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2aUW9. Analysis of horizontal wave number statistical characteristics to the problem of sound propagation in two-dimensional-fluctuating shallow sea with losses

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On the basis of the approach previously proposed by authors a problem of the middle-frequency sound propagation in 2D-fluctuating shallow sea with losses is considered. Statistical characteristics of horizontal wave numbers corresponding to modal ones are studied. Fluctuations of horizontal wave numbers determine statistical features of wave field in random sea medium if wave field is sought by modal expansion. Within the framework of adiabatic approximation we present calculations of sound field statistical moments which demonstrate an effect of transmission loss attenuation along the horizontal distance. There are no references in acoustic literature for this fact. Some estimation has been carried out to explain new effect associated with two reasons that are the medium losses and sound speed fluctuations. They together influence wave numbers of modes in such a way attenuating losses

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INTRODUCTION

The problem of finding the acoustical field statistical characteristics for middle-frequency sound propagation in the ocean with random parameters is of traditional interest, see references 1–4, and at the same time is not enough studied both in theory and experiment. Well-known difficulties to theoretical analysis of this multidimensional stochastic boundary problem (e.g. see Refs. 2,5), make small possibilities to come out of simple approximation limits and seriously restrict original problem analytical description. Numerical simulations at present permit advance farther to understand statistical effects and also they can be very useful for the experiment. In this paper we continue developing the method of statistical simulation, see Ref. 6, applied to the description of 2D sound speed random inhomogeneity influencing the acoustical signal of frequency range 500–1000 Hz that propagates in shallow sea modeling ocean shelf zone. In recent papers devoted to this topic authors make reference the feature of fluctuations inhomogeneity influencing the acoustical signal of frequency range 500–1000 Hz that propagates in shallow sea developing the method of statistical simulation, see Ref. 6, applied to the description of 2D sound speed random field statistics come from the analysis of two first field moments. We also show an example of a simulation in reference to the average field intensity, but the developed approach is not limited only by that and assume obtaining the wide range characteristics of acoustical field for random sea conditions of propagation.

PROBLEM STATEMENT

In Cartesian coordinates \((x,z)\) formulation of 2D problem to describe acoustical pressure field \(p\) excited by linear tonal point source \((x_0,z_0)\) of a frequency \(\omega\) within inhomogeneous ocean medium having constant density \(\rho\) implies solving the following equation and boundary conditions:

\[
\frac{\partial^2 p(x,z)}{\partial x^2} + \frac{\partial^2 p(x,z)}{\partial z^2} + \frac{\omega^2}{c^2(x,z)} p(x,z) = -\delta(x-x_0)\delta(z-z_0), \tag{1}
\]

\[
p(x,h) = 0, \quad p(x,h) + \Omega h(i\omega \rho)^{-1} \left[ \frac{\partial p(x,h)}{\partial z} \right] = 0, \tag{2}
\]

here \(c(x,z) = c_0(x,z)[1 + \varepsilon(x,z)], |\varepsilon| \ll 1\) is a random sound speed (\(c_0\) is its regular part), values \(z = \{h,\Omega\}\) correspond to the position of a sea surface and a bottom with the certain impedance \(\Omega\). We omit arguments \(x_0,z_0\) of a point source location inherent to the function of sound pressure. In the horizontal direction \((x)\) to (1) there are conditions of attenuation at infinity for functions \(p(x,z), \partial p(x,z)/\partial x\), and also they have to be continuous across the possible interfaces separating regular and irregular parts of the medium. Assuming surface and bottom of a sea waveguide are the horizontal we seek solution to boundary problem (1)-(2) by the local mode expansion:

\[
p(x,z) = \sum_i \phi_i(x,z)G_i(x), \tag{3}
\]

where both normalized eigenfunctions \(\phi_i(x,z)\) with the preset domain of a definition \(\Omega \{h,\Omega\}\) and local eigenvalues \(\kappa_i(x)\) depend on \(x\) parametrically satisfying the eigenvalue problem for each section \(x = \text{const}\). After substitution of the expansion (3) into equations (1)-(2) we can reduce the original problem to one-dimensional one in terms of the functions \(G_i(x)\) and represent the solution in general case of an irregular waveguide in handy quadrature form (see Ref. 6). In present paper we neglect both mode coupling and backscattering and within this adiabatic approximation the following form of the solution is valid:

