4aUWb2. Measured scattering of a first-order vortex beam by a sphere: Cross-helicity and helicity-neutral near-forward scattering and helicity modulation

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The wavefield of a traveling wave acoustic vortex beam has an axial null and an angular phase ramp. An appropriately phased four-element transducer array can be used to generate a first order vortex beam [B.T. Hefner and P.L. Marston, J. Acoust. Soc. Am. 106, 3313-3316 (1999)]. The direction of the phase ramp determines the helicity of the beam. Superposition of signals from an appropriately positioned four-element receiver array gives a helicity selective detector and commutation of diagonal source elements can be used to reverse the source helicity [T.M. Marston and P.L. Marston, J. Acoust. Soc. Am. 127, 1856 (2010)]. These techniques were used to investigate the near forward scattering by a small sphere placed on or near a beam's axis. The forward scattering vanishes in the on-axis case [P.L. Marston, J. Acoust. Soc. Am. 124, 2905-2910 (2008)]. As the sphere is moved off axis the scattering to a helicity neutral receiver is found to increase linearly in the displacement with a first order phase swirl as a function of the sphere coordinates. For cross-helicity detection (detection opposite the beam's helicity) as required by symmetry, the signal is approximately quadratic in the displacement with a second-order phase swirl.

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ACOUSTIC VORTICES: TOPOLOGICAL CHARGE AND RELATED PROPERTIES

Helicoidal beams, also known as vortex beams, possess an amplitude null along an axis and an angular phase dependence of the form \( \exp(i\Phi_{\text{rot}}) \) with a non-zero integer \( m \), where \( \Phi \) denotes the angle of field point using cylindrical coordinates having an axis corresponding to the axis of the beam. Such wavefields are an example of a category of wavefields having topological singularities [1-11]. In the case of a vortex beam, the integer \( m \) is sometimes referred to as the “topological charge” of the wavefield. Following the introduction of methods for generating acoustic vortex beams [2], there has been expanding interest in the properties of such acoustic wavefields, including aspects of the scattering [2, 12-16], radiation force [13, 14, 17], radiation torque [2, 7-10, 18], and the evolution of nonlinear beams [3, 4, 6]. In the case of acoustic vortices, however, there has been relatively little attention to the topological properties of the phase of the scattering, even for objects as simple as a sphere, when the object is displaced from the axis of the beam. Notice the integer \( m \) can be either positive or negative, corresponding to the handedness of the beam. A circularly polarized plane wave is the simplest case of an electromagnetic wave having a handedness, and in that case it is conventional to associate the direction of the handedness with the direction of the helicity of the beam. (In the electromagnetic case it is also possible for the beam to possess a topological charge along with circular polarization associated with the spin of photons [11].) In analogy with the simplest electromagnetic case, in the acoustic case the sign of the topological charge will be referred to as the sign of the helicity of a specified contribution to an acoustic wavefield. Hence a beam having \( m = +1 \) will be referred to as an acoustic wavefield having a positive helicity. The wavefield amplitude vanishes as the axis is approached at a rate proportional to \( R|m| \) where \( R \) denotes the radial cylindrical coordinate [1-3].

