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4aSA3. Modeling of wave propagation in drill strings using acoustic transfer matrix method

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In order to understand critical vibrations of a drill bit such as stick-slip and bit-bounce and their wave propagation characteristics through a drillstring system, it is critical to model the torsional, longitudinal, and flexural waves. The objective is to model these waves propagating through the drillstring in a computationally efficient way. Here, a modeling method based on an acoustic transfer matrix between two sets of wave variables at the ends of a cylindrical pipe is proposed. For a drillstring system with multiple pipe sections, the total acoustic transfer matrix is calculated by multiplying all individual matrices of which each is obtained for an individual pipe section. Since drillstring systems are typically extremely long, conventional numerical analysis methods such as FEM require a large number of meshes, which makes it difficult to analyze these drillstring systems. On the contrary, the analytical acoustic transfer matrix method requires significantly low computational costs. For the validation, experimental and numerical data are obtained from a laboratory measurement and by using a commercial FEM package, ANSYS, respectively. They are compared to the modeling results obtained by using the proposed method. It is shown that the modeling results are well matched with the experimental and numerical results.

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INTRODUCTION

A modeling method for predicting structural waves propagating in a drill system is proposed in this article. In order to investigate and prevent critical vibrations such as a stick slip and a bit-bounce, all of the flexural (i.e., lateral), torsional, and longitudinal (i.e., axial) vibration modes of a drill string system are considered in this modeling method. The proposed method is based on an acoustic transfer matrix approach in which a constant cross-sectional drill pipe section is modeled by using an “analytical” acoustic transfer matrix between two sets of wave variables at the two ends of the pipe section. For a drill pipe system with multiple cross-sectional pipe sections, a sectional drill pipe section is modeled by using an “analytical” acoustic transfer matrix approach in which a constant cross-sectional, circular pipe or joint is derived from three uncoupled wave equations (i.e. the longitudinal, torsional, and flexural wave equations [1]). The acoustic transfer matrix description is defined in Fig.1. The acoustic transfer matrix that relates acoustic variables between two axial locations in a constant-cross-sectional drill pipe system is then obtained by multiplying all of the individual transfer matrices. The proposed transfer matrix approach greatly simplifies modeling procedures and results in computationally-efficient models when compared to other approaches such as a finite element method (FEM) where an extensively large number of finite elements are required to model a long pipe system. For the validation, the frequency response functions (FRFs) obtained from the proposed transfer matrices are compared to “experimental” results as well as “numerical” results obtained by using a commercial FEM software package, ANSYS.

UNCOPLED ACOUSTIC TRANSFER MATRIX DERIVED FROM THREE UNCOUPLED WAVE EQUATIONS

An uncoupled acoustical transfer matrix that relates acoustic variables between two axial locations in a constant-cross-sectional, circular pipe or joint is derived from three uncoupled wave equations (i.e. the longitudinal, torsional, and flexural wave equations [1]). The sign convention of the acoustic variables used for this transfer matrix description is defined in Fig.1. The acoustic transfer matrix \( T \) between \( z = 0 \) and \( z = L \) describing the longitudinal, torsional, and flexural wave propagations in the pipe or joint can be obtained as

\[
U_{z=0} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} U_{z=L},
\]

where

\[ U = \begin{bmatrix} u, \beta, M, Q, \beta, T, u, N \end{bmatrix}^T, \quad A_{ij} = a_{ij}, \]

\[
a_{ij} = a_{z-1} - a_{z} = a_{4} = \frac{\delta^{z+2} + \delta^{z-2} + \delta^{z+1} + \delta^{z-1}}{4}, \quad a_{2} = \frac{-\delta^{z+2} - \delta^{z-2} + \delta^{z+1} + \delta^{z-1}}{4E_I \rho L^2}, \quad a_{3} = \frac{-\delta^{z+2} + \delta^{z-2} + \delta^{z+1} - \delta^{z-1}}{4E_I \rho L^2}, \quad a_{4} = -\frac{2}{E_I \rho L^2} \]

\[
B = \begin{bmatrix}
\frac{1}{2} (\delta^{z+1} + \delta^{-1}) & -\frac{1}{2k_J} \frac{1}{H} (\delta^{z+1} - \delta^{-1}) & 0 & 0 \\
-\frac{1}{2k_J} H (\delta^{z+1} - \delta^{-1}) & \frac{1}{2} (\delta^{z+1} + \delta^{-1}) & 0 & 0 \\
0 & 0 & \frac{1}{2} (\delta^{z+1} + \delta^{-1}) & \frac{1}{2k_J} E_A (\delta^{z+1} - \delta^{-1}) \\
0 & 0 & \frac{1}{2k_J} E_A (\delta^{z+1} - \delta^{-1}) & \frac{1}{2} (\delta^{z+1} + \delta^{-1})
\end{bmatrix}
\]

where \( u \) is the longitudinal displacement in the \( z \)-direction, \( N \) is the axial force, \( c_L \) is the longitudinal wave speed defined as \( c_L = (E/\rho)^{1/2} \), \( E \) is the Young’s modulus, \( \rho \) is the density, \( k_L \) is the longitudinal wave number, \( A \) is the cross-sectional area, \( \beta T \) is the torsional angular displacement, \( T \) is the torsional torque, \( c_T \) is the torsional wave speed defined as \( c_T = (G/\rho)^{1/2} \), \( G \) is the shear modulus, \( J \) is the torsional rigidity of the system, \( k_T \) is the torsional wave number, \( n \) is the transverse displacement, \( I_F \) is the area moment of inertia, \( \beta \) is the flexural angular displacement, \( M \) is the bending moment, and \( Q \) is the bending shear force.
COUPLED ACOUSTIC TRANSFER MATRIX DERIVED FROM SHELL EQUATIONS

