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1pUW6. Resolution analysis of the inverse problem for geoacoustic experiment design
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This work explores effects of experiment geometry and array configuration on the resolving power of a continuous ocean bottom geoacoustic inverse problem in a shallow water environment. The uncertainty and resolution of this problem, in which ocean bottom P-wave velocities as a function of depth are estimated from noisy acoustic pressure waveforms received on vertical and horizontal line arrays in the water, can be investigated before the experiment is conducted, allowing one to improve or optimize the problem parameters to best configure an experiment during its planning phase. In this work the resolution results of complete synthetic geoacoustic inversions at varying geometries and array configurations are compared with resolution results at various candidate seabottom profiles, initially using standard techniques of linearized inverse theory. Additional comparisons and analysis address the nonlinearity of the problem, which causes a dependence of the resolution results on the bottom profile being solved for -- which is unknown or only partially known, and Monte Carlo analysis is used to show where the linearity approximation breaks down in the resolution results. [Work partially funded by ONR]

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INTRODUCTION

The ocean geoacoustic inverse problem is the estimation of physical properties of the ocean bottom from a set of acoustic receptions in the water column. This work explores pre-experiment analyses in designing a geoacoustic experiment that estimates a (possibly piece-wise) continuous profile of ocean bottom properties from full-waveform acoustic pressure time-series measured in a shallow-water, short-range geometry.

To address the nonlinearity of waveform inverse problems, common solution methods seen in the underwater acoustics literature are Markov Chain Monte Carlo, simulated annealing, and genetic algorithms, for finding a few layers' properties in a Bayesian framework [1–6]. But locally-linearized inversion approaches are still useful in some of these problems, including waveform inversion, when appropriately applied [7–12]. (One must use a starting model that is close enough to the solution, i.e. within the same local minimum to avoid “cycle skipping”.) Rajan used full-waveform continuous geoacoustic inversion in two papers in the early nineties [11, 12], but analyses of uncertainty, resolution, and regularization were not included in that work. The subject is revisited here, in more detail and with a view to experiment design.

Experiment design and optimization are not themselves new, even in the underwater and ocean acoustic communities. For example, Dosso and Wilmut [13] investigated information content of a Bayesian geoacoustic inverse problem that had one fluid sediment layer over a fluid basement. Their work analyzed improvements in the marginal posterior probability densities of properties in those layers as a function of experiment geometry, frequencies, and other such factors. Barth and Wunsch [14] optimized geometries of an ocean acoustic tomography experiment by maximizing the smallest unregularized eigenvalue of the linearized problem. Experiment design for similar problems in seismology includes work by Curtis and Snieder [15], Curtis [16], and Routh et al. [17]. There have also been a few works in the underwater acoustics community in which the results quantify the resolution of the solution for continuous inversions of the ocean bottom [4, 18–23].

Raytraces could provide some general feel for which experiment geometries and configurations are better for a given problem, but only offer a highly qualitative picture in that role. What is missing is more quantitative information, such as: at what specific resolution will the geoacoustic inversion actually resolve the bottom properties at different depths for the experimental setup one chooses? How will the regularization inherent in geophysical inversions, and specifically the regularization used in this problem, affect the choice of experiment setup? Will a horizontal or vertical array be better in a particular scenario to intersect those rays? How should the hydrophones in these arrays be spread out? Questions like these are not well answered by simply looking at raytraces, but are precisely the topics that a resolution analysis like this can address.

PROBLEM DESCRIPTION

The problem is formulated as frequentist, Tikhonov-regularized, locally-linearized inversion solved by Gauss-Newton iteration; the whole process and mathematical description is readily found in standard inverse theory texts [26, 27]. The linearized frequentist approach provides a convenient tool for optimizing experiment design before measuring any experimental data, in the form of the model resolution matrix. The columns of this matrix produce the “averaging kernels” of the inversion at each depth. These provide the resolution information (to first order) and thus a measure of inversion performance as a function of experiment geometry, an important consideration in experiment design.

