Source bearing and range estimation using an ice-mounted tri-axial geophone

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This paper presents results from Arctic field trials to estimate the bearing, range, and depth of an acoustic source in the water column using seismic particle motion measured at a tri-axial geophone on the sea ice surface. Measurements were carried out on smooth, rough, and ridged annual ice, and on a multi-year ice floe. Impulsive acoustic sources were deployed in the water at a variety of bearings and ranges from 0.2-50 km. Source bearings are estimated by applying polarization filters to suppress shear waves with transverse particle motion and computing the incident power as a function of radial look angle; the inherent 180-degree ambiguity is resolved by requiring prograde particle motion in the vertical-radial plane. Results indicate good bearing estimation (<10-degree average errors) at all ranges with little dependence on ice type. Source range and depth is estimated from the time difference between the water-borne arrival and ice seismic waves. Results are limited due to strong attenuation of the seismic waves, with good range/depth estimation to <1 km for smooth annual and multi-year ice and <0.5 km for rough ice.

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INTRODUCTION

The problem of localizing an ocean acoustic source in ice-covered waters is of considerable interest. This paper presents results of Arctic field trials carried out to study the use of a single tri-axial geophone on the surface of sea ice to localize an acoustic source in the water column. The results of source-bearing estimation for these trials, computed using rotational analysis, have been reported previously [1], [2], and are only briefly considered here. Recent work based on numerical modeling has suggested that source range could be estimated by considering arrival-time differences of various ice seismic waves [3]. In this paper, source range and depth are estimated from the arrival-time differences of the direct water-borne acoustic wave and ice seismic waves including the longitudinal plate ($L_P$) wave, which involves radial-vertical (prograde elliptic) particle motion, and the horizontally-polarized shear ($S_H$) wave [4]. Results show good ability to estimate source range and depth over the relatively limited range over which these ice seismic waves could be detected in the field trials.

FIELD TRIALS

The Arctic field trials considered here were carried out in the Lincoln Sea, north of Elsmere Island, Canada, on three types of ice: smooth, uniform annual ice approximately 2-m thick; rough, ridged annual ice ~2-m thick; and two multi-year ice floes ~5-m thick. Figure 1(a) shows the sensor configuration used at all but one of the multi-year ice sites, which consisted of a line of three tri-axial geophones separated by 20 m (a vertical-component geophone was also included at either end of the line, but these are not considered here). Figure 2(b) shows the source configuration at the two annual-ice sites, which consisted of glass light bulbs imploded under hydrostatic pressure at 50-m depth in the water column at ranges of 200, 500, and 1000 m from the center geophone and at bearings of 0°, 30°, 60°, and 90° with respect to the sensor line. The figure also indicates the position of a 2.5-m high pressure ridge which separated geophones and sources at the rough-ice site. Figure 2(c) shows the source configuration used for one of the multi-year ice floes: in this case SUS (signal, underwater sound) charges were deployed at 18-m depth from 2-50 km range and the same four bearings. At the second multi-year floe, a single tri-axial geophone was deployed and light bulbs were imploded at a single bearing at ranges of 200, 300, 400, 600, 800, 1000 m.

**FIGURE 1.** Geometry of field trials. (a) shows sensor geometry with G indicating geophones (vertical and tri-axial). (b) shows light-bulb source locations (asterisks) for the smooth and rough annual ice sites; the shaded region indicates a pressure ridge present the rough-ice site. (c) shows SUS source locations for one of the multi-year ice sites.
In addition to the water-column sources, a hammer-seismic survey was carried out at each site to estimate wave propagation velocities. Results indicated $L_P$ wave velocities of 3000-3100 m/s and $S_H$ wave velocities of 1700-1740 m/s; full results are given in [1]. Although not measured at the sites considered here, subsequent work yielded attenuation measurements of 1.6 and 2.6 dB/wavelength for $L_P$ and $S_H$ waves in annual ice [1], which indicate these waves attenuate at a very high rate compared to acoustic propagation in water.

**BEARING ESTIMATION**

A rotational analysis to estimate source bearings using ice-mounted geophones was presented in [1], based on measuring ice seismic waves with particle motion oriented radially outward from the source. For such waves, seismic recordings at two orthogonally-mounted horizontal geophones can be combined geometrically to compute the wave power in horizontal look angles from 0-360°, with the angle of maximum power providing the optimal bearing estimate. The 180° ambiguity due to rotational symmetry is resolved by requiring out-going (prograde) particle motion in the vertical-radial plane, yielding a unique bearing estimate. The analysis is enhanced by the application of polarization filters [4] to suppress $S_H$ waves with transverse particle motion which otherwise degrade the analysis. Examples of the results are given in Figs. 2 and 3. Figure 2 shows bearing estimates for the center geophone at the rough annual-ice test site for light-bulb sources at ranges of 200-1000 m. Figure 3 shows results for center geophone at the multi-year ice floe for SUS-charge sources at ranges of 2-25 km. Figure 4(a) summarizes all results in terms of the average bearing-estimation error as a function of source range, averaging over all geophones and bearings, and show results obtained with and without polarization filtering. Average bearing errors are 5-10° and show only a slight increase with range. Finally, Fig. 4(b) shows the average discrimination level (difference between maximum and minimum rotational power over angle) as a function of range. Figure 4 shows that a significant improvement in bearing error and discrimination level is achieved with polarization filtering.

