A quasi-analytic model is derived for the underwater sound signal radiated when an offshore pile is struck on its face by a hammer. The pile is modelled as a semi-infinite cylindrical shell of an elastic solid. The impact generates a pulse of vibration that travels down the pile at the longitudinal sound-speed. At a given distance below the pile face, the radial displacement after the peak has arrived decreases exponentially with time. There are two coupled equations of motion for the axial and radial displacements. A closed form expression is derived for the radiated sound pressure (which is proportional to the radial acceleration) in terms of the Poisson ratio and Young Modulus of the pile material, the hammer velocity, contact area between hammer and pile, pile radius, hammer mass, the pile longitudinal sound-speed, and the sound-speed and density of the external medium. This model is applied to a published scenario for which the radiated sound pressure had been computed using a Finite Element Model, but is found to produce a different result. Some assumptions used in the model are identified that may explain the difference.
INTRODUCTION

Offshore pile driving radiates regular pulses of loud noise underwater, and a substantial amount of data has been presented in the literature on the measured peak pressure of these pulses. The peak pressure at a horizontal range of 10 m can be in the region of 1 atmosphere (100 kPa). Pile driving pulses are “brief, broadband, atonal” and “characterized by a relatively rapid rise from ambient pressure to a maximal pressure value followed by a decay period that may include a period of diminishing, oscillating maximal and minimal pressures” [1]. The frequency of successive pile driving pulses (the blow rate) is usually between 15 and 30 per minute [2]. The individual pulse duration can vary between 15 and 90 ms, and is most likely to lie between 25 and 40 ms [3].

Although the quantity of descriptive data on noise from offshore pile driving is large, there have been few papers that attempt to model the physics of the impact and the consequent sound radiation. It is generally accepted that the major underwater signals originate from radial vibration (bulging) of that portion of the pile that is submerged. Since the bulge travels downward faster than sound travels through water, the first arrival at a hydrophone will originate at a point on the pile a little shallower than itself, and the trailing signal is due to multipaths from portions of the pile both above and below the originating point. A significant paper [4] reported the use of a finite-element model of the sound generated by a simple impact hammer. The results were entirely numerical, and their sensitivities to the various input parameters cannot be ascertained by examining the paper. The dominating effect of the simultaneous arrivals of multipaths from the pile (the “Mach wave”) was included [4].

The objective of the present paper is to present results from analytic models for cylinder vibration and the consequent peak pressure of non-Mach sound radiation, since such models allow the relative importance of the driving parameters to be estimated. Although the effect of Mach waves is not treated explicitly, the results obtained do suggest a method by which it may be estimated.

ASSUMPTIONS

A pipe pile is modelled as a thin vertical cylindrical shell of an elastic material such as steel. Absorption of sound (conversion to heat) in the material is represented by a small loss factor.

The pile is semi-infinite in length. Although other analyses have treated finite lengths and thus include echoes from the pile toe, this aspect is not addressed here. The upper portion of the pile is in air and finite in length. The remainder is submerged in water of infinite depth.

The hammer is a compressible solid vertical cylinder with the same density and Young modulus as the pile. It has a finite mass and therefore length. Reflections from the top of the hammer following impact are however neglected. Since the hammer is compressible, the initial velocity of the pile face is estimated by assuming the interfaces satisfy the principle of momentum conservation.

The hammer strikes the pile instantaneously and uniformly over its face, and does not cause the pile to twist or bend. The ensuing axial and radial displacements will therefore not vary significantly with azimuth (polar angle) around the pile axis.

Only the sound radiated shortly after impact is addressed.

Each of the two external media allows sound waves to radiate from the pile. The effect of the medium on the pile vibration is obtained by assuming the radial velocity of the pile wall generates strain in the external medium, and the consequent stress (pressure) has the same effect on the cylinder as if it were an external pressure applied to a cylinder in-vacuo [5]. The method used here is an approximation to the radiation loading model described by [5]. The wall vibration is thus subject to feedback.