Based on the assumption that the flexural, longitudinal, and torsional wave modes are uncoupled to each other, the uncoupled acoustical transfer matrix is derived from the uncoupled wave equations in the previous section. However, in a real drill pipe system, these wave modes are coupled due to the pipe’s curvature as well as the Poisson’s ratio. Therefore, the coupling effects are considered to derive an accurate acoustic transfer matrix in this section. Here, the coupled acoustic transfer matrix is proposed to be derived from a “thick” cylindrical shell model.

When the thickness of a cylindrical shell is thin, only three governing equations are needed to describe the vibration motion of a shell in terms of displacements, \( u_r, u_\theta \) and \( u_z \). For a “thick” cylindrical shell such as typical drill pipes of which thickness is so thick that shear deformation and rotary inertia cannot be negligible, the rotating angles of \( \beta_e \) and \( \beta_z \), see Fig. 1) cannot be expressed in terms of the displacements, \( u_r, u_\theta \) and \( u_z \). Thus, two additional equations are required to calculate these two rotating angles. The five governing equations [3] to describe the motion of this thick shell structure including shear deformation and rotary inertia are expressed as

\[
\begin{align*}
\frac{k_e^2}{a} \frac{\partial^2 u_r}{\partial \theta^2} + \frac{k_e^2}{a} \frac{\partial^2 u_\theta}{\partial z^2} - \frac{k_e^2}{a} \frac{\partial^2 u_z}{\partial z^2} &- \rho \frac{\partial^2 u_r}{\partial t^2} = 0, \\
\frac{1}{a} \frac{\partial N_{e_r}}{\partial \theta} + \frac{1}{a} \frac{\partial N_{e_\theta}}{\partial z} - \frac{k_e^2}{a} \frac{\partial^2 u_z}{\partial z^2} &- \rho \frac{\partial^2 u_\theta}{\partial t^2} = 0, \\
\frac{1}{a} \frac{\partial M_{e_r}}{\partial \theta} + \frac{1}{a} \frac{\partial M_{e_\theta}}{\partial z} - \frac{k_e^2}{a} \frac{\partial^2 u_r}{\partial r^2} &+ \rho \frac{\partial^2 \beta_e}{\partial z^2} = 0, \\
\frac{1}{a} \frac{\partial M_{e_r}}{\partial \theta} + \frac{1}{a} \frac{\partial M_{e_\theta}}{\partial z} - \frac{k_e^2}{a} \frac{\partial^2 u_\theta}{\partial r^2} &+ \rho \frac{\partial^2 \beta_z}{\partial z^2} = 0
\end{align*}
\]

where \( u_r \) is the displacement in the direction denoted by its subscript (see Fig. 1), \( N_{ij} \) \((i,j = r, \theta, z)\) is the in-plane force and \( k_e^2 \) is the shear coefficient.

Two longitudinal wavenumbers, \( k_{Lz} \) and \( k_{Lr} \), two torsional wavenumbers, \( k_{Tz} \) and \( k_{Tr} \), and four flexural wavenumbers, \( k_{Fz}, k_{Fr}, k_{Fz}, \) and \( k_{Ft} \), can be obtained by solving the eigenvalue problem of the thick shell equations in Eqs. (5) – (9) at a single frequency. The corresponding eigenvector, \( \mathbf{U} = [U_r U_\theta U_z B_e B_z]^T \) can be then obtained at the frequency and the wavenumber. Then, a harmonic wave solution, \( \mathbf{u} = [u_r u_\theta u_z \beta_e \beta_z]^T \) can be expressed by using a superposition of the wave modes represented in terms of the eigenvalues and eigenvectors as

\[
\mathbf{u}(r, \theta, z, t) = \left[ A' \mathbf{U}_r e^{i \omega z} + A' \mathbf{U}_r e^{-i \omega z} + B' \mathbf{U}_r e^{i \omega z} + B' \mathbf{U}_r e^{-i \omega z} + C' \mathbf{U}_r e^{i \omega z} + C' \mathbf{U}_r e^{-i \omega z} + D' \mathbf{U}_r e^{i \omega z} + D' \mathbf{U}_r e^{-i \omega z} \right] e^{i \omega t}.
\]

