear and quadratic dependence on the volume velocities \( U_g \). In most cases the fits were excellent. Dividing the part of the pressure gradient that depends linearly on the volume velocity by this velocity yields the linear coefficient of the pressure gradient at each minimal diameter, whose values are shown in the log-log plot of Fig. 4. The calculated points lie almost on a straight line, which suggests a power law for the dependence of this coefficient on the minimal diameter. However, the trendline shows that the diameter dependence of this power law is not the inverse cube dependence of the Poiseuille law, but closer to inverse 2.59 power. Thus, Fig. 4 shows that the Poiseuille term of Eq. (1) overestimates viscous effects for small diameters and underestimates viscous effects (by as much as a factor of 3) for large diameters. For diameters between 0.0075 cm and 0.02 cm, the Poiseuille formula is a good approximation.

![Figure 4](image-url)

**Figure 4.** Dependence of the linear coefficient of the pressure gradient on the minimal glottal diameter.

It is worth noting that Cisonni et al.\cite{cisonni2013} found sizable discrepancies between their pressure-flow measurements for steady flows with their replica of the uniform glottis and results calculated from the Poiseuille formula, as one would expect from the results of Fig. 4.

**NEW FORMULA FOR FLOW RESISTANCE**
The results of Fig. 4 suggest that it should be possible to emend the second term of Eq. (1) and thus obtain an improved formula for the flow resistance (transglottal pressure divided by the volume flow rate), namely,

$$R_t = \frac{\Delta P_t}{U_g} = \frac{(k_{ent} - k_{ex}) \rho U_g}{2 F_g d^2} + 12 \frac{\mu \Delta x}{l_g d^3} \left( \frac{d}{d_{Poise}} \right)^\lambda,$$

(3)

where $\lambda \approx 0.41$, the difference between the exponent of the power law fit of Fig 4 and the inverse cube dependence of the Poiseuille law, and $d_{Poise} \approx 0.01cm$ is the diameter at which the Poiseuille formula is the best approximation to the linear coefficients of Fig. 4. Our first tests of the “improved” formula of Eq. (3) involve the application of Eq. (3) to each of the 35 pressure distributions that comprise the M5 data set for the uniform glottis.

**FIGURE 5.** Calculated and measured flow resistances for the M5 pressure distributions for the uniform glottis at the four smallest diameters (A) and the four largest (B).

These calculations require the entrance loss coefficients of Fig. 3 and the exit coefficients of Table II of Reference 6. They are compared with the measured pressure-flow ratios in Fig. 5 and with results obtained from the BDZ formula, which assumes $k_{ent} = 1.375$ and $k_{ex} = 0.5$ and $\lambda = 0$ in Eq. (3) (to get the Poiseuille formula). The new
formula gives much better fits to the measured flow resistances at \( d = 0.02, 0.04, \) and 0.08 cm. The results are comparable at \( d = 0.0075 \) and 0.16 cm. The new formula is also better at \( d = 0.005, 0.01, \) and 0.32 cm. It is interesting to note that the BDZ formula overpredicts the glottal resistance at \( d = 0.005 \) cm and underpredicts the glottal resistance at \( d = 0.01, 0.02, 0.04, \) and 0.08 cm, the trend that one would expect for the BZD formula on the basis of its comparison with the M5 derived data of Fig. 4. Additional comparisons with the pressure-flow data from other static vocal fold replicas\(^{2-9,11} \) will also be presented.

**DATA FROM SELF-OSCILLATING REPLICAS**

After formulating the surface wave model, Titze\(^ {12} \) derived a formula for the phonation threshold pressure of a uniform glottis of diameter \( d \) (halfwidth = \( d/2 \)), thickness \( \Delta x \), and length \( l_g \) that may be expressed

\[
P_{th} = \frac{(k_{ent} - k_{ex}) Bc d}{2l_g(\Delta x)^2},
\]

where \( c \) is the speed of the surface wave, and \( B \) is the damping factor for the motion of the vocal fold. He approximated the difference \( k_{ent} - k_{ex} \) by a transglottal pressure coefficient \( k_t \), which was assumed to be constant and approximately equal to 1.1. A consequence of Eq. (4) is that the threshold pressure should approach zero for a very narrow glottal channel, a feature that none of the Chan and Titze data sets\(^ {13} \) possess, as illustrated by the examples of Fig. 6. The key to resolving this discrepancy is to acknowledge the rapid variation of the entrance loss coefficient with diameter at small diameters\(^ {14} \). From the low-pressure side of the surface of Fig. 3, one sees substantial increases in the entrance loss coefficient as the diameter becomes small, which suggest an inverse relationship of the form \( k_{ent} = 2E/d + F \). (In most cases \( k_{ex} \ll k_{ent} \).) Under such assumptions Eq. (4) would be replaced by

\[
P_{th} = P_0 + B^* \frac{cd}{2l_g(\Delta x)^2},
\]