Generation and detection of helicoidal wavefields

The standard procedure for generating a vortex beam is to select the appropriate phase evolution for the excitation of an angular array of transducers [2, 3, 8-10]. The simplest case is the generation of a beam having \( m = 1 \) using four identical transducers placed in identical quadrants and excited with signals having a phase \( \phi_R = 0, \pi/2, \pi, \) and \( 3\pi/2 \) for the source quadrants \( n_S = 0, 1, 2, \) and \( 3, \) respectively. This method was first used to generate ultrasonic vortex beams in water, where it was found that close to the axis the dominant component of the wavefield had a topological charge \( m = 1 \) and the axial null was preserved along the beam’s axis [2]. The components of the wavefield not directly possessing a phase dependence of \( \exp(i\Phi_{\text{rot}}) \) tend to diffract away from the axial region of the beam and diminish in amplitude the larger the axial propagation distance \( z \). Relatively recently [19, 20], a method was introduced for preferentially detecting the component of a wavefield having a specified topological charge, the simplest case being \( |m| = 1 \). Suppose it is desirable to detect the component of a wavefield having \( m = +1 \) with respect to a known axis while rejecting components having \( m = -1 \) or \( m = 0 \). A simple approach is to deploy four receiver transducers having equal sensitivity to the wave but offsetting the positions of the transducers appropriately. In the present investigation it was convenient to deploy the transducers with the axis of each disc-shaped receiver on a circle, which was in turn centered on the axis of the beam. The outputs of this set of transducers was superposed and recorded. To achieve the desired dependence on the topological charge of the wavefield, the axial position was appropriately offset for the transducer at the angular location \( \phi_R = n_R(\pi/2) + \phi_{\text{rot}}, \) \( n_R = 0, 1, 2, 3. \) The constant \( \phi_{\text{rot}} \) specifies the rotational orientation of the array but is unimportant except when the ring of receivers is close to the source transducer. To preferentially detect a beam having \( m = 1 \), the relative axial offsets of the four receivers (relative to the receiver transducer having \( n_R = 0 \)) are \( z_R = -\text{sgn}(m) c n_R(\pi/2)/\omega, \) \( n_R = 0, 1, 2, 3. \) Here \( c \) is the speed of sound in the fluid and \( \omega = 2\pi f \) where \( f \) denotes the frequency of the wave in cycles-per-second. The function \( \text{sgn}(m) \), which is \( \pm 1 \) according to the sign of \( m \), is included to facilitate generalization to the case \( m = -1 \). Notice that in adjacent quadrants the magnitude of the displacement is \( \lambda/4 \) where \( \lambda = c/f \) is the wavelength. A negative displacement of the transducer position corresponds to a displacement closer to the source. Notice that each transducer is at the same distance from the axis of the beam. In practice the arrangement is easiest to align in a vortex beam for beams having a slowly varying radial and axial dependence of the wavefield magnitude. The symmetry properties of the wavefield are such that for an array designed to detect
components having \( m = +1 \), the associated received signals add in phase. By contrast, wavefield components having \( m = -1 \) or \( m = 0 \) (corresponding to no vortex) produce received signals which cancel.

Modulation of the helicity of a beam

When the aforementioned four-transducer array is used to generate an \( m = 1 \) acoustic vortex close to the beam’s axis, there is a technique for rapidly reversing the helicity to produce a similar beam having \( m = -1 \) [19, 20]. The method of inverting the sign of \( m \) is to reverse the sign of the excitation of a single pair of transducers in opposite quadrants. For example, consider a source with \( \phi_S = 0, \pi/2, \pi, \) and \( 3\pi/2 \) for \( n_S = 0, 1, 2, 3 \), which produces a wavefield close to the axis having \( m = 1 \). Now consider a source having the aforementioned sign reversal of the excitation of the opposing pair of transducers with \( n_S = 1 \) and \( 3 \). As a result of this reversal, the phase becomes \( \phi_S = 0, (\pi/2) - \pi, \pi, \) and \( (3\pi/2) - \pi \). This corresponds to an excitation having a phase (modulo \( 2\pi \)) of \( \phi_S = 0, -\pi/2, -\pi, \) and \(-3\pi/2\) such that the helicity of the wavefield is reversed. By reversing the phase at a zero-crossing of the exciting sine wave and by using wide-bandwidth transducers, undesirable transients associated with the reversal may be minimized. In the context of the present research, helicity reversal is useful for demonstrating the helicity selectivity of the previously described receiver array. There are, however, other potential applications such as: (i) the generating of oscillating acoustic radiation torques on symmetric objects associated with the absorption of the energy and angular momentum of a vortex beam [7, 19], and (ii) digital acoustic communications when using a helicity selective array [20].