**FIGURE 2.** Bearing estimation results for light-bulb sources at the rough annual-ice site for ranges and bearings indicated. Solid curves show rotational power as a function of look angle with optimal bearing estimates indicated by arrows and true bearings by dashed lines. Dotted circles indicate 5-dB decrements from the maximum power (solid circle).
FIGURE 3. Bearing estimation results for SUS sources at the multi-year ice site for ranges and bearings indicated. Solid curves show rotational power as a function of look angle with optimal bearing estimates indicated by arrows and true bearing by dashed lines. Dotted circles indicate 5-dB decrements from the maximum power (solid circle).

FIGURE 4. Summary of bearing estimation results for all ice types in terms of (a) bearing error and (b) discrimination level as a function of range. Solid and dashed lines indicate results obtained with and without polarization filtering, respectively.

RANGE AND DEPTH ESTIMATION

Figure 5 shows examples of the three-dimensional particle motion recorded at an ice-mounted tri-axial geophone for a water-column source (imploding light bulb). Figure 5(a) shows the recordings at the center geophone at the smooth annual ice site for a source at 200-m range, 50-m depth, and 0° bearing. The $L_p$ and $S_H$ waves appear on the radial and transverse components, respectively. The direct water-borne acoustic wave ($W$) and subsequent water-
column reflected waves appear on all components, including the transverse component due to scattering and/or local wave conversion near the sensor. Note that the ice seismic waves are of relatively low amplitude and low frequency compared to the acoustic wave. Due to its considerably higher velocity, the $L_P$ wave is well separated in time from the acoustic wave; however, the $S_H$ wave arrives only slightly before and overlaps with the direct acoustic wave. Figure 5(b) shows the arrivals at the same site and geophone for a source at 500-m range. The ice seismic waves are only marginally above the noise level. Seismic waves could not be reliably identified and picked at ranges beyond 520 m at the smooth annual-ice site and beyond 220 m at the rough annual-ice site. Figure 5(c) shows arrivals at the centre geophone at the multi-year ice site for a light-bulb source at 800-m range; the $L_P$ and $S_H$ waves are only just discernable and could not be picked at ranges great than this for light-bulb or SUS sources on multi-year ice.

Since source-transmission instants are not known, the inversion for source range and depth must be based on the differences of picked first-break arrival times for the $W$, $L_P$, and $S_H$ waves. The water-borne acoustic wave can couple continuously into ice seismic waves along the propagation path, but the first (and strongest) seismic-wave arrivals should correspond to the critically-refracted path. Arrival-time differences between the acoustic and seismic waves are modeled here as

$$t_w - t_L = \frac{c_L}{c_w} \left( z + (z - h)^2 \right) + c_w h \left( \frac{z}{c_w \cos \theta_L} - \frac{r - z \tan \theta_L}{c_L} \right)$$

$$t_w - t_S = \frac{c_L}{c_w} \left( z + (z - h)^2 \right) + c_w h \left( \frac{z}{c_w \cos \theta_S} - \frac{r - z \tan \theta_S}{c_S} \right)$$

(Figure 5) Recorded time series (vertical, radial, transverse components) for (a) smooth annual ice for source at 200-m range, (b) smooth annual ice for source at 500-m range, and (c) multi-year ice floe for source at 800-m range. Arrivals of the $W$, $L_P$, and $S_H$ waves indicated with arrows. Each component is normalized independently and acoustic waves are clipped for display purposes. The sampling rate was 2 kHz in (a) and (b), and 1 kHz in (c).
where \(c_w\) represents the water acoustic velocity, and \(\theta_L = \sin^{-1}(c_w/c_L)\) and \(\theta_S = \sin^{-1}(c_w/c_S)\) are the critical angles between water and ice propagation for the \(L_P\) and \(S_H\) waves, respectively. The first term on the right side of Eqs. (1) and (2) represents the travel time \(t_{WP}\) for the water-borne acoustic wave, which is assumed to propagate as a straight ray through the water to the underside of the ice below the sensor, then propagate through the ice as a compressional wave which travels at approximately the \(L_P\) wave velocity. The second terms on the right of Eqs. (1) and (2) represent the travel times \(t_L\) and \(t_S\) for critically-refracted \(L_P\) and \(S_H\) waves, respectively, which represent the first arrival of these wave types at the sensor. Equations (1) and (2) represent a simple model for seismo-acoustic propagation in the water-ice environment and more advanced treatments could be formulated; however, they appear adequate for the limited propagation ranges to which \(L_P\) and \(S_H\) arrival times can be picked in this study.