The data in this problem are the measured or predicted time-series of acoustic pressures received on a set of hydrophones in a vertical line array (VLA). The source signal is a
sinusoidally-shaped pulse of a 100Hz carrier, four periods long. The source spectrum is assumed known in the data, for example via monitor hydrophone placed near the source, allowing to retain a $L_2$ norm for data misfit in the problem. The data remain in the time domain as in examples of linearized full-waveform inversion by Luo, Zhou, and Schuster [7–10]; note they are full time-series waveforms, not travel times. By Parseval’s Theorem, the $L_2$ norm of time-domain data misfit is precisely proportional to the $L_2$ norm of complex-valued frequency-domain (known-source-spectrum) data misfit, as was used for example in Rajan’s waveform inversion papers mentioned above.

The inversion estimates the bottom model profile of P-wave velocity as a function of depth. Other bottom geoacoustic properties such as density, attenuation, and shear properties are based on empirical regression relationships to the P-wave velocity profile [24, 25]; sensitivity analyses of these regression relationships show low sensitivity of the inversion’s objective function to uncertainty in the regressions. The continuous model profile is parameterized by a fine discretization – meaning finer than the resolution of the inversion, which is verified afterwards.

The problem demonstrated here uses a VLA at ranges 0.1–3.0km with 40 receivers at depths 20–176m (essentially the whole water column). The source depth is 5m and the water depth is 200m; the ocean and bottom are one-dimensional.

**Resolution Results**

The plots of averaging kernels in Figure 1 compare the resolution results for different receiver array configurations and lengths. As the impulse response functions of the inversion at each depth and range, the averaging kernels are more narrow and “pointy” when resolution is good, and more spread out as the resolution decreases. They can be shown at any depths, so they are presented every 40m in depth for clarity, alternating in gray and black color to better pick out their features. They are also shown every 0.5km in range, except for the closest range of 0.1km. At close range and the deepest depths, the averaging kernels become “messy” when only using one or two VLAs (the same is true for too short of an HLA). This highlights the problem’s instability at those geometries which is not fully ameliorated by the regularization used. It is especially useful in the experiment planning to see the effects of a given choice of the problem regularization, to see 1.) geometries one might wish to avoid for a given regularization choice, 2.) how changing the placement or number of arrays can fix such problems, or 3.) how changing the regularization choice can do so.

Based on Figure 1, if one were interested in P-wave velocities in the top 100m of the bottom, one could use a single 40-hydrophone VLA at 1.5km range and obtain 15-20m depth resolution. But if much higher resolution than that were desired, spreading the same 40 hydrophones across four VLAs at 0.1, 0.5, 1.0, and 1.5km would provide this (at the cost of more effort!).

The linear approximations to the uncertainty and resolution used thus far have their limits in this nonlinear inverse problem, and it is prudent to explore where the linear approximations break down, and how they break down, in this problem. The random noise added to the data in the synthetic inverse problem is recreated $N$ times for $N$ Monte Carlo runs, and the inverse problem is solved $N$ times. The results of 40 Monte Carlo runs are shown in Figures 2a–2c. Only 40 runs were computed here, so in this particular example one cannot calculate accurate statistics such as moments of the model distribution, but the 40 samples still give useful information about patterns in where the linear approximation falters. Much more detail than that may get lost anyway in the variability of results between different bottom model profiles.

The smooth gray profiles in Figure 2a are the Monte Carlo solutions and the jagged black profile is the “true” bottom model for reference – note however it is the smoothed version of this true bottom model that the frequentist Monte Carlo solutions are estimating. The large
discontinuity greatly reduces the acoustic energy reaching the model profile below it, reducing the sensitivity of the model at those deeper depths, in turn increasing the model solution uncertainty from the inversion. As the model uncertainty increases, the discrepancies between nonlinear effects in the uncertainty and its linear approximation become apparent. More physically speaking, in this example the nonlinearity of the problem causes the velocity in the low sensitivity region below the large discontinuity to be overestimated by the linearized inversion.

The uncertainty and resolution of the solution are inherently linked, so those effects in the model solution uncertainty also show up in the model solution resolution. The frequentist inversion process estimates a smoothed version of the true model, and this smoothed true model is solution dependent. The result in Figure 2b shows that the spread in smoothed true models being estimated is small.