where \( B^* = B F \) and \( P_0 = B c E/l_g(\Delta x)^2 \) are parameters to be fit to the Chan and Titze experiments. The advantage of allowing \( k_{ent} \) to vary is readily apparent in Fig. 6. It is worth noting that the Eq. (5) fits to the data were achieved without a viscous effects term, and it is difficult to distinguish between such a term and the effects of \( k_{ent}(d) \), if there is no independent means of determining \( k_{ent}(d) \), as there was for the M5 pressure distributions. It is likely that such a term would produce an increase in the threshold pressure if the glottal half-width becomes small enough. Such a trend was seen in earlier experiments by Titze, Schmidt, and Titze, but the trend was not reproduced in the 2006 experiments of Chan and Titze. Further measurements would be helpful to address this challenging question.
The entrance loss coefficients, the exit coefficients, and the formula for viscous effects described above can be used with any finite element or lumped element model of the vocal folds, whose driving forces are derived from glottal flow. In Reference 16, a linearized version of the classic model of Ishizaka and Flanagan was used to show that such a model would give a formula similar to Eq. (5) for phonation threshold pressure, namely,

$$ P_{th} = \frac{E + Fd/2}{2I/d_t} - \phi, $$

where $\phi$ is the glottal flow resistance, and $d_t$ is the glottal thickness associated with the lower oscillator. The quantity $\phi$ is a complicated function of the spring constants and damping parameters. In reference 16, the value of $\phi$ was determined for the particular implant under the silicone membrane and the values of $E$ and $F$ were assumed to be the same for all of Chan and Titze’s experiments. Moreover, the relationship developed by Titze and Story between the spring constants $k$ and the elastic shear modulus $G$,

$$ k = \frac{Gl/\Delta_x}{D}, $$(7)

where $D$ is the depth of the oscillating masses in the 2006 and 1995 experiments, was used to calculate the elastic shear modulus for each of the biomaterials inserted under the silicone membrane of the 2006 experiments and to calculate the damping parameters for each of the fluids that flowed under the silicone membrane of the 1995 experiments. These quantities could be measured independently and thus would serve as tests of the formula of Eq. (6).

Such an approach could be generalized to the body-cover model developed by Story and Titze. One would need to make assumptions analogous to Eq. (7) for the spring constants of the two cover masses and of the body mass. It may be feasible to derive an analytic result analogous to Eq. (6) by making the assumption of small amplitudes of oscillation. This approximation linearizes the equations of motion, and it should be valid near the phonation threshold. The results of such an expression could be compared with the recent phonation threshold experiments done by Mendelsohn and Zhang with their 6 body-cover replicas of the vocal folds. Since they measured values of Young’s moduli for their body and cover materials, these numbers could be used to place rather restrictive values on the spring constants. There would seem to be enough data presented in Figs. 2, 4, 5, and 6 of Mendelsohn and Zhang to give rather stringent tests of the approach to the entrance loss coefficients and viscous effects discussed above. If the analytic approach does not prove to be feasible, it would be straightforward to integrate the 3 coupled equations of motion numerically, especially in view of the simplifications expected for small oscillations near threshold. It is likely that most of this calculation will be done before the ICA meeting.

A reasonable amount of success with the Mendelsohn-Zhang data would open another attractive possibility for calculations with the body-cover model of Story and Titze. Murray and Thompson have recently measured a number of properties of their self-oscillating vocal models made from flexible silicone compounds. Two of their replicas were constructed with M5 geometry, so the entrance loss coefficients from Fig. 3 and the exit coefficients from Table II of Reference 6 would be of special relevance. Further, they measured values for the elastic shear modulus $G$ in Eq. (7) and the viscous shear modulus. These would constrain the values of the spring constants and the damping parameters used in the body-cover model calculation. Murray and Thompson measured the onset pressures for the full larynx and the hemilarynx, and values for the oscillation frequency, the maximum glottal width, and the volume flow rate at pressures 10%, 20%, and 30%. These latter pressures should be high enough above threshold so that a linearization procedure would be inadequate. Thus, a numerical solution of the three coupled equations of motion would be required, since nonlinear effects would be necessary. These would most likely include collision effects.

**SUMMARY**

The M5 pressure distributions for the uniform glottis contain a wealth of information about pressure flow relationships. From these one may obtain accurate values of the entrance loss coefficients, which enable one to separate the energy losses on the inferior surface of the vocal folds from those within the glottis. The pressures within the glottis show characteristics that one would expect from the action of viscosity in a narrow channel. With the assistance of the Navier-Stokes equation, pressure changes within the glottis are analyzed to see how they
compare with the predictions of the Poiseuille effect. Although there is a small range of glottal diameters for which the Poiseuille formula is a reasonable approximation, an inverse 2.59 power law gives a better overall description. The entrance loss coefficients and the new power law lead to a new formula for flow resistance, which enjoys considerable success in accounting for data collected with a number of static replicas of the larynx. An extensive project to use information from the M5 pressure distributions to account for measurements of threshold pressure and other properties of self-oscillating replicas is begun.

REFERENCES