2aSA3. Numerical prediction of the vibroacoustic of sandwich panels with add-on damping
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This paper discusses the numerical modeling of the vibroacoustic response of sandwich panels with add-on damping, under two types of excitations: Point load, and acoustic excitation, diffuse acoustic field. The studied damping is of the form of a viscoelastic layer located within the panel. A modal synthesis approach is used for the calculation of the structural response and the Rayleigh's integral is used for the acoustic response (the panel is assumed flat and baffled). Since the panel has a viscoelastic core, a methodology is presented to handle efficiently the modeling of the frequency depended properties of the viscoelastic layer. A direct frequency response is used to validate the proposed approach. Next, a parameters study on the effect of the viscoelastic layer location is presented, In particular, three locations are compared: within the Honeycomb core, within the skins and added to the skin with a constraining layer. The effects of the excitation type on the vibration and acoustic response are also discussed. Key words: Sandwich NIDA, Modal FEM method, viscoelastic damping, acoustic response

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INTRODUCTION

Sandwich panels are widely used in the aerospace industry because of their low cost and low weight while maintaining important features such as rigidity and ease of feasibility compared to most aluminum alloys. These panels are usually constructed from a honeycomb core layer sandwiched between relatively thin facing sheets. These qualities lead however to increased radiation efficiency and lower transmission loss (mainly a consequence of a wide coincidence frequency zone). Because of their high stiffness, the classical use of add-on constrained layer damping is very difficult. Instead, embedding damping within the structure is more promising. In consequence, it is necessary to use appropriate and fast numerical tools to predict the vibroacoustic response of such structures; in particular the frequency depended properties of the viscoelastic layer must be accounted for in such models.

A large body of works has been published on the modeling of sandwich panels. The initial acoustic analysis on sandwich plate was made by Kurtz and Watters [1], in 1959. The aim of their paper was to develop a simple model to predict the sound transmission through sandwich panels. The authors started from the model of the bending of the panel based on the mechanical impedance of each layer. The authors used an equivalent analog circuit for sandwich plate impedance and the propagation velocity of shear waves through the structure. Another model of a sandwich plate with three layers with an isotropic compressible core was developed by Ford, Lord and Walker [2], in 1967, to account for the symmetry behavior of the panel, the core was modeled as an elastic medium in three dimensions. In 1973, Smolensky and Krokosky [3] have corrected the energy expressions cited in [2]. They studied the influence of core properties on the frequency of natural vibration mode along the thickness. One year later, in 1974, Dym and Lang [4] predicted theoretically the TL for unidirectional sandwich panels. Based on the work of Smolenski and Krokosky [3] and on the mechanics of sandwich plates, Dym and Lang conducted an analysis that consists on the formulation of the vibration theory of plates. Following the work of Wilkinson [5] and that of Erickson [6], Renji and al[7], 1996, proposed a model of the modal density of the orthotropic sandwich panel. The core is characterized by a transverse shear equivalent and the term of the inertia rotational is neglected.

In 2000, Tongan Wang and al[8] studied the intrinsic sound insulation property of the sandwich panel independent of the effects of external installation conditions, such as clamped or simply-supported boundary conditions. Therefore, the sandwich panel is assumed to be infinitely large to remove the influence of boundary conditions. According to these formations, the authors described a new acoustic model based on higher order sandwich beam theory. They optimized the acoustic and mechanical properties design with minimal weight. In 2002, Nilsson and al. [9] have proposed a dispersion relation of the sixth order for orthotropic sandwich beam, taking into account the rotational force of inertia, bending and transverse shear of the core. For modeling of the sandwich beam, the authors applied the principle of Hamilton, where the strain energy of the beam is defined as the sum of potential energy due to pure bending of the panel, bending of the skin and pure transverse shear of core. They showed that the various simplifying assumptions are necessary to solve analytically the problem of dispersion. In 2006, Ghinet and Atalla [10] have proposed an approach to solve the dispersion relation of flat composite (general) laminated panels. The dispersion relation is written as a general problem of polynomial complex eigenvalues. Their methodology handles in principle, flexible shear controlled layers such as a constrained layer viscoelastic material. In 2006, Ghinet and Atalla [10] have proposed a vibro-acoustic modeling of finite composite sandwich panels with orthotropic core. Their model is based on a dispersion relation.

