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1pSA9. Omnidirectional acoustic absorber with a porous core - theory and measurements
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An omni-directional acoustic absorber consisting of a porous core and the impedance matching metamaterial layer has been designed and tested in the laboratory. Semi-analytical and numerical models have been developed and validated. The numerical model takes into account the viscous losses in the matching layer. A 1.5 metre demonstrator has been built and tested under acoustic and weak shock excitation. Testing with acoustic excitation showed good agreement between measurement and model, with near perfect absorption between 400 and 1000Hz. The findings suggest that structure is equally effective when wrapped around an object like a column, pipeline or the underside of a vehicle, as it would be when entirely filled with an absorbing porous material.

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The aim of this work is to design, build and test an omni-directional sound absorber consisting of a graded-index metamaterial matching layer and an absorbing porous core. This device is the acoustic analogue of the omnidirectional absorber of EM field described in [1].

1. The Model

The general model of the device is reported elsewhere [2] and is only briefly outlined here. In this work the model is extended to account for a) a hollow porous core and b) possible viscous losses in the matching layer. The cross section of the device consists of an impedance matching metamaterial layer and an absorbing porous layer with thickness $l$ (Figure 1). The incident wave is assumed plane for simplicity.

![Figure 1. Simplified device geometry. 1- matching metamaterial layer, 2 - porous absorbing layer, 3- air.](image)

Outside the cylinder, $r \geq R$, where $R$ is device radius, the field is decomposed into the incident wave with frequency $\omega$ and the scattered wave

$$p_o = \sum_{m=-\infty}^{\infty} \left( J_m(k_0r) + B_mH_m(k_0r) \right) e^{im(\theta+\frac{\pi}{2})},$$  

(1)

here $k_0 = \frac{\omega}{c}$ is wavenumber in air and $c$ is sound speed.

In the matching layer, $R_c \leq r \leq R$, the pressure is represented as a Fourier series

$$p_{m} = \sum_{m=-\infty}^{\infty} p_m(r) e^{im\theta}.$$  

(2)

The matching metamaterial layer is designed so that wavenumber varies as $k(r) = k_0 R_r$ and hence, as shown in [2]. $p_m(r)$ satisfies the following equation

$$\left(R^2k_0^2 - m^2\right)p_m + \rho r \frac{6r^2 - R^2}{2r^2 - R^2} p_m + r^2p_m'' = 0.$$  

(3)

The matching layer is assumed to be composed of small cylinders with a filling fraction increasing towards the centre of the device. Inside the porous layer, $R_c - l \leq r \leq R_c$, the pressure is

$$p_p = \sum_{m=-\infty}^{\infty} \left( A_mJ_m(k_pr) + D_mH_m(k_pr) \right) e^{im\theta},$$  

(4)

where $k_p$ is wavenumber in porous material. Finally in the inner part of the device, $r \leq R_c - l$, the pressure is

$$p_i = \sum_{m=-\infty}^{\infty} Q_mJ_m(k_0r) e^{im\theta}.$$  

(5)
Boundary conditions of pressure and radial velocity component continuity is set on all surfaces which allows to derive a set of equations for scattering coefficients $B_m$ and hence find the pressure distribution around the device. If a rigidly backed porous core is considered then Neumann boundary condition is set at $r = R_c$ and equation (5) is not needed.

As suggested in [2] the absorbing core outer radius $R_c$ calculated to match the absolute value of the porous material characteristic impedance $Z_p$ and that of the metamaterial layer at a single frequency within the working range can be found as

$$R_c = \frac{R}{\sqrt{2}} \frac{\sqrt{8q^2}}{\sqrt{8q^2 + 1} - 1},$$  

(6)

where $q = \frac{|Z_p|}{\rho c}$.

2. Optimization of the Prototype Device

2.1 Absorbing Material Properties

Porous Rockwool has been chosen as an absorbing material for the device core. Its impedance and complex wavenumber have been measured in the impedance tube. The porosity value (0.96) has been deduced from the density measurements. The equivalent model [3]-[4] parameters have been adjusted to fit the data (wavenumber is shown in Figure 2) in the working frequency range of the device which is defined as 500Hz-1000Hz. The properties used for the modelling are: tortuosity 1.04, flow resistivity 33726 Pa s/m², characteristic viscous length $24 \times 10^{-6}$ m, characteristic thermal length $47 \times 10^{-6}$ m, thermal permeability $1 \times 10^{-9}$ m². It should be noted that measured values of flow resistivity strongly depended on the sample and ranged between 12086 Pa/s m² – 63799 Pa/s m². These variations are due to the inhomegeneity of the Rockwool. The calculations of the absorber radius $R_c$ have been performed assuming impedance matching at 500 Hz.

FIGURE 2. Frequency dependence of the wavenumber of Rockwool. Solid lines –data, dashed line –model predictions with parameters listed in text.
2.2 Approximation of the Filling Fraction Profile in the Matching Layer

The cylindrical scatterers used to create a profile of wavenumber in the matching layer have finite size. This means that a smooth profile would be unachievable in practice and inevitably its approximation will have to be used. In Figure 3 the staircase approximation of a filling fraction profile defined by equation (9) from [2] is shown.

![Graph showing the approximation of the filling fraction profile](image)

**FIGURE 3.** Ideal profile of the filling fraction in the matching layer (equation (9) from [2]) and its staircase approximation. Parameters of 6 layers are given in Table 1.

