1pSA1. Visco-thermal dissipations in heterogeneous porous media
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Semi-phenomological models have been widely used since the 90's for modeling visco-thermal dissipations of acoustical energy through porous media. These dissipations are taken into account by two complex frequency-dependent functions (the dynamic density $\rho_{eq}(\omega)$ and the dynamic bulk modulus $K_{eq}(\omega)$) which are analytically derived from macroscopic parameters. Other models were derived for modeling perforated plates [J. Sound Vib. 303 (2007)], double porosity media [J. Acoust. Soc. Am., 114 (1) (2003)] or, more recently, porous composites made of porous inclusions in a substrate porous media [Acta Acoustica 96 (2010)]. So far, this later model is not able to consider the shape of the host and client media. This model can neither be extended to limiting cases of perforated plate model nor double porosity model. Based on a modified equivalent fluid model, this work proposes a unified model which accounts, analytically, for the shape of the inclusions, might they be porous or not. This model enables to describe the acoustic behavior of any kind of composite media from perforated plates to arbitrarily shaped porous composites including configurations of porous inclusions in solid matrix or double porosity media. In addition, possible pressure interactions between the substrate material and the inclusions are accounted for.

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INTRODUCTION

The acoustical modeling of porous media has been the subject of numerous investigations for some decades. Zwikker and Kosten introduced in 1949 the hypothesis of decoupling between visco-thermal and thermal dissipation effects [1]. A few years later (1956), Biot introduced a model to account for the elastic effect in addition to the potential visco-thermal effects [2, 3]. From these seminal works, refined models for visco-thermal dissipations and reformulations of Biot’s equations have been proposed since the 90s. Considering long-wavelength compared to the characteristic size of the pores, the acoustic propagation and dissipation through a rigid porous media may be represented macroscopically by an equivalent fluid. Semi-phenomenological models have been widely used since the 90s for modeling this equivalent fluid [4, 5, 6, 7].

The models above are not valid when some heterogeneities are of the same order of magnitude as the acoustic wavelength. Therefore, other models were derived for modeling perforated plates [8], double porosity media [9] or, more recently, porous composites made of porous inclusions in a substrate porous media [10].

Based on a modified equivalent fluid model, this work proposes a unified model which analytically accounts for the shape of the inclusions whether they be porous or not. This model enables to describe the acoustic behavior of any kind of composite media from perforated plates to arbitrarily shaped porous composites including configurations of porous inclusions in solid matrix or double porosity media.

In this work, porous media are considered rigid and motionless.

First, the principle of the semi-phenomenological models is briefly recalled. Then, some derived models are reviewed. Third, the principle of the new model is introduced and two examples are presented.

Acoustical Modeling of Porous Media

Using semi-phenomenological models, the visco-thermal dissipations of acoustical energy through porous media are taken into account by two complex frequency-dependent functions (the dynamic density $\tilde{\rho}_{eq}(\omega)$ and the dynamic bulk modulus $\tilde{K}_{eq}(\omega)$) which are analytically derived from macroscopic parameters. Acoustic wavelengths are considered larger than the characteristic size of the pores and we considered the decoupling between visco-inertial and thermal effects.

The most popular semi-phenomenological models are the Johnson-Champoux-Allard (JCA) and the Johnson-Champoux-Allard-Lafarge (JCAL) models [5, 6, 7]. Equations of the JCAL model are presented below.

Visco-inertial effects:

$$\tilde{\rho}_{eq}(\omega) = \frac{\rho_0 \alpha}{\phi} \left[ 1 - j \frac{\omega_v}{\omega} \tilde{G}(\omega) \right]$$ .................................................. (1)

$$\tilde{G}(\omega) = \sqrt{1 + \frac{1}{2} j M \frac{\omega}{\omega_v}}$$ .................................................. (2)

$$M = \frac{8 k_0 \alpha}{\phi \Lambda^2}; \quad \omega_v = \frac{\nu \phi}{k_0 \alpha}$$ .................................................. (3)

