Pneumatic vibration isolation is the most widespread effective method for creating vibration-free environments that are vital for precise experiments and manufacturing operations in optoelectronics, life sciences, microelectronics, nanotechnology and other areas. The modeling and design principles of a dual-chamber pneumatic vibration isolator continue to attract attention of researchers. On the other hand, behavior of systems of such isolators was never explained in the literature in sufficient detail. After a brief summary of the theory and a model of a single standalone isolator, the dynamics of a system of isolators supporting a payload is considered with main attention directed to three aspects of their behavior: first, the static stability of payloads with high positions of the center of gravity; second, role of gravity terms in the vibration transmissibility; third, the dynamic stability of the feedback system formed by mechanical leveling valves. The direct method of calculating the maximum stable position of the center of gravity is presented and illustrated by three-dimensional stability domains. A numerical method for feedback stability analysis of self-leveling valve systems is provided, and the results are compared with the analytical estimates for a single isolator. The relation between the static and dynamic phenomena is discussed.
INTRODUCTION

Pneumatic vibration isolators have been used for precision vibration control for several decades. They help meet stringent vibration control requirements in demanding application areas such as optoelectronics, life sciences and nanotechnologies. Besides excellent vibration isolation characterized by natural frequencies as low as 1 Hz, they provide high load capacity that enables researchers to isolate large setups including elements like cryostats, high-power lasers, atomic force microscopes, scanning tunneling microscopes, optical assemblies and precision acoustical equipment.

The basic theory and mathematical models of a pneumatic vibration isolator had been developed by Cavanaugh [1], Bachrach and Rivin [2], DeBra and Bryan [3]; see also the textbook by Rivin [4]. In these works, the model of a double-chamber damped pneumatic isolator as a three-parameter system was established based on thermodynamic analysis of the gas volume. This model was further refined by Erin, Wilson and Zapfe [5,6], Lee and Kim [7], Pu, Luo and Chen [8]. In recent years, considerable work was devoted to improving low-frequency properties of pneumatic isolators by means of active time delay control [9-11]; another approach to suppressing low-frequency resonance of a pneumatic isolator can be found in [12,13].

Typically pneumatic isolators are used in sets of three or more, supporting a platform such as an optical table loaded with vibration-sensitive equipment. Leveling valves are used for maintaining the position of the platform. Dynamics of these platforms can present some challenges to effective and stable set-up, especially for large elevated payloads. The behavior of systems of low-frequency pneumatic isolators was never explained in the literature in sufficient detail. This paper considers, after a brief summary of the model of a single standalone isolator, dynamics of a system of pneumatic isolators supporting a payload. Attention is directed to three aspects of the system behavior: first, static stability of payloads with high positions of the center of gravity; second, role of gravity terms in the vibration transmissibility; third, dynamic stability of the feedback system formed by mechanical leveling valves.

MATHEMATICAL MODEL OF A PNEUMATIC VIBRATION ISOLATOR

A pneumatic chamber, also known as an acoustic cavity, is the “heart” of a pneumatic vibration isolator. The linear stiffness of the pneumatic chamber is given by the following equation:

\[ K = \frac{\gamma p_0 A^2}{V}. \]  

(1)

Here \( p_0 \) is the total pressure, \( A \) is the piston area, and \( V \) is the internal volume, see Figure 1(a). Equation (1) is derived from the assumption that the compression – expansion process of air in the chamber is adiabatic. Hence, \( \gamma \) is the adiabatic constant, or ratio of specific heats, equal to 1.41 for air. The adiabatic hypothesis has been subject to some scrutiny [14]; both theoretical solution and experimental evidence show that, for typical sizes of air chambers of pneumatic vibration isolators, thermal conductivity of air is not significant, and adiabatic equations may be used at all frequencies, even for slow quasi-static processes. A linear model of the pneumatic chamber is adopted in this paper. Heerjes and van de Wouw [15] found that the non-linear characteristics of the chamber affect the dynamic compliance and transmissibility functions when the displacements are not small compared to the length of the chamber; such cases are excluded from this analysis.