The parameters of the layers are summarised in Table 1. The inner size of the first layer was chosen to approximately match \( R_c \). The total size of the device should be calculated as an inner radius of the last (6\(^{th}\)) layer plus its thickness, i.e. 71.7+2.31=74.01cm.

<table>
<thead>
<tr>
<th>Inner radius, cm</th>
<th>Filling fraction</th>
<th>Effective layer thickness, cm</th>
<th>Cylinder radius, cm</th>
<th>Number of cylinders in the row</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.43</td>
<td>0.44</td>
<td>1.69</td>
<td>0.64</td>
<td>235</td>
</tr>
<tr>
<td>64.12</td>
<td>0.37</td>
<td>1.86</td>
<td>0.64</td>
<td>220</td>
</tr>
<tr>
<td>65.98</td>
<td>0.29</td>
<td>2.08</td>
<td>0.64</td>
<td>202</td>
</tr>
<tr>
<td>68.06</td>
<td>0.21</td>
<td>1.82</td>
<td>0.48</td>
<td>238</td>
</tr>
<tr>
<td>69.88</td>
<td>0.15</td>
<td>1.82</td>
<td>0.40</td>
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</tr>
<tr>
<td>71.70</td>
<td>0.09</td>
<td>2.31</td>
<td>0.40</td>
<td>199</td>
</tr>
</tbody>
</table>

The normalised pressure amplitude at the surface of the device with approximate matching layer profile is compared with that for the device with the ideal profile in Figure 4. It can be seen, that the results are very close. This justifies the approximation of the profile by six rows of cylinders with varying filling fraction in the design. It can be observed that in all cases the presence of the matching layer leads to a significant decrease of front reflections. At 500Hz, the porous cylinder reflects more than 50% of the incident radiation, while the device reflects only 13%.
3. Measurements and Comparisons with Model

3.1 Physical Prototype and Test Setup

Due to the practical constraints the dimensions of the device were limited to 1.5m diameter and 1.25m height. To reproduce the matching layer profile two aluminium plates of 5mm thickness were cut by laser according to a CAD model. In order to evaluate the performance of the device as an omni-directional acoustic absorber the following measurement procedure was followed. First, a measure of the acoustic excitation in free field was required to serve as a reference point. This was defined by taking the frame of the device, installing it in the anechoic chamber and attaching a microphone to each of the 40 supporting cylinders. An omnidirectional source positioned at 1.9m from the device surface was used for the excitation. The sound pressure due to a pink noise excitation for each of the 40 microphone positions on the surface of the empty cylinder structure was measured. The base and top plates were included in these measurements so that their influence could be removed. The next step was to determine the performance of the porous absorber alone. This data then provides a further reference point which is expected to improve upon with the use of a matching layer. The mineral wool was applied in strips to the frame structure to form a cylinder of the same size as the full device. Then, the sound pressure at the same 40 microphone positions was recorded. The only difference between the two described tests was the mineral wool layer. Finally, the last test was to repeat this measurement procedure for the omni-directional absorbing device, i.e. graded-index layer backed by a 10cm thick mineral wool layer. Photographs showing the construction of the device and the test setup are shown in Figure 5 and Figure 6.
3.1 Results and Comparisons with the Model

In addition to the semi-analytical model described in the previous section, a virtual prototype with dimensions identical to that of the physical one was produced using COMSOL. FE model took into account viscous losses in air using the model described in [5] and could therefore be used for future development especially if smaller elements are used in the matching layer (this could significantly increase the role of the air viscosity and compressibility, especially at lower frequencies). The comparisons of the semi-analytical and FE model predictions are shown in Figure 7. The source is assumed to be 1.9 m away from the device and positioned on the right. A good agreement is observed for the tested configuration. For the sake of brevity, in the following the experimental results are compared with FE model only.

Polar plots of the normalized sound pressures on the surface of the mineral wool cylinder and the “black hole” (sound incident from right) are shown in Figure 8. These plots show the average value over the 1/3 octave bands. In the frequency range considered the device diameter is comparable to sound wavelength (their maximum ratio equal to 3.49 is achieved at 800Hz). However the device still strongly outperforms Rockwool absorber of the same size in terms of absorption for frequencies above 200Hz whilst providing a similar level of shielding behind the device in all plots. In fact, for the 1/3 octave bands 400 to 800 Hz, the sound pressure measured on the surface of the device is almost identical to that measured with no device present suggesting near total absorption. The mineral wool only begins to match the performance of the omnidirectional absorber at 1000Hz.
A good agreement between the model and the measurements is achieved for frequencies higher than 250Hz as demonstrated in Figure 9. At lower frequencies the disagreement is most likely due to the fact that the equivalent fluid model does not fit Rockwool impedance/wavenumber data.

4. Conclusions

An omnidirectional absorber device comprised of a graded index metamaterial matching layer and an absorbing porous core has been designed, built and tested. The graded index array was created using 6 rows of sub-wavelength in size cylinders with varying filling fraction. The design was based on a semi-analytical scattering model and an FE model which accounted for the viscous and thermal losses in the matching layer. It has been predicted theoretically and confirmed experimentally that the matching layer offers a significant improvement in absorption. It has also been shown, that the device is considerably more effective than a porous absorbing core of the same size but without a matching layer even when device radius is comparable to the wavelength of sound. The device with a central cavity (hollow absorbing core) is as effective as that with the full core - this suggests that the absorber can be arranged around existing structures and hence would not require a significant amount of space.

ACKNOWLEDGMENTS

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REFERENCES