2aSA4. A dynamic response of a laminated windshield with viscoelastic core – numerical vs. experimental results

Kaiss Bouayed* and Mohamed-Ali Hamdi

*Corresponding author's address: ESI Group, 20 Rue du Fonds Pernant, COMPIEGNE, 60200, COMPIEGNE, France, kaiss.bouayed@esi-group.com

The dynamic response of a laminated windshield with a viscoelastic core is computed using a simplified modal method combined with a quadratic sandwich finite element. The method is based on a modal expansion of the displacement field using a constant young modulus of the core layer. The frequency dependence of the complex modulus of the core is taken into account using the residual dynamic stiffness matrix. The method is applied to predict the frequency response of two types of laminated windshield using a standard and acoustic PVB cores. Numerical results are compared in a first step with those obtained using the direct solver of Nastran software, and in a second step with experimental results obtained by a laser vibrometer. Comparisons show a very good agreement between experimental and numerical results and demonstrate the efficiency of the simplified modal solving method and the developed parabolic sandwich element. The method will be applied to compute the coupled vibro-acoustic frequency response of a full vehicle body integrating a laminated windshield and glass surfaces.

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INTRODUCTION

Passive damping technology using viscoelastic materials is classically used to reduce noise and vibration in different industries such as automotive, aerospace, etc... [1]. The mechanical properties of such kind of material are generally frequency dependent, and necessitate a complex representation of mechanical modules. The complex representation simplifies the analysis of the dynamic behavior of mechanical systems damped by viscoelastic materials. Using this representation in a finite element formulation, leads to systems with complex matrices. In this case, computing the frequency response of a structure using the classical direct method is a hard task especially for large size problems and large frequency band analysis. For this reason, several authors developed new methods to solve complex frequency dependent problems. For example, Poulain and Balmès [2] proposed a pseudo-modal representation using a real modal basis and adding dynamic corrections computed at higher end of the model frequency band. In reference [3], Balmès proposed a method to construct dynamically equivalent models with frequency independent matrices, from reduced models with non linear frequency dependence. Other authors such as Abdou et al. [4] proposed an asymptotic numerical method to compute the forced harmonic response of viscoelastic structures.

In this paper, an efficient simplified modal approach is presented to compute the dynamic response of sandwich structures with a viscoelastic core. The proposed approach is based on a modal reduction using a real modal basis and takes into account the frequency dependence of the viscoelastic material modulus. In the first section, the theoretical formulation of the simplified modal method is presented. Then in the second section, the dynamic response of two types of industrial laminated windscreen of vehicle, using standard and acoustic PVB cores, is computed using the proposed method and associated results are compared with Nastran results using a direct solver and with experimental results.

SIMPLIFIED MODAL METHOD

The simplified modal approach presented in this paper is combined with a layer-wise sandwich shell finite element developed by Bouayed et al. [5].

Since the complex Young modulus of the polymer core is frequency dependant, the equation of motion in the frequency domain can be written as following:

\[ [K(\omega) - \omega^2 M] U = F \]  \hspace{1cm} (1)

Where \( K \) and \( M \) are respectively the stiffness and mass matrices of the sandwich, \( U \) is the displacement vector and \( F \) is the dynamic load vector.

There are two methods to solve the above system (1): the first one is a direct method (or nodal) where the displacement vector is calculated for each frequency step, while the second one is a modal method based on the modal expansion of the displacement field. The direct method has the advantage of being more precise, but it requires a lot of memory and computing time especially for large size problems.

The modal method uses a finite modal expansion of the nodal displacement vector and is hence approximate and less precise especially at high frequencies. However it has the advantage of being very fast because the system to be solved is reduced by a modal projection. The latter method is most suitable for solving large size problems encountered in industrial applications.