Equation (1) illustrates one of the most valuable properties of a pneumatic isolator, namely, its ability to adjust the stiffness to the load. Since the pressure increases with the supported weight, the natural frequency tends to stay almost constant in a range of supported loads.

Another element that affects the stiffness of a real pneumatic isolator is the diaphragm, sometimes called “rolling diaphragm,” that seals the air volume and allows free motion of the piston in process of re-leveling. Equation (1) is still applicable with \( A \) meaning an “effective” piston area which covers the space up to half-span of the inflated diaphragm, see Figure 1(b). Speaking about the linear stiffness of the diaphragm, distinction should be made between two ranges of piston displacement. When the displacements are small, the diaphragm works like a linear elastic spring with very low damping. This range spans about 10 microns of relative displacement amplitudes, which covers typical displacements in a laboratory environment. This value of stiffness should be used when analyzing vibration isolation performance (transmissibility) of the isolator; it is roughly proportional to the square root of the...
load [14]. At larger displacements, the diaphragm starts to “roll,” thereby providing higher friction and lower stiffness.

Damping is necessary in isolation systems in order to limit the resonance amplification. To this effect, a double chamber “self-damping” design is used in state-of-the-art pneumatic vibration isolators. The schematic is shown in Figure 1(c). The air volume is divided into two chambers: the upper one, \( V_1 \), sometimes called a compliance chamber, and the lower one, \( V_2 \), called a damping chamber or a surge chamber. The two chambers are connected by a flow resistance orifice that is designed to provide a laminar viscous flow of gas between the chambers.

If the pressure in the upper chamber deviates from the equilibrium value \( p_0 \) by a small quantity \( p_1 \), and the pressure in the lower chamber deviates from \( p_0 \) by a small quantity \( p_2 \), then the volume flow of gas from \( V_2 \) to \( V_1 \), \( q_2 \), is described, in the linear approximation, by the equation

\[
p_1 - p_2 = -c_{12} \dot{q}_2.
\]

(2)

Here \( c_{12} \) is the viscosity coefficient depending on the viscosity of the gas and the geometry of the orifice. To create a linear analytical model of the two-chamber pneumatic isolator, complement (2) with the linear adiabatic flow equations for both chambers:

\[
p_1 = -\frac{\rho p_0}{V_1} (q_1 - q_2), \quad p_2 = -\frac{\rho p_0}{V_2} q_2.
\]

(3)

Change of volume of the upper chamber, \( q_1 \), is caused by the small motion of the piston, \( u_1 = q_1/ A \). Variation of the force acting on the isolator results from the change of pressure and the additional force exerted by the diaphragm: \( P = -A p_1 + K_d u_1 \), where \( K_d \) is the linear stiffness of the diaphragm (see Figure 1 for the sign convention). Introducing equivalent displacement for the second chamber, \( u_2 = q_2/ A \), one obtains from (2), (3) the following system of equations for the mechanical model of a two-chamber isolator:

\[
P = K_1 (u_1 - u_2) + K_d u_1,
\]

\[
K_1 (u_1 - u_2) - K_2 u_2 = C_{12} \dot{u}_2,
\]

(4)
where \( K_1 = \frac{p_0 A^2}{V_1}, \ K_2 = \frac{p_0 A^2}{V_2}, \ C_{12} = c_{12} A^2 \). This corresponds to the schematics shown in Figure 2. Here \( K_1 \) represents the stiffness of the upper chamber, \( K_2 \) represents the stiffness of the lower chamber, and \( C_{12} \) represents the viscosity of the gas moving through the orifice.

**FIGURE 2.** Mechanical model of a two-chamber pneumatic isolator.

Dynamic stiffness of the two-chamber pneumatic isolator, \( K(\omega) \), has two limiting values: at low frequencies \( (\omega \rightarrow 0) \) the flow resistance is low, and \( K(\omega) \) assumes the lowest value, \( K_l \), which corresponds to both chambers acting as one: \( K_l = \frac{p_0 A^2}{V_1 + V_2} + K_j \); at high frequencies the flow resistance grows, and \( K(\omega) \) approaches the highest limiting value, \( K_h \), which corresponds to the top chamber acting alone: \( K_h = \frac{p_0 A^2}{V_1 + K_j} \).