The above papers were based mainly on analytical or semi-analytical methods and usually assume the panel flat and of infinite extent. But there are also several papers discussing the use of the finite element method to model the vibroacoustics of sandwich structures. Two classes of approaches were used. The first is based on the development of specific sandwich finite element [12]. For example, Kamel and Atalla developed [11] a finite element for Sandwich panels and use it to study the vibration and acoustic performance of laminated Steel (Polymer sandwiched between two steel layers). Recently, K.Bouayed [12] presented an other finite element shell for sandwich panels with viscoelastic layer and validated it using laminated glass used in car’s windshields. The second class of models use classical FE modeling of the sandwich structure (plate-solid-plate or solide modeling for all layers) and several variants to efficiently account for the frequency depended properties of the viscoelastic material using modal method structures [13][20]. The first modal approach is named complex modulus developed by DiTaranto [13], 1965, and Mead [14], 1969, they characterise a viscoelastic materials by a complex frequency and temperature dependant of complex shear modulus. In 1980, Johnson and Kienholz [15][16], propose the modal strain energy method (MSE). This method assumes that the damped structures can be represented in terms of the normal modes of the undamped system. Then an iterative MSE method is used to make the approach suitable for the frequency dependent properties of viscoelastic materials. In 1996, Lesieure [17], Hughes and McTavish [18], proposed the Anelastic Displacement Fields (ADF), the Golla-Hughes-McTavisich proposed the GHM model [18].
develop an iterative version of the MSE method, in conjunction with a complex based model reduction. In the same year, Poluin and Balmes\cite{20}, proposed an augmented real-based modal reduction technique. This method analyses the validity of reduced models obtained by projection on bases of real valued Ritz vectors.

The approach presented in this paper belongs to the second class. It describes a simple numerical methodology to predict the vibroacoustic response of sandwich-composite panels with embedded damping, under various excitations: point load and diffuse acoustic field. The method is based on a modal synthesis approach, to describe the vibration response of the panel and an analytical methodology to predict rapidly the acoustic response. A numerical example is presented to compare the accuracy of this method by comparison with a direct response solution.

**DESCRIPTION OF THE USED METHODOLOGY**

First we consider the FE model of the sandwich panel. Assume the system is excited by a deterministic load (point load for example). The frequency response solution is obtained by solving:

$$-\omega^2 [M] \{u\} + [\tilde{K}(\omega)] \{u\} = \{F\} \tag{1}$$

Here, $[M]$ denote the total mass matrix of the system, $[\tilde{K}(\omega)]$ the frequency dependent stiffness matrix including damping, $\{F\}$ the force vector and $\{u\}$ the displacement vector (response). Assuming the damping dominated by the Viscelacti Layer (VEL), the stiffness matrix can be written as the sum of $[\tilde{K}_v(\omega)]$ the stiffness matrix of the VEL and $[K_m]$ the stiffness matrix of the other layers (skins and honeycomb):

$$[\tilde{K}(\omega)] = [K_m] + [\tilde{K}_v(\omega)] \tag{2}$$

The frequency dependency of the VEL matrix is due to the dependency of the layer’s shear modulus $G(\omega)$ and loss factor $\eta_v(\omega)$:

$$[\tilde{K}_v(\omega)] = [\tilde{K}_v(G(\omega),\eta_v(\omega))] = [K_v(\omega)](1 + j\eta_v(\omega)) \tag{3}$$