For a hydrophone at a given horizontal range from the pile and depth beneath the water surface, the problem of determining the pressure waveform from the moment the leading edge of the downward travelling pulse crosses the water surface until it reaches great depth is beyond the scope of the present paper. Instead, it is assumed that the peak pressure occurs a short time (the travel time for the horizontal range) after when the leading edge is at the same depth as the hydrophone. For the purpose of modelling, the pulse is considered to be fixed at that depth. The aperture of the pile that determines the waveform and hence peak pressure is the whole pile beneath the water surface.

The radial displacement algorithm that is utilised is linked to two simple models (near-field and far-field) for sound radiation from a cylinder. Modelling of Mach-wave multipaths that arrive simultaneously due to the supersonic speed of the sound source is included in the near-field model, but neglected in the far-field model.

It is assumed that reflection of underwater sound waves by the water surface has negligible effect on peak pressure. Most pile driving noise spectra have dropped to no more than 10% of their peak by a frequency of around

4 kHz. To replicate such data, sampling with a time resolution of 0.1 ms would suffice. If source and hydrophone are at a horizontal range and depths such that a few samples (say 4) of the direct arrival are taken before the surface reflection arrives then the peak pressure will not be affected by the reflection. This criterion requires a delay of 0.4 ms (which corresponds to a path difference in water of 0.6 m). In this paper, results for peak pressure are presented only if the path difference between direct arrival and surface reflection is at least 0.6 m.

PILE RADIAL VIBRATION

Using the assumptions listed above, the simultaneous equations of motion for axial and radial displacement (u and w) in a cylindrical shell [6] have been formulated for initial conditions that correspond to impact by a hammer [7]. The radial displacement is treated as a function of depth z and time t: w = w(z,t). The following expression for the Fourier Transform (FT) of the radial displacement (W) has been obtained [8]:

\[ W(z, \omega) = \chi \exp\left( -i \omega H/V_1(\omega) - i \omega z/V_2(\omega) \right) /S_2(q_y) \cdot S_2(q_h)(\Omega + i\omega) \]  

(1)

where H is the length of the air-exposed portion of the pile, 

\[ V_1(\omega) \text{ and } V_2(\omega) \text{ are the phase velocities of propagation in the air-exposed and submerged portions of the pile respectively:} \]

\[ V_n(\omega) = q_n S_n(q_y) / S_n(q_h), \ n = 1, 2 \]

(2)

where

\[ S_n(q) = \sqrt{q^2 + \omega^2 + i \omega \rho_n c_n / \rho_y h}, \ n = 1, 2 \]

(3)

in which h is the pile wall thickness, \( q_y \) is the longitudinal sound speed in the material \( \sqrt{Y/\rho_y} \), Y is Young modulus of the pile material, \( \rho_y \) is the density of that material, \( q_h = q_y / \sqrt{1 - \nu^2} \), \( \nu \) is its Poisson ratio, \( \chi = v v_0 q_h / a \); a is the cylinder radius, and \( v_0 \) is the initial velocity of the pile face (typically around 12% less than the hammer impact velocity, due to compressibility of the hammer):

\[ \Omega = \Lambda Y / M q_h, \text{ where } \Lambda (2 \pi a h) \text{ is area of contact between pile and hammer, and } M \text{ is mass of hammer.} \]

The case described in [4] has been applied to Eq. (1). The hydrophone depths reported in [4] ranged from 4.9 to 10.5 m, and magnitudes of the spectrum at those depths are shown in Figure 1. Although the spectra were computed with a Nyquist frequency of 50 kHz, the maximum frequency displayed has been reduced to 10 kHz for clarity (the results at higher frequencies did not reveal any unexpected features). High damping in the air-exposed portion (H was 5.4 m for [4]), especially between the two radial resonance frequencies near 2 kHz, causes a deep minimum there. In addition, the spectra decay with depth, most noticeably between 1 and 4 kHz.