### STATIC STABILITY OF VIBRATION ISOLATED SYSTEMS WITH HIGH CENTERS OF GRAVITY

State-of-the-art pneumatic isolation systems are equipped with leveling valves that are used to set up the table in precise horizontal position and maintain this position against external perturbations or changes in the payload. The system has three leveling valves, notwithstanding the number of the isolators. This means that in systems having more than three isolators some of the isolators must be slaved together, i.e., their air volumes connected. When disturbed, they can freely trade air between themselves, so they do not offer any resistance to rotation about their common center. Standard ways of connecting the isolators in typical configurations are shown in Figure 3. In a static position, the pressure is the same in each isolator of the group; therefore each isolator bears the same load and has the same stiffness. A group of slaved-together isolators is therefore statically equivalent to one isolator placed at their geometrical center with the stiffness equal to the sum of individual stiffnesses. Consequently, there are always three effective support points and the system is always statically determinate.

There are two types of instability in pneumatic vibration isolation systems. First, the system can become statically unstable because the center of gravity is too high. Second, the system can become dynamically unstable because the self-leveling valves have the gains set too high. This section is concerned with static stability.

High performance vibration isolation systems are characterized by very low stiffness of the isolators. State-of-the-art pneumatic vibration isolation systems can have natural frequencies of vertical motion as low as 1 Hz. That gives rise to potential static instability if payloads have high centers of gravity. A general criterion for static stability states that a mechanical system in a position of static equilibrium is stable if any small deviation from this position causes the potential energy to increase.

To analyze the static stability, calculate the changes in potential energy due to small deviations from the equilibrium described by a small vertical motion, \( w_0 \), and a small rotation around a horizontal axis by an angle \( \phi \) with components \( \phi_x \) and \( \phi_y \). Only quadratic terms should be taken into account since the linear terms cancel out due to conditions of equilibrium. The change in the gravitational energy is

\[
U_g = -\frac{1}{2} mgH(\phi_x^2 + \phi_y^2),
\]

where \( m \) is the total mass of the system. The change in the elastic energy of isolators is

\[
U_e = \frac{1}{2} \sum_{j=1}^{n} k_j w_j^2,
\]
where $k_j$ are vertical static stiffnesses and $w_j$ are vertical displacements of the isolators, $j = 1, \ldots, n$. Note that $k_j$ is defined by the total air volume of the isolator and does not include the stiffness of the diaphragm. The change in the potential energy can be reduced to the form

$$
U = \frac{1}{2}(I_x - mgH)\varphi_x^2 + \frac{1}{2}(I_y - mgH)\varphi_y^2 - I_{xy} \varphi_x \varphi_y,
$$

where $I_x$, $I_y$, and $I_{xy}$ are defined by positions of isolators and pressures in each group. The system is statically stable if $U > 0$ for any $\varphi_x$ and $\varphi_y$. The analytical criterion is easily derived using the Sylvester’s rule. The resulting stability domain for the center of gravity is a dome-shaped area over the “stability triangle” in the horizontal plane. Note that the height $H$ in the previous equations is measured from the diaphragm level of the isolator. It may be more convenient to use, instead, the height $H_b$ from the bottom surface of the isolated platform, $H_b = H - d_b$, where $d_b$ is the distance from the bottom surface to the diaphragm level. The examples of the stability domains for one standard model of a pneumatic isolator are shown in Figures 4 and 5. The geometry of these domains can be visually described as a combination of parabolic and pyramid shapes.