In consequence:

$$[\tilde{K}(\omega)] = [K_m] + [K_v(\omega)](1 + j\eta_v(\omega)) \tag{4}$$

The use of direct frequency response solution being computationally prohibitive, an approximate solution is used using the normal modes of a representative eigenvalue problem. For this, assume the VEL has a constant (initial guess) shear modulus $G_v$ and zero damping. Let us denote by $[K(G_v)]$ the associated total stiffness matrix of the system, by $[K_m]$ skins and by $[K_v(G_v)]$ the stiffness matrix of the VEL calculated for $G_v = G_0$. The normal modes equations reads:

$$[K(G_v)] \{u\} = \omega^2 [M] \{u\} \tag{5}$$

The solution leads to an initial approximation of the natural frequencies $\Omega_m^2 = \{\omega_1^2 \ldots \omega_m^2\}$, and mass normalized modes of the panel, $[\phi(G_v)] = [\phi_1 \ldots \phi_m]$. In these two equations, $nm$, denotes the number of kept modes. Defining the associated transformation $[K(G_v)] \{u\} = \omega^2 [M] \{u\}$, Eq. (1) transforms into:

$$(\Omega_m^2 - \omega^2 [I]) \{q\} + [\phi]^\top ([\tilde{K}(\omega)] - [K(G_v)]) [\phi] \{q\} = [\phi]^\top \{F\} \tag{6}$$
Denoting the generalized force by $\{\vec{F}\} = [\phi]^T \{F\}$ and using Eq. (3), we write:

$$\left(\left[\Omega_n^1\right] - \omega^2 [I]\right)\{q\} + [\phi]^T \left(\left[K_v^1(\omega)\right] - \left[K_v^2(G_v)\right]\right)[\phi]\{q\} + j\eta_v(\omega)[\phi]^T \left[K_v^3(\omega)\right][\phi] = \{F\}$$

Assuming the VEL homogeneous, its stiffness matrix is proportional to its shear modulus $[K_v^1(\omega)] = G(\omega)/G_v [K_v^2(G_v)]$ and in consequence, Eq. (7) becomes:

$$\left(\left[\Omega_n^1\right] - \omega^2 [I]\right)\{q\} + \left(\frac{G(\omega)}{G_v} - 1\right)\left[\bar{K}(G_v)\right]\{q\} + j\eta_v(\omega)\frac{G(\omega)}{G_v}[\bar{K}(G_v)] = \{\vec{F}\}$$

With $[\bar{K}(G_v)] = [\phi]^T \left[K_v^2(G_v)\right][\phi]$. Equation (8) has been implemented using DMAP in a MATLAB script.

It leads to a small system of equations $(nm \times nm)$ that can be solved rapidly for the modal response of the system. Two parameters control the accuracy of this system: (i) The initial selection of shear modulus and (ii) the number of kept modes $(nm)$.

Once the vibration response of the panel is obtained, its vibration and acoustic response can be estimated. The space averaged quadratic velocity is approximated using the nodal values of the normal velocity (assuming an approximation over $N$ nodes):

$$\langle \vec{v}^2(\omega) \rangle = \frac{1}{2N} \sum_{i=1}^N |v_i|^2$$

To estimate the radiated acoustic power, the panel is assumed flat and baffled thus authorizing the use of Rayleigh’s integral. The radiated power is given by:

$$P_{rad} = \frac{1}{2} \text{Re} \left( \int_S p\vec{v} \, dS \right)$$

With the parietal pressure:

$$p(M) = j\omega \rho_c \int_S v_r(P) G(M, P) dS(P)$$

Here $G$ denotes the Baffled free-field Green’s function, $G(M, P) = e^{-jkr}/(2\pi r)$, with $k$ the wavenumber and $r = |PM|$ the distance between point $M$ and $P$. Dividing the radiation surface into small patches (mesh), $S = \bigcup S_i$ and assuming the velocity constant over each patch, the radiated power is given by (algebraic manipulations are not detailed here):

$$P_{rad} = \frac{1}{2} \text{Re} \left( \int_S p\vec{v} \, dS \right) = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N v_i \text{Re}(Z_j) v_j S_j + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \text{Re}(Z_j)$$

with $r_{ij}$ the distance between the center of patches $i$ and $j$, $\rho_0$ the density of the fluid, $c_0$ the speed of sound and

$$\text{Re}(Z_j) = \frac{\rho_c c_0 k \sin kr_j}{2\pi r_j} S_j \text{ for } i \neq j \text{ , } \text{Re}(Z_i) = \rho_c c_0 \left(1 - \cos ka\right)$$

These classical approximations are implemented in MATLAB and used to compute the radiated power for various types of excitations (mechanical, plane wave, diffuse acoustic field).