\[
G_i(x) = a_i \left( (\kappa_i(x)\kappa_i(x_0))^1/2 \right) \exp\left( i \int_{x_0}^x d\zeta \kappa_i(\zeta) \right), \tag{4}
\]

where \(a_i = i \phi_i(x_0,z_0)/(2\rho)\). Notice that in axisymmetric case of cylindrical coordinates \((r, z, \psi)\) (4) reduces to
\[
G_i(r) = a_i[2\pi \kappa_i(r) r]^{-1/2} \exp \left\{ i \int_0^r d\zeta \kappa_i(\zeta) \right\}, \quad \text{(in this case point of an observation } r > 0 \text{ and is far from the source located at } r = 0). \]

Original set (1)-(2) are the stochastic equations and all the considered functions are random ones defined by an ensemble of realizations. So, to obtain pressure field (3) statistical characteristics one must calculate eigenvalues \( \kappa \), eigenfunctions \( \varphi \), and functions \( G_i(x) \) for representative sample of random realizations. In known papers (e.g. see Refs. 1, 3, 4 and 7) analysis of the random sound field is performed on the basis of similar representations (3)-(4) but instead of equation (4) authors studied statistics of modal amplitude omitting exponent. This consideration is adequate if values \( \kappa \) are the real ones, but in the case of lossy medium the important fact of mutual influence the fluctuations and absorption is ignored. This effect described in Ref. 6. That is why we assume everywhere the sea as a dissipative medium; particularly there is a typical situation for shallow sea that the bottom involves absorption and considerably influences the sound propagation. In this case eigenvalues will be the complex \( \kappa = \kappa + i\kappa' \), and imaginary part \( \kappa' \) not to be negligibly small value.

SIMULATION OF 2D RANDOM INHOMOGENEITIES

The key moment for statistical simulation is a construction of an efficient algorithm to calculate random realizations of 2D field \( \varepsilon(x, z) \). In this paper we assume Gaussian fluctuations and randomizing the spectral representation to simulate homogeneous Gaussian field (see Ref. 8). Let \( \varepsilon(x, z) \) be a random real field defined via its correlation function

\[
\Psi(\xi) = \int d^2\kappa \cos(\kappa \xi) f(\kappa), \quad (5)
\]

here \( f(\kappa) \) is a spectral density (here \( \xi \) and \( \kappa \) are points of 2D space \( \mathbb{R}^2 \)). Let us divide space \( \mathbb{R}^2 \) by \( M \) nonoverlapping regions \( D_m \) and assume the random points \( \kappa_m \) \((m=1, M)\) are distributed within these regions having the probability density

\[
f_m(\kappa) = f(\kappa) F_m^{-1}, \quad F_m = \int_{D_m} d^2\kappa f(\kappa), \quad \kappa \in D_m. \quad (6)
\]

Then, it is obvious that validity of equation (5) yields the following identity:

\[
\Psi(\xi) = \sum_{m=1}^{M} F_m \left\langle \cos((\kappa_m \xi)) \right\rangle, \quad (7)
\]

where angular brackets correspond to the ensemble statistical average. This identity indicates that the correlation function is not varied if we divide the phase space of vectors \( \kappa_m \). It is well-known that for the random field with discrete spectrum \( \{\kappa_m\} \) \((m=1, M)\) one can construct the random model by the way

\[
\varepsilon(x, z) = \sum_{m=1}^{M} F_m^{1/2} \left[ \chi_m \sin(\kappa_m^{(1)} x + \kappa_m^{(2)} z) + \eta_m \cos(\kappa_m^{(1)} x + \kappa_m^{(2)} z) \right], \quad (8)
\]

where \( (\chi_m, \eta_m) \) are independent in ensemble standard Gaussian values. Representation (8) can obviously be used to approximate constructing a field with continuous spectrum (Ref. 8). This representation is the basis for our simulation algorithm. For our simulation it is convenient taking a correlation function of the anisotropic Gaussian form

\[
\Psi(x, z) = \sigma^2 \exp \left[ -\frac{1}{2} \left( \frac{x}{L_x} + \frac{z}{L_z} \right)^2 \right]. \quad (9)
\]
This form of correlation function is comfortable in terms of analytics since one can explicitly obtain the spectral density $f(\kappa)$ is necessary to simulate random realizations (8). In (9) values $L_x$ and $L_z$ are the spatial correlation scales in directions $x$ and $z$ correspondingly. Space partition on regions $\{D_m\}$ has been performed in accordance with the taken spatial scales of the correlation function. Correctness of this partition is specially checked out in numerical experiments.