**GRAVITATIONAL TERMS AND VIBRATION TRANSMISSIBILITY**

As the system approaches static instability, its lowest natural frequency tends to zero. This shows that performance analysis of a high-CG system must take into account the gravitational components of the system potential energy and, correspondingly, of the stiffness matrix, which is sometimes ignored in texts on vibration isolation. Without going into details, this dependence will be illustrated here by an example. Consider the system described in the caption to Figure 4 assuming that the CG is placed directly above the center of the platform. Assume that the mass is distributed symmetrically, so that $J_x = J_y = m\rho^2$, $J_{xy} = 0$. Horizontal stiffness is approximated by $k_{hor} = m\rho^2(1 + 0.3i\omega/\omega_{hor})$, $\omega_{hor} = 2\pi \cdot 1 \text{ s}^{-1}$. Figure 6 plots the calculated transmissibility in the horizontal x-direction from the floor to the center of gravity of the isolated platform. The graphs show that the difference between vibration transmissibilities calculated with gravitational terms taken into account and those calculated with gravitational terms omitted can be very substantial.
FIGURE 4. Stability domain for the CG of isolated payload. The light blue plane represents the bottom surface of the isolated platform (optical table). The isolators ($A = 0.017 \text{ m}^2$, $V_1 = 0.42 \cdot 10^{-3} \text{ m}^3$, $V_2 = 0.91 \cdot 10^{-2} \text{ m}^3$, $d = 0.061 \text{ m}$) form a rectangle 0.914 m by 1.321 m, two of them slaved along the long side of the table. Isolators are drawn not to scale. Total load 1820 kg. Maximum height $H_b = 0.657 \text{ m}$ over the bottom surface of the table. Restriction on maximum admissible load on each isolator is not taken into account.

FIGURE 5. Same as Figure 4, but the isolators are slaved along the short side of the platform. Maximum height $H_b = 0.556 \text{ m}$.

SELF-LEVELING SYSTEMS BASED ON MECHANICAL FEEDBACK

Pneumatic vibration isolators can have very low linear stiffness. So, even small changes of the load could lead to unacceptably large deviations in the position of the isolated platform, up to encountering hard stops. In order to avoid this, most state-of-the-art pneumatic vibration isolators are equipped with self-leveling mechanisms ensuring stationary position of the payload. In this Section, the mechanism of self-leveling is described using a linear model.
of the mechanical leveling valve. The main objective of this analysis is to find the criterion of stability for oscillations of height controlled systems.

![Graph](image)

**FIGURE 6.** Thick line: transmissibility in the x-direction from the floor to the CG for a symmetric payload on four isolators with a CG placed directly above the center, see Figure 4. Total load 1820 kg. \( L = 0.914\text{m}, \ W = 1.321\text{m} \). The CG is placed 0.635 m above the diaphragm level of the isolators. Dashed line: the same calculated without gravitational terms. Thin line: nominal horizontal transmissibility of the isolators. (a) low moments of inertia, \( \rho_c = W/4 \). (b) high moments of inertia, \( \rho_c = W/2 \).

Consider a single standalone two-chamber pneumatic isolator shown in Figure 1 (c) supporting a mass \( m \) and equipped with a mechanical valve that allows certain volume of air \( q_v \) at a flow rate of \( q_v' \) into the lower chamber in response to the displacement of the payload:

\[
\dot{q}_v = -Gu_u,
\]

where \( G \) is the gain of the valve. Equations (4) must be augmented as follows:

\[
P_l = K_l(u_1 - u_2) + K_g u_1,
K_i(u_1 - u_2) - K_2(u_2 - u_v) = C_{12} \dot{u}_2,
\dot{u}_v = -G_0 u_u,
\]

where \( u_v = q_v/A \), \( G_0 = G/A \). One should have in mind that the re-leveling motion happens in the displacement range much wider than a few microns vibration amplitude typical for a laboratory environment. The re-leveling valves usually have dead bands of tens to hundreds of microns. Therefore, for a safe analysis we neglect the stiffness of the diaphragm, \( K_d \), and the corresponding loss factor. Damping can be normalized to the time constant, \( \tau \), of the pressure equalizing between the chambers, as follows: \( \tau = C_{12}/(K_1+K_2) \).

To analyze the stability of this motion, one must research the position of the roots of the characteristic polynomial in terms of the Laplace parameter \( s = i\omega \), that is, the solutions of the following characteristic equation:

\[
m s^2 (1 + \tau s) + K_i + K_h \cdot \tau s + \frac{G_0 K_l}{s} = 0.
\]

The motion is asymptotically stable if all the roots reside in the left half-plane of the complex \( s \) plane. Application of the classic Routh-Hurwitz criterion leads to the following exact criterion of stability:

\[
G_0 < \tau \omega_1^2 N,
\]

where \( \omega_1^2 = K_1/m \), \( N = V_2/V_1 \).