**FIGURE 1.** Magnitude of the radial displacement spectrum (W) at depths in water of 4.9 and 10.5 m (for the [4] case).
The Inverse Fourier Transforms (IFT) of $W(z,\omega)$ have been calculated at both $z = 4.9$ and $10.5$ m and the magnitudes of the resulting waveforms $|w(z,t)|$ are shown in Figure 2. The waveforms rise rapidly to peaks of around 170 and 152 microns and then decay quasi-exponentially with a time constant of 3 ms. Compared with the shallower waveform, the deeper waveform is 1 ms later, and its amplitude is 11% smaller.

**FIGURE 2.** Magnitude of the radial displacement waveforms at depths in water of 4.9 and 10.5 m (for the [4] case).

**UNDERWATER RADIATED SOUND PRESSURE**

The sound pressure radiated into the surrounding fluid by a vertical cylinder is computed using two models for acoustic radiation from a vibrating cylinder. The first model assumes that the cylinder vibration is independent of depth, and the second allows for it to vary with depth. Both models assume the external medium to be fluid, homogeneous and unbounded. The sound speed and density of the external medium are denoted by $c$ and $\rho$.

**Depth-Independent Cylinder Vibration**

An infinite vertical cylinder vibrates radially with a depth-independent (uniform) amplitude. The expression for $P(r,\omega)$, the FT of the radiated pressure $p(r,t)$, at horizontal range $r$ from the cylinder axis [modified here to correspond to a time dependence of $\exp(+i\omega t)$] is [9]:

$$P(r,\omega) = -\rho \, c \, \omega \, W(\omega) \, \frac{H_0^{(2)}(\omega \, r / c)}{H_1^{(2)}(\omega \, a / c)} . \quad (4)$$

This is referred to as the Morse model. For the [4] case, $r = 12$ m, and the corresponding FT pressure magnitudes are shown in Figure 3. The spectrum $W(z,\omega)$ in Eq. (4) is the same as that shown in Figure 1 for both $z = 4.9$ and $10.5$ m (and $H = 5.4$ m). Since this model will give the exact pressure spectrum only if $W$ is independent of depth, these results are only approximate; they assume that the cylinder vibrates for all $z$ with the value computed at $z$. 
FIGURE 3. Magnitude of the Morse sound pressure spectra at depths in water of 4.9 and 10.5 m (for the [4] case). Obtained by applying the depth-independent Morse model [9] to \( W(z,o) \), although the model assumes that each value of \( W \) applies for all \( z \).

The IFTs of \( P(12,z,o) \) have been computed for both \( z = 4.9 \) and 10.5 m, and the resulting waveforms \( |p(12,z,t)| \) are shown in Figure 4. The peak pressure is 160 kPa at 4.9 m, and 120 kPa at 10.5 m, a decrease of 25%.

FIGURE 4. Magnitude of the Morse sound pressure waveforms at range 12 m and depths in water of 4.9 and 10.5 m (for the [4] case). Obtained by taking the IFTs of the spectra whose magnitudes are shown in Figure 3.

Results have been obtained for the Morse peak pressures for hydrophones at a range of 12 m and depths from 1 to 12 m in steps of 0.1 m, and are shown by the red curve in Figure 5. For each of these calculations the source depth was set to the hydrophone depth. The individual data points labelled “Reinhall” are the peaks of the individual waveforms at nine hydrophone depths, as read from Fig. 11 in [4]. The Reinhall results increase significantly with depth, whereas the Morse pressures decay with depth due to the decay in \( W(z,o) \), and are also too high. Being too high is to be expected, since this model assumes the cylinder to be infinitely long and vibrating with uniform phase.
Depth-Dependent Cylinder Vibration

A method that allows for vibration to vary with depth down a cylinder has been presented [10]. This method takes the spatial FT of the vibration’s depth dependence, yielding a spectrum as a function of vertical wavenumber (γ):

\[ \tilde{f}(\gamma, \omega) W(0, \omega) = \int_0^\infty W(z, \omega) \exp(-i\gamma z) \, dz \]  

(5)

The component of this spectrum at each wavenumber is invariant with depth (just as a spectral component of a temporal waveform is invariant with time). The sound pressure component as a function of wavenumber is computed, and for the scenario of no azimuthal dependence, \( P(r, z, \omega) \) has a simple dependence on the spatial IFT [10]. The expressions presented by [10] cannot however cater for a radial displacement \( w(z,t) \) that also varies with time due to example to motion of the sound source. Since a description is required of \( P(r, z, \omega) \) at various depths of the pulse below the pile face, and thus at various times, it is necessary to re-compute \( P \) at each such time.