To take into account the frequency-dependence of the complex modulus of the core, the complex stiffness matrix of the structure is decomposed in the sum of two matrices \( K_0 \) and \( \Delta K(\omega) \):

\[ K(\omega) = K_0 + \Delta K(\omega) \] \hspace{1cm} (2)

where \( K_0 \) is the frequency-independent stiffness matrix calculated with a constant averaged modulus \( E_{20} \) of the viscoelastic core, \( \Delta K(\omega) \) is the residual stiffness matrix calculated with a residual Young’s modulus which the real part is equal to \( \Delta E_2(\omega) \) as expressed in equation (3):

\[ \Delta E_2(\omega) = E_2(\omega) - E_{20} \] \hspace{1cm} (3)

The modal basis is obtained by solving the following eigenvalue problem:

\[ [K_0 - \omega^2 M] W = 0 \] \hspace{1cm} (4)

System can be solved using standard eigenvalue solvers, leading to a real modal basis composed by eigenvalues and eigenvectors \( \{\omega_i^2, W_i\} \).
The solution of system (1) can be approximated by a modal decomposition:

\[ U \approx \sum_{i=1}^{N_{\text{modes}}} \alpha_i W_i \]  

(5)

where \( N_{\text{modes}} \) is the number of retained modes and \( \alpha_i \) are generalized coordinates.

Considering the decomposition (2), the system (1) can be projected on the modal basis \( W \) whose columns are composed by the first \( N \) modes including rigid body modes. This projection is used to write the system in the following reduced form:

\[ ([\omega_i^2] + W^T \Delta K(\omega)W - \omega^2[I])\alpha = W^T F \]  

(6)

\([\omega_i^2]\) is a diagonal matrix with dimensions \( N_{\text{modes}} \times N_{\text{modes}} \) containing eigenvalues of problem (4), \( [I] \) is the \( N_{\text{modes}} \times N_{\text{modes}} \) identity matrix.

Since the polymer core is an isotropic material, the dynamic stiffness matrix can be obtained by the product of the complex modulus that depends on the frequency with a matrix built using a unitary Young's modulus. At each frequency step, the reduced system (6) is solved by updating the residual stiffness matrix. After determining the generalized coordinates \( \alpha_i \), the displacement field is reconstructed using the modal expansion (5).

**EXAMPLE OF VALIDATION: LAMINATED WINDSHIELD**

The industrial case treated here is a laminated vehicle’s windscreen (Renault Mégane III), provided by Saint-Gobain. The laminated windscreen is composed by two elastic glass skins intercalated by a viscoelastic PVB core which mechanical properties depend on frequency and temperature according to the WLF law (FIGURE 2) [6]. This frequency dependence will be taken into account by using the simplified modal approach to calculate the dynamic response of the free-free windscreen.

![FIGURE 1. Meshing of the windscreen with Q8 elements](image)

The free-free windscreen is excited by a unitary force \( F \) placed at node n°1 and orthogonal to the mid-surface of the windscreen as shown in FIGURE 1. Dynamic responses of two types of windshield, using standard and acoustic PVB cores, are computed using the simplified modal approach briefly presented in the previous section and considering physical and mechanical properties given by TABLE 1.
Results of the developed simplified modal method will also be compared with experimental data obtained at the University of Technology of Compiègne (UTC – France).

To measure the frequency response of the windscreen, a laser vibrometry system Polytec PSV-400 has been used. It is mainly composed by a sensor head with an integrated laser interferometer, a signal controller and a data management system (PC).

![Graph showing storage shear modulus G’ and loss factor vs frequency for Standard PVB and Acoustic PVB.](image)

**FIGURE 2.** Storage shear modulus $G'$ (a) and Loss factor (b) of the PVB at 20°C

<table>
<thead>
<tr>
<th>Physical and mechanical properties of glass and PVB</th>
<th>Young’s modulus (MPa)</th>
<th>Poisson’s coefficient</th>
<th>Density (kg/m³)</th>
<th>Thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>$72 \times 10^3$</td>
<td>0.22</td>
<td>2500</td>
<td>internal layer: 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>external layer: 2.1</td>
</tr>
<tr>
<td>PVB</td>
<td>WLF</td>
<td>0.49</td>
<td>1000</td>
<td>Standard: 0.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Acoustic: 0.86</td>
</tr>
</tbody>
</table>

**Standard Windshield**

The acceleration of the standard windshield is recomposed at nodes 1 and 2 shown in **FIGURE 3**. The results obtained with the present modal approach are compared to those obtained with Nastran using a direct nodal approach and to experimental results. The constant averaged modulus used to calculate the modal basis is equal to 542 MPa for the standard windscreen. The modal basis used here is calculated up to 1000 Hz. The choice of the
constant core’s modulus and the size of the modal basis has been fixed after a sensitivity studies conducted on reference [7].

