3pSAa3. Underwater sound transmission through thin soft elastomers containing arrays of pancake voids: Measurements and modeling

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Measurement of underwater sound transmission through thin (~750 micron) layers of the soft elastomer polydimethylsiloxane (PDMS) containing microfabricated arrays of pancake-shaped cavities is presented. Cavities are 120 microns in diameter and 5 microns in height with a nominal lattice spacing of 300 microns. A sound transmission minimum is found at 282 kHz which agrees with predictions of a finite-element model of the array and the value for monopole resonance frequency of an air-filled single pancake cavity in unbounded PDMS. This resonance is a factor of 0.62 lower than the null that would occur for spherical cavities of equivalent volume. The width of the null is also significantly broader than that which would be obtained with spherical voids. Modeling results incorporate careful measurements of attenuation for both shear and compression waves in PDMS done in a separate effort. Acoustic transmission variation as a function of lattice spacing and the number of layers is discussed. [Work sponsored by the Office of Naval Research]
INTRODUCTION

Several studies have examined the scattering of sound from air inclusions in soft media by both experimental and theoretical means.\textsuperscript{1-5} The goal of these studies has been to manipulate sound propagation with strong resonant scatterers that are small in comparison with an acoustic wavelength. The scatterers are in ordered lattices either one or multiple layers deep\textsuperscript{1,2} or randomly distributed.\textsuperscript{3} The motivation for using soft (low-shear modulus) elastic media, or liquids with finite yield stress is that the novel acoustic properties of an air-bubble in a liquid appear to be largely maintained without the drawback of inherent drift and unsteadiness.\textsuperscript{4}

Recently, inclusion shape has been found to be important in situations where the shear modulus is on the order or greater than the bulk modulus of the gas. For situations where the shear modulus dominates, the resonant frequency of a cavity can be significantly lowered by deforming the cavity into an oblate shape.\textsuperscript{5} This is but one method of tailoring the resonant frequency of individual scatterers by custom microfabrication.

The present study builds upon the inclusion shape-sensitivity analysis by modeling underwater acoustic plane wave transmission through a single planar array of pancake-shaped air inclusions located in the mid-plane of a thin layer of polydimethylsiloxane (PDMS).\textsuperscript{6} Experimental verification of transmission minima is made by fabricating the array using soft lithography techniques\textsuperscript{7} and ultrasonic transducers to transmit underwater sound in the frequency range (100 kHz to 3.5 MHz).

GEOMETRY AND FINITE-ELEMENT ANALYSIS

We focus on the geometry depicted in Figure 1 which consists of an array of pancake (or disk-shaped) air inclusions with 120 micron diameter arranged in a rectangular array with lattice constant 300 microns. Each inclusion is 5 microns high. The single layer is positioned in the mid-plane of a 750 micron thick layer of PDMS (a soft elastic medium). The resulting PDMS sheet is submerged in water. We analyze sound transmission by considering a unit amplitude plane wave to impinge on the PDMS sheet at only normal incidence. The magnitude of the transmission coefficient is then simply the amplitude of the complex pressure in the forward direction.

The computational method used is the finite-element method for acoustic-structure interactions. The PDMS is modeled as an elastic medium described by a Young’s modulus (E), Poisson ratio (ν) and density (ρ). The elastic medium is loaded by the acoustic pressure in the water on either side of the PDMS sheet and the acoustic pressure inside the air cavity. Significant computational savings can be obtained by invoking symmetry for this normal incidence situation. As shown in Figure 1, it suffices to model a single quadrant of the pancake inclusion. The symmetry boundary conditions are roller constraints on the edges of the elastic medium (no shear or normal displacement) and an acoustic hard condition on the edges of the water domain (no normal velocity). The acoustic parameters used for the water are ρ\textsubscript{w} = 1000 kg/m\textsuperscript{3} for the density and c\textsubscript{w} = 1482 m/s for the sound speed. The acoustic parameters for the air inside the cavity are ρ\textsubscript{a} = 1.2 kg/m\textsuperscript{3} and c\textsubscript{a} = 343 m/s.

Essential parameters in the model are the compression and shear wave attenuation of the PDMS. The PDMS used in the experiments is RTV-615 from Bayer Silicones mixed with a standard 10:1 monomer to hardener ratio. We present a straightforward model of attenuation with simple frequency dependence based on separate materials characterization work. The full details are left to a separate study. For the computations to be presented, we have used a complex wave speed model for the compression attenuation which was measured to be relatively small in the band 200-700 kHz. The complex wave speed model represents an attenuation in nepers/meter which varies linearly with frequency. While not realistic for a wide frequency band, this was an adequate approximation over the bands reported. The compression wave speed used was c\textsubscript{p} = 1020 + i 5.065 m/s. The shear modulus was measured to be approximately G = 1.0 + i 1.0 MPa. Shear waves are therefore damped fairly quickly as they radiate from the inclusion.\textsuperscript{5} The density of the PDMS is 1042 kg/m\textsuperscript{3}. Based on this information, a complex Young’s modulus and Poisson’s ratio were calculated as input to the finite-element model. We note that Poisson’s ratio is very close to \( \frac{1}{2} \), but not exactly \( \frac{1}{2} \) as this would imply an infinite compression wave speed in the PDMS (perfectly incompressible elastic solid).
MICROFABRICATION AND ACOUSTIC TESTING

The 750 micron sheet of PDMS is comprised of a single unpatterned sheet of spin-coated and cured PDMS that is bonded to a sheet in which the inclusions are open to the surface (the inclusions are 5 micron deep depressions in the surface). The individual sheets are half the thickness of the overall sheet. The bonding of the layers is accomplished by means of a plasma oxidation cleaning process. The inclusions were created using standard methods of soft lithography. Specifically, a laser pattern generator was used to create a mask which was used in a photolithography process to create an SU-8 mold. The mold was then filled with the mixed liquid PDMS and then degassed and cured in an oven.