$$\nu = \frac{\eta}{\rho_0}$$ .................................................. (4)
Thermal effects:

\[ \tilde{K}_{eq}(\omega) = \frac{\gamma P_0 / \phi}{\gamma - (\gamma - 1) \left[ 1 - j \frac{\omega t}{\omega} \tilde{G}'(\omega) \right]} \]

\[ \tilde{G}'(\omega) = \sqrt{1 + \frac{1}{2} \frac{j M' \omega}{\omega_t}} \]

\[ M' = \frac{8 k_0'}{\phi \Lambda^2} \quad \omega_t = \frac{\nu' \phi}{k_0} \]

\[ \nu' = \frac{\kappa}{\rho_0 C_p} \]

where the macroscopic parameters are \( \phi \) the open porosity, \( \sigma \) the static airflow resistivity, \( \Lambda \) the viscous characteristic length, \( \Lambda' \) the thermal characteristic length, \( \alpha_\infty \) the high limit frequency of tortuosity, \( k_0' \) the static thermal permeability and \( k_0 \) the static viscous permeability (\( = \eta / \sigma \)).

For classical porous materials, the saturating fluid is air that is taken into account by its density \( \rho_0 \) and compressibility \( \gamma P_0 \) (other parameters are \( \kappa \): the thermal conductivity, \( C_p \): the heat capacity, \( \gamma \): ratio of specific heat, \( \eta \): the dynamic viscosity, \( P_0 \) the atmospheric pressure).

**Derived Models**

**Perforated Plates**

Atalla and Sgard suggested to model perforated plates and screens using rigid frame porous models [8]. This model is a JCA model for which the macroscopic parameters have been properly sized:

\( \phi \) the perforation rate,

\[ \Lambda = \Lambda' = r, \]

\[ \sigma = \frac{8 \eta}{\phi r^2}, \]

\[ \alpha_\infty = 1 + \frac{n \epsilon}{L}, \]

where \( r \) is the perforation radius, \( L \) is the thickness of the plate, \( \epsilon \) is the correction length and \( n \) is a factor depending on upstream and downstream materials.

**Double Porosity Media**

Adding meso-perforations in an appropriately chosen porous media can lead to an enhancement of the sound absorbing performance for a given frequency range. This kind of material is called double porosity medium. Olny and Boutin offered a model for considering dissipation effects at the meso-scale (ie. the meso-perforations) and at the micro-scale (ie. the porous matrix) [9]. This model can also consider the pressure diffusion effects when there is a strong permeability contrast between the meso- and the micro-scale. The resulting dynamic quantities of such a material are given by Eqs. 9 and 10.
\[ \tilde{\rho}_{eq} = \frac{1}{(1 - \phi_{meso}) \tilde{\rho}_{eq_{micro}}} + \frac{1}{\tilde{\rho}_{eq_{meso}}} \]  

\[ \tilde{K}_{eq} = \frac{1}{(1 - \phi_{meso}) F_d \tilde{K}_{eq_{micro}}} + \frac{1}{\tilde{K}_{eq_{meso}}} \]  

where \( F_d \) is the dynamic diffusion function.

\[ \tilde{\rho}_{eq} = \frac{1}{(1 - \phi_{meso}) \tilde{\rho}_{eq_{micro}}} + \frac{\phi_{meso}}{\tilde{\rho}_{eq_{micro}}} \]  

\[ \tilde{K}_{eq} = \frac{1}{(1 - \phi_{meso}) \tilde{K}_{eq_{micro}}} + \frac{\phi_{meso}}{\tilde{K}_{eq_{micro}}} \]  

where \( F_d \) is the dynamic diffusion function.

**Composite Model**

Gourdon and Seppi suggested an extension of the double porosity model for modelling material containing specific porous inclusions [10]. This model considers a host and a client material but does not account for their shape. In other words, they consider a porous material which is diluted in a porous substrate. Therefore, the resulting macroscopic quantities are derived using mixing laws.