**APPROXIMATE ESTIMATIONS**

From the original modal representation (3) the following expression for the second pressure field moment is valid:

$$\left\langle |p|^2 \right\rangle = \sum_l \left\langle |G_l|^2 |\varphi_l|^2 \right\rangle + \sum_{l,m(l\neq m)} \left\langle (G_l \hat{G}_m^*) (\varphi_l \varphi_m^*) \right\rangle.$$  \hspace{1cm} (10)

We represent here the average field intensity in the form of noncoherent and coherent sums. Let us estimate how small fluctuations of medium inhomogeneities can change the sound energy pattern both in vertical and horizontal directions. The first we mark that average values of functions $G_l$ and $\varphi_l$ equal to their non-perturbed values and relative fluctuations of these functions are enough small. So, the first (noncoherent) sum in (10) is close to the deterministic value (i.e. if there are no fluctuations in the medium). Essential variations (in reference to deterministic problem) of the second sum can be due to cumulative action of statistics coming from exponential factors (4). Neglecting statistical variation in eigenfunctions $\varphi_l$ let us focus attention on average values of these factors in (10) for $l \neq m$:

$$\left\langle G_l \hat{G}_m^* \right\rangle = \exp \left[ i \int_{x_0}^x d\zeta (\kappa_l - \kappa_m^*) \right] \exp \left[ i \int_{x_0}^x d\zeta \left( (\kappa_l) - (\kappa_m^*) \right) \right] \times \exp \left\langle -\frac{1}{2} \int_{x_0}^x \left[ d\zeta (\delta \kappa_l^* - \delta \kappa_m^*) \right]^2 \right\rangle \exp \left\langle -\frac{1}{2} \int_{x_0}^x \left[ d\zeta (\delta \kappa_l^* - \delta \kappa_m^*) \right]^2 \right\rangle.$$  \hspace{1cm} (11)

Obtaining (11) we use a well-known relation for the exponential index in the form of the Gaussian random value and also take into account the small value of $\kappa_l \delta$. $\delta$ denotes random variations of real and imaginary eigenvalue parts. General conclusions already come from the analysis of this expression (11). However, simplifying the analysis assume that horizontal scale $L_x$ of random inhomogeneities is enough large while vertical scale $L_z$ is small comparatively the sound wavelength. Then, integrals in (11) can be reduced to approximate ones owing to the theorem of averages. Also it was shown in Ref. 9 that eigenvalue dispersions are equal to each other while correlations between different $\kappa_l$ and $\kappa_m$ are equal to $2/3$ of $\kappa_l$-dispersion. As a result we can represent (11) in simpler approximate form ($l \neq m$)

$$\left\langle G_l \hat{G}_m^* \right\rangle = \exp \left[ i(x-x_0) \left( (\kappa_l) - (\kappa_m^*) \right) \right] \exp \left\langle \frac{(x-x_0)^2}{3} \left( \delta \kappa_l^* \right)^2 \right\rangle \exp \left\langle \frac{5(x-x_0)^2}{3} \left( \delta \kappa_m^* \right)^2 \right\rangle.$$  \hspace{1cm} (12)