Just as the nonlinear aspects in the model uncertainties and smoothed true models became apparent below the large discontinuity, the averaging kernels increasingly spread out with depth below too. As seen in Figure 2c, at shallower depths the resolution is very consistently represented by the linear averaging kernels, whereas increasingly with depth the averaging kernels smear out, showing that the true resolution of the nonlinear problem is broader than that reported by the individual averaging kernels.

Overall then, the results demonstrate that as model uncertainty increases, the nonlinearity
of the problem has an increasing effect on quantifying that uncertainty and the resolution of the inversion solution, manifested as increased spread in the profile errors bounds and in the resolution kernel widths. It is interesting to note that while the bias in the solution model profile increased with increasing uncertainty, the bias in the smoothed version of the “true” model and in the locations of the resolution kernels was comparatively rather small, a pertinent point regarding the use of the linearized approximation in the resolution analysis.

Since the geoacoustic inversion investigated in this thesis is a nonlinear inverse problem, the uncertainty and resolution results are dependent on the bottom model. One must know how the analysis is affected by this issue, including as a function of how well the bottom model may be known a priori. As with many nonlinear geophysical inverse problems a fully comprehensive analysis of this issue may not be feasible. However, the focus of this work is the practical interest in planning a geoacoustic experiment, so to that end a phenomenological approach is taken here, to consider these questions generally regarding a particular resolution analysis at hand. Some information may be known about the bottom a priori, as from cores or sub-bottom profiler or previous experiments, and these can constrain a suite of bottom models for which to compare results.

In comparing variations in resolution results across some different bottom models, overlaying averaging kernel plots becomes chaotic, so we overlay plots of the resolution matrix diagonals, which line up with averaging kernel peaks as long as they are away from an unstable region such as the one at close range above. Figure 3a shows one of the averaging kernel plots from Figure 1 and its associated resolution diagonal. Figure 3b shows six bottom model profiles used in the resolution analysis and synthetic inversions to compare changes in resolution results – #1 was used for results in earlier figures. Figure 3c shows resolution diagonals for the six profiles in 3b. Close inspection can reveal effects of discontinuities and so on, but the key is that overall the resolution results are not grossly different here. Results at other ranges in this problem show the boundary of the unstable region in roughly the same place for all the six
profiles here.

**FIGURE 3:** Variations in resolution results across a set of different bottom models. Overlaying averaging kernel plots for multiple problems becomes chaotic, so we overlay plots of the resolution matrix diagonals, which line up with averaging kernel peaks as long as away from unstable region at close range. (A) This is the 3.0km 1-VLA averaging kernel plot from Figure 1 and its resolution diagonal (dashed line). (B) Six “true” bottom model profiles used for synthetic inversions to compare changes in resolution results – #1 was used for results in earlier figures. (C) Resolution diagonals for the six profiles. Close inspection of the resolution diagonals can reveal effects of discontinuities and other features, but the key is that overall the resolution results are not grossly different across these six bottoms.

**SUMMARY AND CONCLUSIONS**

In summary then, this work has explored the results and the feasibility of a resolution analysis of an example continuous geoacoustic inverse problem. The example discussed decisions about geometry based on a desired resolution, choice of problem regularization, and hardware available. The focus was on a linearized analysis but with a Monte Carlo investigation to consider limits in where that approximation of local linearity is valid, and what happened outside the valid domain. The sensitivity of experiment geometry planning conclusions based on this resolution analysis to variations in the unknown (or partially-known) “true” bottom model was explored using six different bottom models.

The techniques and conclusions in this work are considered appropriate when inverting for piece-wise-continuous velocity profiles (as compared to say just a couple of homogeneous layers) in a full-waveform geoacoustic inverse problem. This work is focused on theoretical and computational demonstrations for pre-experiment planning, and there was no analysis of real-world experimental data. However, the conclusions and considerations discussed within are applicable in planning the types of experiments currently undertaken in the geoacoustic community. The approach and the mathematics in this type of analysis can theoretically scale to a higher dimensional bottom model, and/or more complicated environments and experiment geometries than the comparatively simple demonstration problem used here.
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