High frequency backscattering from sandy sediments: single or multiple scattering

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As the sand grain size approaches the acoustic wavelength, the underwater backscattering strength increases. Laboratory measurements show that the shallow-grazing angle backscattering strength increases as the third power of the normalized grain diameter, until it reaches a saturation level. In this regime, it has been shown that the attenuation of the sound in the sand increases as the fourth power of frequency and the speed of sound decreases with increasing frequency. The most likely explanation for the attenuation and speed dispersion is multiple scattering [Schwartz and Plona, Journal of Applied Physics, vol. 55, 1984, and Kimura, J. Acoust. Soc. Am. EL, vol. 129, 2011]. The multiple scattering model is applied to the backscattering process, and it is shown to be consistent with the published backscattering data. [Work supported by the Office of Naval Research, Ocean Acoustics Program].

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INTRODUCTION

Bottom roughness and volume inhomogeneities are the usual mechanisms associated with bottom backscattering. This paper concerns a different mechanism that is often ignored. Samples of published backscattering strength $BS$ of sandy sediments, at a grazing angle of 10 degrees, plotted as a function of the normalized grain size, defined as the acoustic wavenumber in water multiplied by the mean grain diameter, $kd$, are shown in Fig. 1. The data span $kd$ values from 0.01 to 30. The laboratory data are from Nolle et al. 1963 and Thorne et al. 1988. It is noticed that, between $kd$ values 0.2 and 2, the laboratory data may be fitted with a straight line with a slope of 30 dB per decade. For $kd$ values less than 2, the in-situ measurements, shown as hollow symbols, fall above or close to the straight line. It does not apply at $kd$ values greater than 2, most likely because the grains become large compared to the acoustic wavelength and a different scattering regime is entered. For $kd$ values less than 2, the line may be regarded as a lower bound. Theoretical basis for it will be developed below.

![Figure 1](image)

**FIGURE 1.** A few published backscattering strength $BS$ measurements of sandy sediments, at a grazing angle of 10 degrees, plotted as a function of the normalized grain size, defined as the acoustic wavenumber in water multiplied by the mean grain diameter, $kd$.

SCATTERING THEORY

Spherical grains and the single scattering assumption

One approach is to consider the scattered energy from each sand grain using the single scattering approximation. Let the sand grain be approximated as a spherical scatterer. In the Rayleigh regime, the scattering cross section of a fluid sphere is given by Johnson 1977 in terms of the acoustic wavenumber $k$, the sphere diameter $d$, and a material constant $c$, as,

$$
\sigma_{fs} = \frac{\pi}{64} cd^2 (kd)^4 \quad kd << 1
$$

(1)

Given that the sediment has a porosity $\beta$, then the average number of grains per unit volume is given by,

$$
N = \frac{1 - \beta}{\pi d^3}
$$

(2)

Neglecting any mutual interference effects, the scattering cross section per unit volume, for low values of $kd$, is given by,

$$
\sigma_{vfs} = \sigma_{fs} N
$$

(3)
For a rigid sphere, the material constant $c$ has a value of approximately 0.31, and for water-saturated sand, the porosity is approximately 0.36. Using these values, the result is,

$$\sigma_{f0} = \sigma_{fs} N = 0.019k^4 d^3 \quad kd \ll 1$$ \hspace{1cm} (4)

### Scattering cross section with multiple-scattering

Another approach is to use the multiple-scattering model for high-frequency sound attenuation in granular media and the conservation of energy. Following Schwartz and Plona 1984, Kimura 2011 measured the attenuation of sound in water-saturated sand and glass beads, and found that the attenuation coefficient, in decibels per meter, due to multiple-scattering may be represented by an expression of the form,

$$\alpha_{l(SCA)} = 0.0131k^4 d^3 \quad \text{dB/m} \hspace{1cm} (5)$$

Assuming that all the energy lost to multiple-scattering is reradiated, then the fractional energy loss and the effective scattering cross section per unit volume $\sigma_{v(SCA)}$ are both given by,

$$\sigma_{v(SCA)} = \frac{\alpha_{l(SCA)}}{10 \log(e)} = 0.003k^4 d^3 \hspace{1cm} (6)$$

This has the same frequency and grain diameter dependence as the single-scatter expression for spherical scatterers shown in Eq.(4), but with a much smaller coefficient. Eq.(6) is the preferred expression because it is based on conservation of energy, while that of Eq. (4) is not properly constrained.