Now we see the second exponent in (12) is equal to 1 for noncoherent terms in (10) whereas for the coherent terms this factor tends to zero with $x$. Thus we conclude (comparatively deterministic problem) that from the certain distances $x \geq x_1$ the noncoherent sum contribution is not change while the contribution of coherent sum terms become smaller due to the square-law function in exponential index. So, average intensity of pressure field $\left\langle |p|^2 \right\rangle$ become smoother both in vertical and horizontal coordinates. Such behavior of the functions demonstrates that $|p|^2$-fluctuations are not small in reality. Similarly, we can make some conclusions concerning the statistics of $|p|^2$ influenced by the imaginary part of eigenvalue $\kappa_l$. The third exponent in (12) has always positive index, but if $l = m$ it is larger (equal to 2) than for $l \neq m$ (equal to 5/3). So due to fluctuations in imaginary parts of $\kappa_l$ and $\kappa_m$ all the terms in (10) grow with distance $x$. At that time the noncoherent part grows faster additionally smoothing statistical
moments of the field along the distance. It is obvious that fluctuations in eigenvalue imaginary parts can influence the field such a way for the distances $x_2 \gg x_1$. Physical pattern is clear: for the lossy bottom there is the effect of a weaker attenuation in transmission loss. So the intensity level at far distances lies above the deterministic problem intensity level. Performed calculations for typical parameters of the ocean shelf show that the relation $x_2/x_1 \sim 10$ if the bottom absorption is $\beta = 10^2$.

**EXAMPLE OF SIMULATION**

As an example of simulations we consider a model of the randomly-inhomogeneous Pekeris waveguide. It is shown in Fig. 1 for some distant from the source section $x = x_1 = 5$ km. Let tonal point source radiates in this waveguide at frequency 500 Hz, and sound speed fluctuations have the intensity $\sigma_\varepsilon^2 = 10^{-6}$ and following scales: vertical scale $L_z = 1$ m, horizontal scale $L_x = 1000$ m. At taken frequency in the specified shallow sea model 11 propagating modes are excited in average.

![FIGURE 1. Model of a random waveguide with horizontal boundaries and lossy penetrable bottom for some distance $x_1 = 5$ km from a source. $H - h = 50$ m, $H - z_0 = 10$ m, $\rho/\rho_1 = 2$, $n_0 = c_\delta(h)/c_1 = (1500/1600)(1 + i\beta)$, $\beta = 0.01 (0.34$ dB/(km Hz)). Intensity of sound speed fluctuations $\sigma_\varepsilon^2 = 10^{-6}$ .](image)

![FIGURE 2. Transmission loss for the waveguide model in Fig. 1 and the layered one. Red curve corresponds to waveguide with the layered fluctuations having vertical scale $L_z = 1$ m and black oscillating curve is a transmission loss for the same but deterministic waveguide ($\varepsilon = 0$). Blue curve corresponds to our waveguide with $L_z = 1$ m, $L_x = 1000$ m.](image)
In calculations we use the factor $x^{-1}$ (or $r^{-1}$, see remarks after equation (4)) for intensity describing cylindrical field divergence. In Fig. 2 there is an average intensity transmission loss curve (blue) for above indicated waveguide model in comparison with the similar 2 curves in the case of layered Pekeris waveguide (if horizontal scale $L_z \to \infty$). From Fig. 2 one can see phenomenon of a weaker relaxation in average intensity law (transmission loss) established at first in Ref. 9 and also described in Ref. 6 for the case of layered sound speed fluctuations in waveguide with lossy bottom. Blue curve in Fig. 2 calculated here for the waveguide model in Fig. 1 within adiabatic approximation validates this result though it is not very pronounced now. This fact is in harmony with estimations performed above (see (12)). Estimations indicate that this phenomenon appears stronger for larger horizontal correlation scale of fluctuations. In the limiting case of $L_x \to L_z \to 0$ random field of sound speed approaches to delta-correlated one and discussed result is minimal.

CONCLUSIONS

In present paper we consider some estimations and numerical results to the problem of middle-frequency sound propagation in shallow sea with 2D random inhomogeneities of sound speed and lossy penetrable bottom. Consideration performed within the framework of developing approach that is statistical simulation and by the adiabatic approximation. Estimations and simulation confirm that there is a phenomenon of a weaker relaxation in average intensity law along a distance in the waveguide case with lossy bottom and 2D sound speed fluctuations in water column. This fact was studied previously for the case of shallow sea layered fluctuations. In present paper we have shown that the phenomenon of propagation loss weaker relaxation is defined to a considerable degree by the horizontal scale value of fluctuations. The larger this scale value the more pronounced this phenomenon of the weakening of losses. So in the case of quasi-layered fluctuations weakening of losses for average sound field intensity can reach 15 dB at distances $x > 10$ km.

REFERENCES