For the DAF excitation, the transmission loss of the panel is computed using:
\[ TL = 10 \log \left( \frac{1}{\tau} \right) \]

\[ \tau = \frac{\int_X \tau(\theta, \phi) \sin \theta \cos \theta d\theta d\phi}{2\pi \int_X \sin \theta \cos \theta d\theta} \]

\[ \tau(\theta, \phi) = \frac{P_r(\theta, \phi)}{P_x(\theta, \phi)} ; \quad P_x(\theta, \phi) = \frac{1}{2} \rho c \cos \theta \]

In the above equation, \( |p|, \theta, \phi \) denote the amplitude and headings (angles of incidence) of the plane waves composing the DAF.

### EQUIVALENT PROPERTIES

Using the vibration response of the panel, equivalent properties such as bending stiffness coefficients and composite damping can be calculated. The equivalent (composite) damping can be calculated in two ways. The first uses the strain energy of the VEL and the strain energy of the resting layers (skins and HC). The second approach uses SEA power balance approach. The former is rigorous while the second assumes validity of SEA assumptions. It has however the advantage of being simple and compatible with test methods. The equivalent bending stiffness coefficients are calculated using an estimation of the wavenumbers of the panel along its two transverse directions \( x \) and \( y \). Using the normal velocity field over one of the faces (excited) of the panel, a wavenumber transform is used to approximate the wavenumber components (dispersion curves) of the panel at any given frequency over a \( N_x \times N_y \) mesh:

\[ \hat{v}(k_x, k_y) \equiv \frac{1}{2N_x N_y} \sum_{x,i} \sum_{y,j} v(x_i, y_j) \exp(-jk_x x_i - jk_y y_j) \]

The used mesh is selected to have at least 3 nodes per wavelength (two nodes are at least separated by half a structural wavelength). In MATLAB, Eq. (15) is implemented using the Fast Fourier algorithm (FFT2). shows an example of used mesh and obtained dispersion curve.

Once the two wavenumber components are obtained, the effective components of the bending stiffness of an equivalent orthotropic panel are identified using:

\[ D_x(\omega) = \frac{m \omega^2}{k_x(\omega)} ; \quad D_y(\omega) = \frac{m \omega^2}{k_y(\omega)} ; \quad D_{xy}(\omega) = \sqrt{D_x(\omega)D_y(\omega)} \]

In the above equation, \( m = \rho h \), denotes the surface mass of the panel. These equivalent properties (bending stiffness components, the equivalent mass per unit area and equivalent (composite) damping) are used to replace the sandwich composite panel by an effective orthotropic panel and use a Transfer Matrix Method (TMM) to estimate its acoustic response (Transmission loss for example).

### NUMERICAL VALIDATION

The present methodology is validated using a simple 0.4 m x 0.4m sandwich panel. It consists of a 0.1 mm thick viscoelastic layer sandwiched between two 6.35 mm steel panels. The structure is assumed baffled and excited by a point load. The properties of the skin and viscoelastic core are given in table 1 and figure 1. Various indicators are used to validate the accuracy of the method by comparison with an exact FEM/BEM analysis of the same structure using NASATRAN for the structural response (use of direct response solution) and an in-house BEM code for the acoustic response.
TABLE 1. Panel configurations and the materials properties used for the numerical validation.

<table>
<thead>
<tr>
<th>Skins</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1=h_3=6.35\text{mm}$</td>
<td>$h_2=0.1\text{mm}$</td>
</tr>
<tr>
<td>$\rho = 7800 \text{ kg/m}^3$</td>
<td>$\rho = 1100 \text{ kg/m}^3$</td>
</tr>
<tr>
<td>$E = 211 \text{ GPa}$</td>
<td>$G = 6895 \exp(0.579 \ln(f)+1.136) \text{ Pa}$</td>
</tr>
<tr>
<td>$\nu = 0.31$</td>
<td>$\nu = 0.49$</td>
</tr>
<tr>
<td>$\eta = 0.01$</td>
<td>$\eta = \exp(0.02 \ln(f)+0.008)$</td>
</tr>
</tbody>
</table>

Figure 1: Mechanical properties of the viscoelastic layer

Figure 2 shows the comparison between the presented methodology (modal approach) and full direct response solution (direct approach) Three indicators are compared, the input mobility, the space averaged quadratic velocity and the radiated acoustic power. Note that static correction was included in the modal prediction of the input mobility. In addition, a constant value of $G_0= 1 \text{ MPA}$, corresponding to the mean value of $G$ over the studied frequency range (10 Hz to 2 kHz) was used to compute the modal basis. Still, a perfect agreement is observed between the two methods.