Given approximate knowledge of wave propagation velocities and ice thickness, Eqs. (1) and (2) can be applied to invert measured arrival-time differences for the source range and depth, \(r\) and \(z\). The equations are nonlinear and cannot be solved analytically. However, it is straightforward and efficient to evaluate the data misfit for a grid of possible source ranges and depths, which, within a Bayesian formulation, provides a sampling of the posterior probability density (PPD) for uncertainty analysis. Assuming the residual errors are independent Gaussian-distributed random variables, the log-likelihood misfit function is given by

\[
E(r,z) = \sum_{i=1}^{n} \frac{(d_i - d_i(r,z))^2}{2\sigma_i^2},
\]

where \(d_i\) and \(d_i(r,z)\) represents the measured and modeled data (arrival-time differences), respectively, and \(\sigma_i\) is the standard deviation of the \(i\)th datum. Here arrival-time picking uncertainties are assumed independent and estimated to be 1 ms for sources at \(\leq 220\) m range and 2 ms for greater ranges. Standard deviations for arrival-time differences were then computed as the square root of the sum of squares of these uncertainties. The posterior probability was computed at 1-m range and depth increments over a search region of 0–1000 m in depth and 0–2000 m in range, over which the prior probability was considered constant.

Figure 6 shows contours of the PPD as well as marginal probability distributions over source range and depth for the 200-m range source at the smooth annual-ice site [signals shown Fig. 5(a)]. Since Eqs. (1) and (2) are quadratic in \(r\) and \(z\), two solutions are possible, and appear as two maxima (modes) in the probability distribution, one of which is in excellent agreement with the true source location while the second is much deeper. Both PPD modes lie along an inclined ridge of high probability indicating a positive correlation between source range and depth. Note that although the marginal probability distributions peak at the deeper mode since it is wider in range and depth, the optimal (highest-probability) solution occurs at the shallower mode, corresponding to the true source location. Localization solutions for all sources at ranges \(\leq 220\) m resemble those shown in Fig. 6. In some applications prior information could preclude a secondary deep PPD mode (e.g., if the source, such as a marine mammal, is unlikely at this depth). In particular, the depth of the deep PPD mode in Fig. 6 greatly exceeds the water depth at the site (~120 m), but is considered here to more fully explore the inverse problem.

Secondary PPD modes were not observed for any sources at ranges at ranges >220 m. Figures 7 and 8 show localization solutions for sources at 500-m and 800-m range at the smooth annual-ice and multi-year ice floe sites, respectively [signals shown in Fig. 6(b) and (c)]. Uni-modal PPDs are obtained with peaks near the true source range and depth and reasonably small uncertainties indicated by the marginal probability widths. Figure 9 summarizes the localization results in terms of absolute depth and range errors for all sources for which ice seismic-wave arrivals could be identified and picked. Average depth and range errors, respectively, are 18 m and 26 m at the smooth-ice site, 25 m and 32 m at the rough-ice site, and 26 m and 30 m at the multi-year ice floe.

**SUMMARY**

This paper presented results of Arctic field trials for three-dimensional localization of an ocean acoustic source using a tri-axial geophone mounted on the sea-ice surface. Recordings of light-bulb and SUS sources were made at sites on smooth and rough/ridged annual ice and multi-year ice floes. Source-bearing estimation, based on rotational analysis and polarization filtering, provided excellent results with average bearing errors <10° for sources at ranges from 200 m to 50 km (reported previously). Source range and depth estimation was based on Bayesian inversion of arrival-time differences between the water-borne acoustic wave and critically-refracted ice seismic (\(L_P\) and \(S_H\))
FIGURE 6. Probability and marginal probability distributions for source location as a function of range and/or depth for a source at 200-m range and 50-m depth at the smooth annual-ice site (true range and depth indicated by dotted lines).

FIGURE 7. Probability and marginal probability distributions for source location as a function of range and/or depth for a source at 500-m range and 50-m depth at the smooth annual-ice site (true range and depth indicated by dotted lines).

FIGURE 8. Probability and marginal probability distributions for source location as a function of range and/or depth for a source at 800-m range and 50-m depth at the multi-year ice site (true range and depth indicated by dotted lines).
waves. For short-range (≤ 220-m) sources the PPD was bi-modal; however, the optimal (most-probable) estimates were in good agreement with the true source locations. For longer-range sources a unique (uni-modal) solution was obtained. Average errors in source depth and range were ~20–30 m for all source ranges and ice types. The limiting factor appears to be relatively short range over which critically-refracted ice seismic waves could be identified and arrival times picked: ~500 m on smooth annual ice, ~200 m on rough, ridged annual ice, and ~800 m on multi-year ice.

REFERENCES