The frequency of the experiments (100 kHz to 3 MHz) and the inclusion/lattice geometry were selected as compromise between layer width constraints imposed by the soft-lithography equipment available, and the need for a layer that was many acoustic wavelengths wide in order to minimize acoustic diffraction around the sample. This diffraction around the sample in the underwater test tank would contaminate the transmission loss measurements.

The experimental data was collected using matched pairs of flat-focused immersion broadband transducers, one operating as a source and one as a receiver (in a through-transmission configuration). One pair (Panametrics, Olympus NDT Inc., Waltham, MA) operated at a center frequency of 1/2 MHz with an active element diameter of 1 inch. Another pair (Ultran Group, State College, PA) operated at a center frequency of 1 MHz with an active element diameter of 0.75 inches. A pulser (5077PR, Panametrics, Olympus NDT Inc.) was used to excite the sources with a short broadband pulse, and the matching pair received the signal that was then fed to a 200 MHz digital oscilloscope (Lecroy Wavesurfer 424) and saved. Both the source and receiver were positioned approximately 10 cm from the sample face, which is within the far-field (a^2/λ), and data was collected with the sample present and the sample absent. The active area of inclusions in the samples was 3 X 3 inches^2. Transmission loss curves as a function of frequency were then generated by calculating abs(fft(sample) / fft(water)).

RESULTS AND DISCUSSION

Finite-element modeling and experimental results are presented in Figures 2 and 3. Figure 2 shows the presence of a predicted transmission minimum at 333 kHz. The measured transmission minimum is close by at 282 kHz. Both values are close to the predicted monopole resonance frequency for a single pancake oscillator in unbounded PDMS. Agreement of measured vs modeled transmission is quite good over the frequency ranges measured. Interestingly, although the pancake array is only one layer deep, transmission minima at higher frequencies appear much like in the case of multiple layers of inclusions in PDMS (with inclusions aspect ratios that are O(1)).

Higher-order minima in reported in Leroy et al. were attributed to Bragg resonance with multiple layers. Agreement between theory and model between 3 and 3.5 MHz is good. Transmission is largely unaffected by the pancake-inclusions for frequencies near 3 MHz for which the compression wavelength in PDMS is roughly 3 times the diameter of a pancake.

Figure 3 shows the calculated axial displacement in the PDMS as well as the acoustic pressure amplitude in the water for 333 kHz. A monopole type resonance in which the top and bottom surfaces of the pancake move in opposition is very apparent. A weak transmitted signal in the forward direction can be seen as can an interference between the incident and reflected waves in the backward direction.

To examine the role played by inclusion shape, a single-layer array of cylindrical inclusions was also fabricated in which the lattice constant and inclusion volume are the same as the pancake array, but the inclusion aspect ratio is O(1). The height of each inclusion was equal to its diameter. Agreement between measured and calculated transmission is again very good with a minimum occurring in the equivalent cylinder at approximately 600 kHz. This frequency is in good agreement with the resonant frequency for a spherical bubble oscillation

\[
 f = \frac{1}{2\pi R_{eff}} \sqrt{\frac{3\rho_w c_w^2 + 4\mu_{PDMS}}{\rho_{PDMS}}} = 549 \text{ kHz},
\]

where the effective spherical radius is \( R_{eff} = 18.9 \) microns and the other parameters are as described previously. The pancake therefore resonates at a frequency approximately 60% of what a spherical inclusion of equivalent volume resonates at.

Although not reported here, modeling results demonstrate that strong gains in transmission blocking can be obtained by stacking/bonding the single-pancake layer sheets as was found in Leroy et al. The acoustic wavelength
for the maximum blocking in the single-layer pancake case was roughly 6 times the overall sheet thickness (which was mainly limited to 750 microns for convenience of handling). By economizing sheet thickness, and stacking sheets, it therefore appears that good sound transmission blocking can be obtained for overall sheet thicknesses that are small in comparison with an acoustic wavelength.

![Diagram](image)

**FIGURE 1.** (a) Example view of the lattice consisting of pancake-shaped inclusions in PDMS. The inclusion diameters and lattice constant are shown with the finite-element symmetric analysis region. (b) 3D view of the finite-element analysis region where only ¼ of the pancake inclusion must be modeled.

![Graph](image)

**FIGURE 2.** Comparison of measured vs modeled transmission through the PDMS lattice with pancake-shaped air inclusions.
FIGURE 3. Axial displacement field inside the PDMS plotted with acoustic pressure in the water for 333 kHz. The pancake-shaped air inclusion resonates with its top and bottom surfaces 180 degrees out of phase. The transmitted pressure is at a minimum for this frequency. The axial displacement is in meters and the acoustic pressure is in Pascals. The incident field has amplitude 1 Pascal.

FIGURE 4. Comparison of pancake array transmission results with an array of circular cylinder-shaped inclusions in which the height equals the diameter and the volume is the same as the pancake inclusion volume. The lattice constants are the same in both cases. Both cases are single layer sheets. The pancake-inclusion sheet has a transmission minimum at a lower frequency than the sphere case (and a deeper minimum).

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REFERENCES