If \( r \) is sufficiently large that the large-argument asymptotic value of the Hankel function \( H_0^{(2)}(\omega \alpha / c) \) can be used for most values of the integrand in the expression for \( P(r, z, \omega) \), then an approximate simple expression for the pressure spectrum can be obtained using the method of stationary phase to simplify the integral [10]:

\[ P(r, z, \omega) = i \rho \exp(-ikr) \omega^2 \tilde{f}(0, \omega)W(0, \omega) / \pi kr H_1^{(2)}(\omega \alpha / c) \]  

(6)

An argument of at least 5 makes the approximation reasonable, and since \( r = 12 \) for the [4] case, this assumption is reasonable for a minimum \( \omega/c \) of 5/12 m\(^{-1}\), which in water corresponds to minimum frequency of 100 Hz. Thus only a very small portion of the spectrum is rendered in significant error by this approximation. This model is referred to as the Junger model. The first argument of the spatial FT is zero since in the stationary-phase approximation, if the direction of a sound wave is perpendicular to the cylinder axis, then the only value of the spatial FT that is required is \( \gamma = 0 \) [10].

Computed results for \( P(12, z, \omega) \) for the [4] case are shown in Figure 6, again for both 4.9 and 10.5-m depths. Compared with the results in Figure 3, the peaks of the spectra are about 75% less, and their bandwidths are also somewhat smaller.

**FIGURE 5.** Peak pressure of waveforms computed for hydrophones at a range of 12 m and at depths from 0.1 to 12 m (for the [4] case). KEY: ‘Reinhall’ refers to results read from [4], ‘Morse’ is a generalization of the results shown in Figure 4, ‘Junger’ is a generalization of the results shown in Figure 7, and ‘Mach Estimate’ is a hypothetical estimate of the Mach Wave peak pressure.
FIGURE 6. Magnitude of the sound pressure spectra at depths in water of 4.9 and 10.5 m (for the [4] case). Obtained by applying the Junger far-field depth-dependent radiation model [10] to \( W(z, \omega) \) as per Eq. (1).

The corresponding pressure waveforms have been obtained by taking the IFT of \( P(12, z, \omega) \), and the results are shown in Figure 7. The peak pressures at the two depths are 27 and 26 kPa, which are respectively 83% and 78% less than the corresponding results in Figure 4.

FIGURE 7. Magnitude of the sound pressure waveforms at depths in water of 4.9 and 10.5 m (for the [4] case). Obtained by taking the IFT of the spectra whose magnitudes are shown in Figure 6.

Results have been obtained for the Junger peak pressures for hydrophones at a range of 12 m and depths from 0.1 to 12 m in steps of 0.1 m, and are shown by the black curve in Figure 5. For each of these calculations the source depth was again set to the hydrophone depth. These results are quasi-independent of source /hydrophone depth, since this model takes the wavenumber FT along the (infinite) length of the pile, which is independent of depth. The whole Junger curve is lower than the smallest Reinhall peak pressure.

**Estimation of Mach-Wave Pressures**

In attempting to estimate the pressures produced by the simultaneous arrivals of multipaths from different depths on the pile, the first point to note is that the travel time of a pulse from an impact at height \( H \) above the water surface to a hydrophone at \((r, \xi)\) is given by

\[
t(r, \xi) = \frac{(H + z)}{q} + \sqrt{(z - \xi)^2 + r^2}/c
\]  

(7)
where $z$ is the depth on the pile from which the pulse emanates. If $q > c$ then at any $\zeta$ this function has a minimum, which occurs at

$$z_1 - \zeta = -r/\sqrt{q^2/c^2 - 1}$$  \hspace{1cm} (8)

