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2pSA2. Miguel Junger: Legacy contributions to the field of structural acoustics
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The field of structural acoustics, i.e. the theory of acoustic radiation and scattering from elastic structures, developed primarily in the last half of the 20th century in response to Navy needs. Miguel Junger and colleagues at Cambridge Acoustical Associates under the sponsorship of the Office of Naval Research Sound and Structures Program made seminal contributions to the field. This presentation reviews some of this research which ultimately was included in the book, "Sound, Structures and their Interaction."

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MIGUEL JUNGER LEGACY CONTRIBUTIONS TO THE FIELD OF STRUCTURAL ACOUSTICS

Introduction

Structural acoustics is the study of the vibrations, radiation, and scattering of sound by elastic structures responding to forces while subjected to the reactive loading of the ambient fluid medium. Today we celebrate the significant achievements of Miguel Junger as the pioneer investigator in this field. As founder of Cambridge Acoustical Associates he was an inspirational mentor to those of us employed there as well as many others who learned from his publications and presentations throughout the years.

When Lord Rayleigh published the first modern text on acoustics, The Theory of Sound, [1] he divided the work into two volumes, one dealing with the vibrations of structures in vacuo and the other with various aspects of acoustics propagation. Most subsequent theoretical textbooks on acoustics followed this precedent. But there are problems where the interaction of sound and structures must be dealt with simultaneously and these are the subject of Miguel’s monograph, Sound, Structures and their Interaction [2] for which I was fortunate enough to have been asked to co-author. Most of the chapters were written by Miguel, some by me, with comments and suggestions by the other except for the last chapter entitled, “High Frequency Formulation of the Acoustics and Structural Vibration Problems.” The latter was a joint effort and some of its results are the subject of this presentation.

Application of the Sommerfeld-Watson Transformation to Structural Acoustic Problems

Before introducing the Sommerfeld-Watson transformation (SWT) it is useful to state it in the form of an equation that relates the sum of an infinite series to a contour integral. Assume that a function $f(z)$ has isolated poles and the $\lim_{|z|\to\infty}|f(z)| \to 0$ and suppose that $C$ is a contour that is deformed such that all poles of $f(z)$ are contained in $C$ then

$$\sum_{n=-\infty}^{\infty} (-1)^n f(n) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{\sin \pi z} \, dz \tag{1}$$

Solutions for the vibration and radiation fields of force excited or scattering from fluid loaded elastic bodies with separable geometries are readily obtained in the form of wave-harmonic or normal mode series. Unfortunately, these solutions are effectively expansions in powers of $ka$ and converge rather slowly for large $ka$, $k = \omega/c$, the acoustic wave number and $a$ the radius of the spherical or cylindrical shell. To circumvent this situation the Sommerfeld-Watson transformation (SWT) can be used to find alternative representations of the solutions. The basic characteristic of this approach is that it converts the slowly converging wave harmonic series into a rapidly converging sequence of residue terms each corresponding to an integrand’s complex poles and leads to a better physical understanding of the waves propagating along these structures.

As an example of the approach discussed above, we consider the high frequency vibrational response of a point excited infinite fluid loaded elastic cylindrical shell and the solution for the radial velocity response is given by Eq. (12.93) of [2] as

$$\ddot{u}(z,\phi) = \frac{F}{(2\pi)^2 a} n \sum_{n=0}^{\infty} e^{in\phi} \cos(n\phi) \int_{-\infty}^{\infty} \frac{e^{jz}}{Z_s(n,\Omega) + \tilde{z}_a(n,\Omega)} \, dy \tag{2}$$

Using the SWT and some further manipulation the latter summation Eq. (1) can written be written as
\[
\dot{w}(z, \vartheta) = \frac{F}{(2\pi)^2 a} \int_{-\infty}^{\infty} \frac{\cos(\phi - \pi)}{\sin(\vartheta \pi)} e^{ivz} dv \int_{-\infty}^{\infty} e^{i\vartheta^2} \mathcal{Z}(v, \omega) + \mathcal{Z}(v, \Omega) dv
\]  
(3)

For \(0 \leq \beta < 2\pi\), the integrand in Eqn. (3) vanishes exponentially on the semicircle \(v = Re^{i\vartheta}, 0 < \vartheta < \pi\).