Figure 2: Validation of the presented approach in the case of point load excitation

Figure 3, shows the same comparison for the Diffuse Acoustic Field Excitation. This time, the Transmission Loss predicted using the two methods is shown are shown. Again an excellent agreement is observed. And, as expected, a significant reduction of computational time is achieved thanks to the use of the modal based methodology.

To illustrate the accuracy of using the identified equivalent properties, the TL of the panel computed using the TMM is compared in Figure 4 to both the direct method (reference) and modal method. The equivalent (composite) damping is calculated using the strain energy approach. Since the results of the TMM approach are presented in 1/3 octave band (wave based approach), the 1/3 octave TL curve obtained using the modal approach is also presented. Again an excellent agreement is observed between the two methods.
Finally to illustrate the application of the method, we consider the effect of a VEM locations within a 0.9m x 0.6m sandwich-Honeycomb (HC) panel. The properties of the skin, core and VEM are given in table 2. The frequency dependency of the VEM shear modulus and loss factor are given in Figure 6. Three VEM locations (Figure 5) are compared to the base configuration (baseline) : (1) VEM located at the center of the HC; (2) VEM located at each skin-HC interface and (3) VEM located at the center of each skin. The three configurations have the same mass; that is the two layers of the VEM are used in case 1 (the thickness of the VEM layer is double the ones used in cases 2 and 3). In all cases the mass added by the VEM is approximately 7% the mass of the bare panel. Figure 7 shows the comparison for a DAF excitation. As expected, configurations 1 and 2 gives the best performance even if the TL remains lower than mass law of the full panel. VEM added mass being small, the observed engacement of the TL is directly related to the damping of the panel’s modes.

**FIGURE 3.** Validation of the presented approach in the case of a diffuse acoustic field excitation

**FIGURE 4.** Comparaison of TL : modal method vs TMM method

**FIGURE 5.** Differents configurations
TABLE 2. Mechanical properties of the materials used in the viscoelastic layer (VEM) location study

<table>
<thead>
<tr>
<th></th>
<th>Skins</th>
<th>Core (HC)</th>
<th>VEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1 = h_2 = 0.8$ mm</td>
<td></td>
<td>$h_2 = 25.7$ mm</td>
<td>$h_{VEM} = 0.125$ mm</td>
</tr>
<tr>
<td>$\rho = 1560$ kg/m$^3$</td>
<td></td>
<td>$\rho = 48$ kg/m$^3$</td>
<td>$\rho = 1000$ kg/m$^3$</td>
</tr>
</tbody>
</table>
| $E = 44$ GPa           |       | $E_z = 139$ MPa; | $v = 0.45$
| $\nu = 0.31$          |       | $G_{xy} = 41$ MPa; | $G$ (Figure 6) |
| $\eta = 0.01$         |       | $G_{yz} = 24$ MPa; | $\eta_v$ (Figure 6) |
| $\eta = 0.01$         |       | $\nu_{xy} = 0.35$ |     |

FIGURE 6. Mechanical properties of the viscoelastic layer

FIGURE 7. Effect of the viscoelastic layer location on the TL of the panel

CONCLUSION

A simple modal synthesis based approach is presented for the calculation of the vibroacoustic response of sandwich panels with embedded damping (viscoelastic layer). Comparison with a direct response solution, for a panel under two types of excitations, show that the approach handles well the frequency dependent properties of the viscoelastic layer. The approach was also used to estimate the properties of an equivalent orthotropic panel (bending stiffnesses and composite damping). The use of these properties within a TMM methodology was shown using the same panel configuration to lead to good TL predictions. However, the presented configuration is simple and a parameters study is needed to fully assess the accuracy and limitations of the method.
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