The superposition coefficients in Eq. (10) at \( z = 0 \) can be rewritten in a matrix form as

\[
\mathbf{C}_u = \mathbf{T}_u \mathbf{X}_{ue}.
\]

where

\[
\mathbf{X} = \begin{bmatrix} u_r & u_\theta & M_r & \beta_e & T \end{bmatrix}^T.
\]

When \( z = L \), Eq. (10) leads to

\[
\mathbf{X}_{ue} = \mathbf{T}_u \mathbf{T}_u \mathbf{C}_u.
\]
By plugging Eq. (11) into Eq. (14), the acoustic transfer matrix $T$, between $z = 0$ and $z = L$, that describes the longitudinal, torsional, and flexural waves propagating in the pipe or joint, can be obtained as

$$T = T_L T_T T_F^{-1}. \quad (15)$$

### EXPERIMENTS AND FINITE ELEMENT ANALYSES FOR VALIDATION

An experiment and finite element (FE) analyses are conducted to validate the proposed transfer matrix approaches. Figure 2 shows the experimental setup. The material properties and the inner and outer diameters of the drill pipe are presented in Table 1. As shown in Fig. 2, the two drill pipe sections, with the same cross-section, which are connected with a joint in the middle, is hanged by using two steel cables. A Brüel & Kjær (B&K) Type 8206 impact hammer is used to excite the left end of the drill pipe. A B&K PULSE system (Model: 3560-B-130) is used to record acceleration data with a PCB Piezotronics “triaxial” accelerometer (Model: 356A24). For each axial or transversal excitation case, acceleration data at each of the 2 measurement points, $z = 2.13$ m and 4.84 m as indicated in Fig. 2, is recorded for 8 seconds at the sampling frequency of 1600 Hz. Then, the FRFs at the 2 measurement locations are estimated from the measured acceleration spectra.

The results of finite element (FE) analyses obtained by using a commercial FE software package, ANSYS, are also used to validate the proposed method. In particular, torsional wave cases are validated only with the FE results since it is difficult to generate pure torsional waves experimentally. The material properties and the inner and outer diameters listed in Table 1 are used to build a FE model of the pipe system in Fig. 2.

### RESULTS AND DISCUSSION

For the “longitudinal” excitation cases described in Fig. 2, the FRFs, in Fig. 3, estimated by using the proposed transfer matrix methods and the ANSYS analyses agree well with the experimental FRF results except the valley locations in Figs. 3(a) and 3(b) at approximately 160 Hz and 430 Hz. The measured anti-resonance amplitudes are expected to be inaccurate due to the low signal to noise ratio (SNR) at these anti-resonance frequencies and the high sensitivity of accelerometer placement error on the anti-resonance amplitudes. Although the first resonant amplitude at approximately 280 Hz is consistent throughout all of the results, the second resonant amplitude at approximately 470 Hz is underestimated with all of the predicted results. This may be caused by the overestimation of the damping value at this second resonance frequency where the resonance amplitude is significantly sensitive to the damping value. For the “torsional” excitation case, the FRFs obtained from the proposed transfer matrix methods agree well with the ANSYS analysis results in Fig. 4.

For the “flexural” excitation case in Fig. 5, at low frequencies (e.g., below 100 Hz), the boundary condition in the experiment cannot be assumed as a free-free boundary condition since the drill pipe is hanged by the two steel cables.
cables as shown in Fig. 2, while the predicted results are based on the free-free boundary condition. Therefore, there are some discrepancies between the measured and predicted results in the low frequencies. However, above 100 Hz, the discrepancies become negligible, resulting in the predicted FRF results matched well with the experimental results. At high frequencies above 250 Hz, the FRFs predicted from the coupled acoustic transfer matrix are better fitted to the experimental results than those from the uncoupled acoustic transfer matrix.

FIGURE 3. Experimental and predicted FRF results for case of “longitudinal” excitation ($L = 9.74$ m): (a) $z = 0.219$, (b) $z = 0.5 L$.

FIGURE 4. Experimental and predicted FRF results for “torsional” excitation ($L = 9.74$ m): (a) $z = 0.219$, (b) $z = 0.5 L$.

FIGURE 5. Experimental and predicted FRF results for “flexural” excitation ($L = 9.74$ m): (a) $z = 0.219$, (b) $z = 0.5 L$.

CONCLUSION

In this paper, the uncoupled and coupled acoustic transfer matrices are derived in order to analytically estimate the longitudinal, torsional, and flexural waves propagating through long drill strings in a computationally-efficient way. The uncoupled transfer matrix is here derived from the three uncoupled wave equations. Since the wave modes are weakly coupled in reality, the acoustic transfer matrix including the coupling effects is derived from a thick circular cylindrical shell model. The proposed methods are validated experimentally and numerically.
Through the comparison between the measured and predicated results, it is shown that the simplistic uncoupled transfer matrix approach can be used to accurately predict the critical vibrations of a drill pipe system such as stick-slip and bit-bounce below 500 Hz for the drill string considered in this article although the coupled transfer matrix approach can generate the better results than the uncoupled transfer matrix approach for estimating the torsional and flexural vibration.

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