In the $r$-$z$ plane this corresponds to a straight line: at $r = 0$, $z = \zeta$; and if $\phi$ is the depression angle of this line relative to the horizontal $r$-axis, then $\tan \phi = dz/dr$ and thus $\sin \phi = c/q$. For the [4] case as an example, $\phi = 17^\circ$. At a given $(r, \zeta)$, the first arrival originates from position $(0, z_1)$ on the pile, where $z_1 = \zeta - r \tan \phi$. Successive pulses arrive simultaneously from points both above and below $z_1$. For the Morse model for which $V$ is effectively infinite, $\phi = 0$ and $z_1 = \zeta$.

For source depths ($z$) from 0.1 to 12.5 m (the seafloor depth in the [4] case), the arrival times at range 12 m and hydrophone depths from 4.9 to 10.5 m (the number has been reduced from 9 to 4 for clarity) are shown in Figure 8. At a hydrophone depth of 7.7 m for example, the first pulse arrived at 10.3 s and originated from a depth of 4.0 m.

![Figure 8](image)

**FIGURE 8.** Arrival times after impact at range 12 m as a function of source depth on a pile for each of five hydrophone depths. The hydrophone depths range from 4.9 to 10.5 m in steps of 1.4 m, as indicated in legend. Impact occurred 5.4 m above water surface.

The feature of the curves in Figure 8 that is relevant to simultaneous arrivals is the region around the minimum. For $\zeta = 7.7$ m for example, the arrival time is 10.7 s for a pulse from the surface, decreases to 10.3 at the minimum ($z = 4.0$), and then increases and attains its surface value at $z = 7.6$ m; this arrival is referred to as the “surface-coincident” arrival. Pulses that emanated from the pile at both $z = 0$ and $z = 7.6$ m arrived simultaneously, and the continuum of pulses that emanated at intervening depths all arrived within a time span of 0.4 s.

The results for source depth of both the first and surface-coincident arrivals have been computed for the [4] case and are shown as functions of hydrophone depth in Figure 9. The former is given by Eq. (8), while the latter is given by

$$z_c(\zeta) = 2(\zeta - \sqrt{r^2 + \zeta^2} c/q)/(1 - c^2/q^2)$$  \hspace{1cm} (9)

The two curves extrapolate to $\zeta = 3.7$ m at $z = 0$ (for $r = 12$). Thus for $\zeta < 3.7$ m, simultaneous arrivals would not occur at a range of 12 m (although they would at a shorter range).
FIGURE 9. Source depths of both the first ($z_1$) and surface-coincident arrivals ($z_c$), as functions of hydrophone depth.

Since there is little variation in arrival time for source depths between zero and the surface-coincident depth ($z_c$), it is hypothesized that the Mach-wave pressure is due to an equivalent finite uniform (virtual) cylinder with length equal to $z_c$ but with unknown vibration $W$. To overcome $W$ being unknown, the Mach-wave pressure at any hydrophone depth is estimated to equal the Morse pressure at the same depth multiplied by the ratio $z_c/D$, where $D$ is the seafloor depth. The results are shown by the green curve in Figure 5, and are seen to be a reasonable estimate of the data reported by [4].

CONCLUSIONS

In order to predict peak pressure of the pulse radiated in water when a cylindrical shell is struck by a hammer, a model of the vibration of has been linked with both a “Morse” uniform-infinite cylinder and a “Junger” depth-dependent cylinder. In the first case, the predicted peak pressure is higher than the results previously reported from an accurate Finite-Element (FE) model, due to the uniform cylinder being infinite in length. The reported FE peak pressure, which was dominated by the “Mach-wave” associated with supersonic speed of a sound source, increased as the hydrophone depth increased, whereas the Morse pressure decreased over the same interval. In the second case, the predicted peak pressure is approximately independent of hydrophone depth, and everywhere less than the smallest Mach-wave pressure. A reasonable result is obtained from the hypothesis that the Mach-wave pressure is the product of the Morse pressure and the ratio of surface-coincident depth to seafloor depth.

REFERENCES