Without considering the shape of the two complementary parts, this model can neither be extended to limiting cases of perforated plate model nor double porosity model.
New Unified Model

Based on a modified equivalent fluid model, the principle of the unified model is to take into account, analytically, for the shape of the inclusions might they be porous or not. This model enables to describe the acoustic behavior of any kind of composite media from perforated plates to arbitrarily shaped porous composites including configurations of porous inclusions in solid matrix or double porosity media as depicted Fig. 3.

Considering a shaped material with a meso-scale larger than the micro-scale of the filling porous medium, it can be modelled as a semi-phenomelogical model for which the properties of the filling-fluid are modified. Eqs. 1, 4, 5, 8 are replaced by Eqs. 13, 14, 15, 16, respectively.

\[
\tilde{\rho}_{\text{shaped}}(\omega) = \tilde{\rho}_{\text{micro}}(\omega) \alpha_\infty \left[1 - j \frac{\omega}{\omega} \tilde{G}(\omega)\right] \\
\nu = \frac{\eta}{\tilde{\rho}_{\text{micro}}(\omega)} \\
\tilde{K}_{\text{shaped}}(\omega) = \frac{\tilde{K}_{\text{micro}}(\omega)}{\gamma - (\gamma - 1) \left[1 - j \frac{\omega}{\omega} \tilde{G}'(\omega)\right]^{-1}} \\
\nu' = \frac{\kappa}{\tilde{\rho}_{\text{micro}}(\omega) C_p}
\]

Considering porous composite, this unified model should be used for the two complementary mesoscopic parts. Then the mixing law, Eqs. 11 and 12, is applied. Finally, four semi-phenomelogical models are employed to model a composite material with two complementary shaped porous media (cf. Fig. 4).

\[
\tilde{\rho}_{\text{eq}, 1}, \tilde{K}_{\text{eq}, 1} = \tilde{\rho}_{\text{shape}, 1}, \tilde{K}_{\text{shape}, 1} + \tilde{\rho}_{\text{eq}, 2}, \tilde{K}_{\text{eq}, 2} \\
\tilde{\rho}_{\text{eq}, 2}, \tilde{K}_{\text{eq}, 2} = \tilde{\rho}_{\text{shape}, 2}, \tilde{K}_{\text{shape}, 2} + \tilde{\rho}_{\text{eq}, 1}, \tilde{K}_{\text{eq}, 1}
\]

Figure 3: Illustration of types of materials considered by the new model. Parts in black represent impervious regions. Thick lines show where the shape is taken into account.

Figure 4: Principle of porous composite model with shape consideration.
Example 1

The first example compares the new model which takes into account the shape and the composite model presented by Gourdon and Seppi [10]. The composite cylindrical sample is made of a host and a client material as depicted Fig. 5. The mesoscopic characteristic size (29 mm) must be larger than the micro-scale (≈100 μm). Therefore, the shape of such straight tube does not allow significant additional dissipation. This example allows to validate the new model on a simple case where the shape is not one of the main influencing parameters. The parameters of the host and client materials are given in Tab. 1 (client filling and host filling).

![Figure 5: Example 1: Sound Absorption Coefficient. Impedance tube conditions. 19 mm-thick.](image)

Example 2

This second example shows what could be the interest of this new model. One considers two complementary mesoscopic shaped materials with a high tortuosity. The macroscopic parameters of each shape and each filling porous medium are computed and given in Tab. 1.

Fig. 6 shows the sound absorption coefficient of each material (host, client and the composite assembly) with or without the consideration of the shape. This example shows that the effect of the high tortuosity of the shape allows an additional acoustical energy dissipation in the low frequency range.


<table>
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<th>Parameters</th>
<th>Client</th>
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<th>Host</th>
<th>Filling</th>
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<td>$10 \times 10^{-10}$</td>
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</table>

**CONCLUSION**

A new composite model which takes into account the shapes of the materials has been presented. Its main interest is to unify the classical derived models from perforated plates to arbitrarily shaped porous composites including configurations of porous inclusions in solid matrix or double porosity media. The second issue addressed by this work is the opening for designing new concept of composite materials using the benefit of their shape.

**REFERENCES**