Stability analysis of a multi-support system of pneumatic isolators presents a more complicated task. Stability depends on the height of the center of gravity and the moments of inertia of the supported load. Vertical, horizontal
and rotational degrees of freedom should be taken into account. The condition of stability is derived from dynamic equations of the payload, considered as a rigid body together with the table supported by the isolators.

As an example, consider again the system described in the caption to Figure 4, assuming that the CG is placed directly above the center of the platform. Horizontal stiffness is again approximated by $k_{hor} = m \omega_{hor}^2 (1 + 0.3 i \omega_{hor} / \omega_{hor}^2)$. Figure 7 plots the maximum stable gain as function of the CG height for the two cases. The system is unstable for all values of gain, including zero, if the height of the CG is beyond that determined by static stability. For low-profile payloads with small radii of inertia the maximum gain is the same as that determined from Equation (7), $G_0 = 2.24 \text{ s}^{-1}$. This is the case when the lowest natural frequency corresponds to the vertical mode of vibration. In other cases rotational (rocking) modes have lower natural frequencies, and the maximum gain is lower. The circle and square symbols denote results obtained by a simplified method, applying formula (7) for the single-degree-of-freedom case with the lowest natural frequency of the undamped uncontrolled system in place of $\omega_c$. In most cases the simplified method yields the results close to the results of the full analysis. The deviations are due to horizontal damping that is not taken into account by the simplified procedure.

![Figure 7](image)

**FIGURE 7.** Stability limits of the leveling valves’ gain for a symmetric payload on four isolators with a CG placed directly above the center. Total load 1820 kg. $L = 0.914 \text{ m}$, $W = 1.321 \text{ m}$ as in Figure 4. Maximum height is 0.718 m from the diaphragm level as in static case. Red line – high moments of inertia, $\rho_c = W/2$. Blue line – low moments of inertia, $\rho_c = W/4$. Maximum gain equals 2.24 $\text{ s}^{-1}$ in accordance with Equation (7). Symbols show results of Equation (7) using the lowest eigenfrequency of the undamped uncontrolled system.

Stability limits such as those shown in Figure 7 can be used directly for design of precision linear height control systems with proportional valves. If the valves are not proportional, as it frequently happens in practice, these data give qualitative understanding of the relationship between the system design parameters, such as payload geometry and placement of isolators, and the stable area of valve operation.

**CONCLUSIONS AND DISCUSSION**

The analytical and numerical procedures outlined in this paper, as well as graphical illustrations, clarify the design and application of pneumatic vibration isolators and, especially, systems of such isolators. It has been shown that the stability domain for the center of gravity of a pneumatically isolated system is a dome-shaped spatial area that can be visually described as a combination of parabolic and pyramid contours. An analytical procedure has been established for constructing such domains. A role of gravitational factors in the vibration transmissibility has been illustrated. Dynamic stability of the self-leveling pneumatic systems has been analyzed with emphasis on the
interaction of static and dynamic parameters such as feedback gains, position of the center of gravity and moments of inertia.

A note should be made about the analytical approach. There are three distinct operation ranges of the pneumatic isolators in terms of displacement amplitudes. The displacement range of precision isolation in typical laboratory settings covers only a few microns. The diaphragm is not “rolling” and its full stiffness should be taken into account when estimating the vibration transmissibility. The leveling valves are closed within their dead band. In the displacement range from few microns up to the dead band of leveling valves (usually tens or hundreds of microns) the diaphragm is “rolling,” thereby providing a diminishing resistance to displacement of the piston. This displacement range is critical for the static stability analysis; the stiffness of the diaphragm can be neglected for a safe design. Finally, the range beyond the dead band of the leveling valves is important for the dynamic stability analysis of the feedback re-leveling system. In this work, traditional linearized models were adopted, thereby neglecting the non-linearities introduced by dead bands and increased stiffness in the small displacement range. Although this “piecewise linear” approach is deemed acceptable for approximate practical estimates of the system stability, more intricate nonlinear models may be necessary for more advanced tasks such as designing active control systems based on pneumatic isolators.

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REFERENCES