### Skin depth

With reference to Eqs. (2.17) and (2.87) in Tolstoy and Clay 1987, and approximating the sediment as a fluid, the effective penetration depth, or skin depth, $d_s$ of the acoustic energy is given by,

$$d_s = \left[ W_b W_w \int_{0}^{\infty} \exp(2i\gamma_b z) dz \right]^2 \hspace{1cm} (7)$$

The pressure transmission coefficients $W_b$ and $W_w$ are for crossing from water into the bottom and back, respectively, at the chosen grazing angle.

$$W_b = 2 \frac{\rho_b \gamma_w}{\rho_b \gamma_w + \rho_b \gamma_b} ; \quad W_w = 2 \frac{\rho_w \gamma_b}{\rho_b \gamma_w + \rho_w \gamma_b}$$

where $\alpha = k \cos \phi$, $\gamma_w = k \sin \phi$ and $\gamma_b = \sqrt{k_b^2 - \alpha^2}$ \hspace{1cm} (8)

In the above equations, $\phi$ is the grazing angle of the incident sound wave, measured relative to the horizontal, $\alpha$ is the horizontal component of wavenumber, $k_b$ is the complex wavenumber in the sediment, $\gamma$ is the vertical component of wavenumber in water, and $\gamma_b$ is the vertical component of wavenumber in the sediment. From Kimura 2011, it is shown that the sound speed in the sediment, hence the wavenumber, is frequency dependent. The sound speed at high frequencies $c_l$ is reduced by the multiple-scattering effect relative to its unmodified value $c_{lo}$, and has the form,

$$c_l = c_{lo} \left( 1 - 0.015(kd)^{2.45} \right) \hspace{1cm} (9)$$

This expression clearly is not valid for very large values of $kd$, since a negative speed is unphysical. Plots in Kimura 2011 indicate that it is valid for $kd$ values up to 3. A plot of the sound speed is shown in Fig. 2.
Backscattering strength

The backscattering strength $BS$ is estimated from the scattering cross section per unit volume $\sigma_{\text{v(\text{SCA})}}$, multiplied by the effective penetration depth $d_s$, and divided by the solid angle over which the scattered energy is radiated, which is $2\pi$ steradians, since the energy is radiated into the upper half-space. This will be called the multiple-scatter model of backscattering strength because it uses the attenuation and sound speed variation that is predicted by the multiple-scattering model of sound propagation in granular media, from Kimura 2011. It is expected to be valid for $kd$ values up to 3 in line with the expression for the sound speed in Eq. (9).

$$BS = 10 \log \left( \frac{\sigma_{\text{v(\text{SCA})}} d_s}{2\pi} \right) \quad kd < 3$$

(10)

COMPARISON WITH MEASUREMENTS

The result for a grazing angle of 10 degrees is shown in Fig. 3. At low values of $kd$, the model matches the 30 dB per decade trend quite well. It is interesting to note that the deviation from the 30 dB per decade trend at high values of $kd$ is also predicted by the model. As $kd$ increases, initially, the backscattering strength increases at a higher rate as the sound speed drops and approaches that of the sound speed in water, because of the increase in penetration depth. The saturation effect is caused by the rapidly increasing attenuation, which effectively reduces the penetration depth and negates the effect of any further increase in volume scattering strength. The post saturation reduction is due to the negative sound speed dispersion, as shown in Fig. 2, which further reduces the effective penetration depth. These results confirm the central role of multiple-scattering process in this regime. However, the model is incomplete because it ignores the effect of shear waves.
FIGURE 3. A few published backscattering strength $BS$ measurements of sandy sediments, at a grazing angle of 10 degrees, plotted as a function of the normalized grain size, defined as the acoustic wavenumber in water multiplied by the mean grain diameter, $kd$, compared to the multiple-scatter model of backscattering strength.

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REFERENCES