Using this fact and residue theory, we find that

\[
\dot{w}(z, \vartheta) = -\frac{F}{2\pi a} \sum_j \int_{-\infty}^{\infty} \frac{\cos \vartheta \cot \pi \vartheta}{\sin \vartheta \pi} e^{ivz} dv
\]  
(4)

In the above equation \(v = v_j\) are the complex zeros of the denominator \(\mathcal{Z}(v, \omega) + \mathcal{Z}(v, \Omega) = 0\), that occur in the upper half plane. Although not actually calculated here, an example of the roots and their locations in the complex \(\nu\) plane for a particular value of \(\Omega\) are shown in Fig. 1 which are calculated and reproduced from a paper by Rumerman [3]. Note the roots are labeled \(\nu_b, \nu_c,\) or \(\nu_{cr}\) and these represent the flexural (or bending) wave in the shell, compressional, and “creeping waves” in the shell, respectively. The creeping waves are so called since they are slower than the corresponding acoustic waves in the fluid medium - note that the real part of the roots are slightly larger than \(ka\) implying a phase speed less than the acoustic speed as it circumnavigates the shell.

For the sake of brevity in this presentation we only treat the case of the in vacuo response of the cylindrical shell, that is we set \(\mathcal{Z}(v, \omega) = 0\), and in the high frequency limit \((ka)^2 \gg 1\), the structural impedance can be approximated by

\[
\mathcal{Z}(v, \Omega) \approx -i\omega m \left[1 - \frac{1}{\Omega^2} \frac{\beta^2}{\Omega^2} \left(\Omega^2 + v^2\right)^2\right]
\]  
(5)

The zeros in the upper half plane are

\[
v_1 = \left(\frac{\beta}{\Omega} - \sqrt{\nu_b}\right) = a\left(k_f^2 - \nu_c^2\right)^{1/2}
\]  
(6a)

\[
v_2 = i\left(\frac{\beta}{\Omega} + \sqrt{\nu_c}\right) = ia\left(k_f^2 + \nu_c^2\right)^{1/2}
\]  
(6b)

We can now rewrite Eqn. (4) explicitly as the sum of two integrals

\[
\dot{w}(z, \vartheta) = \frac{i\omega m}{2\pi a} \left[\int_{-\infty}^{\infty} \cos \vartheta \nu_1 e^{iv\vartheta} dv - \int_{-\infty}^{\infty} \cos \vartheta \nu_2 e^{iv\vartheta} dv\right]
\]  
(7)
After quite a bit of manipulation the above expression can be put in the following form

\[ \dot{w}(z, \phi) = \frac{F}{z_p} \left\{ \sum_{m=0}^{\infty} H_0(k_f R_m) + H_0(k_f R_m') \right\}^{1/2} \{\sum_{m=0}^{\infty} K_0(k_f R_m) + K_0(k_f R_m')\} \quad (8) \]

\( R_m = \sqrt{z^2 + a^2(\phi + 2mn)^2} \) is the helical distance measured on the surface of the cylinder from the drive point to the observation point in the direction of increasing \( \phi \), and \( R_m' = \sqrt{z^2 + a^2(2\pi r - \phi + 2mn)^2} \) again measures helical distance, but this time measured in the direction of decreasing \( \phi \).

The high frequency response of a point excited submerged spherical shell can be developed in a similar way starting with the expression

\[ \dot{w}(\theta) = \frac{F}{4\pi a^2} \sum_{n=0}^{\infty} \frac{2n+1}{Z_S+z_a} P_n(\cos \theta) \quad (9) \]

In Eqn. (9) we introduce the new index \( m = n - \frac{1}{2} \) so that it can be rewritten as

\[ \dot{w}(\theta) = \frac{F}{2\pi a^2} \sum_{n=0}^{\infty} \frac{(n-\frac{1}{2})^p v_\frac{1}{2}(-\cos \theta)}{Z_S(n-\frac{1}{2} \Omega^2) + z_a(n-\frac{1}{2} \Omega^2)} \quad (10) \]

and now apply the SWT to this expression. We again ignore the effect of fluid loading and obtain the final expression

\[ \dot{w}(\theta) = -\frac{1}{Z_P} \left\{ \frac{P_{v_1-\frac{1}{2}}(-\cos \theta)}{\cos v_1 \pi} - \frac{P_{v_2-\frac{1}{2}}(-\cos \theta)}{\cos v_2 \pi} \right\} \quad (11) \]

This solution can eventually be shown to correspond to flexural disturbances propagating on the shell in opposite directions from the north pole, i.e. \( \Theta = 0 \).

References