The measurements are performed on a series of five standard windscreens. After measures treatment of each windshield, a dispersion of results is observed at the frequency responses. For this reason, the envelope containing the measurements of different windscreen has been considered for comparison with numerical results. These dispersions may be caused by the manufacturing process, measurement error or environmental conditions such as temperature.

![Figure 3](image-url)  

**FIGURE 3.** Frequency response of the standard windscreen at nodes 1 (a) and 2 (b)

FIGURE 3 shows the comparison between experimental results, Nastran results obtained with the direct approach method, and the present simplified modal approach using the Q8 element developed by Bouayed et al. [5]. FIGURE 3(a) represents the acceleration at point 1 and FIGURE 3(b) the acceleration at point 2. These two figures show that below 200 Hz, a good agreement is observed between Nastran and the proposed modal approach.

These figures show that the dynamic responses calculated are located within the envelope of experimental measured responses. However, we find generally, at two points of comparison that the difference between numerical and experimental results increases with frequency. This could be explained by the uncertainty observed on measures, which is translated here by the width of the envelope, which increases with frequency. At very low frequency (<10 Hz), the difference between the two results is more important. This could be caused by two facts: numerically, global matrices are not well conditioned at low frequencies and experimentally, the laser vibrometer is not able to detect the vibration of the windscreen under the first flexible mode.
Acoustic Windshield

For the acoustic windshield, the comparisons between numerical and experimental results are shown in FIGURE 4(a) and FIGURE 4(b) at the same nodes 1 and 2 respectively.

For the comparison between calculations and measurements, higher and lower envelope curves of the dynamic responses corresponding to the two tested acoustic windcreens have been plotted. FIGURE 4 shows that numerical results obtained at nodes 1 and 2 by both Nastran (direct nodal solution) and the proposed simplified modal method using the developed Q8 shell element, are in very good agreement with measurements. The difference between calculated and measured curves has the same order of magnitude than the dispersion level observed between the two windshields. At low frequencies, the same behavior is observed as in the standard case.

As a general observation, it could be highlighted that the frequency response of the acoustic windshield is much more damped than the response of the standard one. This proves the efficiency of the acoustic PVB core which induces higher damping leading to high reduction of the vibration level. So at temperature near to 20°C, the use of an acoustic windshield can better isolate the interior compartment of the car from outside noise and subsequently provide better passenger comfort.

![Graph (a)](image1)

![Graph (b)](image2)

**FIGURE 4.** Frequency response of the acoustic windshield at nodes 1 (a) and 2 (b)
The good agreement generally observed between numerical and experimental results confirm the accuracy of the new developed shell finite element and the efficiency of the proposed simplified modal approach allowing quick calculation of the dynamic frequency response of sandwich structures integrating a viscoelastic core having frequency dependant mechanical properties.

CONCLUSION

A simplified modal method has been developed to compute the dynamic response of sandwich structures taking into account the frequency dependence of the complex modulus of the viscoelastic core. This method is combined with a sandwich finite element presented in reference [5].

Dynamic responses of two types of vehicle's windscreen, using standard and acoustic PVB cores, computed with the proposed simplified modal method are in good agreement with those obtained by the direct nodal method implemented in Nastran Software. The proposed method has the advantage to reduce drastically the computing time and memory size.

In comparison with the measured frequency responses of standard and acoustic windscreens, the results obtained by the simplified modal method, show a good consistency since the error between the two results is in the same range of the dispersion observed on experimental results. This confirms the validity of the proposed method in predicting the dynamic behavior of sandwich structures with viscoelastic core.

The model of the windshield will be applied to compute the coupled vibro-acoustic frequency response of a full vehicle body integrating a laminated windshield and glass surfaces.

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REFERENCES