**FIGURE 1.** Demonstration of helicity modulation and helicity selective detection associated with the transmission of two tone bursts. The vertical axis is the combined output of the array illuminated directly by the transmitted beam. The initial burst is transmitted in the cross-helicity configuration relative to the array followed immediately by a burst in the co-helicity configuration. The received signal is weak in the cross-helicity configuration and large in the co-helicity configuration.
Verification of helicity selectivity of a receiver array

To verify the selectivity of the aforementioned receiver array, the following experiment was performed. A four-element transducer generated a beam having a vortex with \(|m| = 1\) near the axis. Triangular transducer elements were used instead of square elements used by Hefner and Marston [2] so as to improve the quality of the beam. The transducer was in water and driven at a frequency of 123 kHz. A water tank was used having a capacity of approximately 300 gallons so that though there were weak non-ideal reflections from the sides of the tank, there were no essential complications discussed here and below. The aforementioned helicity selective ring of four transducers was positioned on the beam axis at a distance of 122 cm from the source. Figure 1 shows an example of the time record from the superimposed output of the four receivers. The initial burst of 30 cycles had a helicity opposite the selected helicity of the detector, followed by a burst of 30 cycles having the helicity for optimum detection. The record shows that following an initial transient response the detected signal in the cross-helicity configuration approached a weak steady signal. This was followed by a much stronger signal in the co-helicity configuration nearer the end of the record. If instead the signal is displayed for only one of the receiver elements it is found the recorded signal is large and nearly constant in amplitude away from the regions for transition in the source excitation. Following this verification of the source and receiver properties, the transducers were used to investigate some topological properties in the scattering by a sphere.

Topological properties in the off-axis near-forward scattering by a sphere

When a symmetric object such as a sphere is placed on the axis of a vortex, it follows immediately from symmetry considerations that the forward and backward scattering on the axis vanishes [2, 12, 15]. It also follows that the topological charge of the scattering is the same as that of the incident wave when a symmetric object is on the axis [2, 12]. In the specific case of a Bessel vortex beam these properties also follow from a detailed analysis of the scattered wave [12-14]. The scattering is more complicated if the sphere is displaced from the axis of a vortex beam. Though in some special cases the magnitude of the scattering has been computed for a sphere displaced off of the axis of a Bessel vortex beam [15, 16], more relevant to the present investigation is the spatial evolution of the phase of the scattering. The beam here is not a Bessel beam and computational evaluations of the complex scattering is unavailable. To investigate the topological properties of the scattering, a small acrylic sphere was hung using a thin fishing line in a vortex beam with \(|m| = 1\) generated as previously described. The sphere was suspended using a two-dimensional stepper motor positioning system, which could reposition the sphere in a raster pattern at a fixed distance from the plane of the source transducer. The diameter of the sphere was 25.4 mm so that \(ka = 6.6\) where \(a\) denotes the radius of the sphere and \(k = \omega/c\). This \(ka\) is sufficiently offset from the principal sphere resonances that any elastic response of the sphere does not complicate the interpretation of the scattering [21]. The scattering of plane acoustic waves by acrylic spheres has previously been experimentally investigated and modeled in the relevant frequency range [21]. In the simplest of the measurements a single helicity-neutral receiver transducer was placed on the beam’s axis such that if the sphere is displaced slightly from the axis, the near-forward scattering is detected. As expected from symmetry, the detected scattering is weak for an on-axis sphere having a magnitude approximately linear in the offset for small offsets from the axis, which itself is an analytic property of beam amplitudes having \(|m| = 1\) near the axis [1-3]. Of greater importance is the evolution of the phase of the detected scattering, which was experimentally found to primarily contain a phase ramp having an angular dependence approximately of the form \(\exp(\pm i \phi)\) superposed on the radial phase evolution of a spreading wave. Here \(\phi\) is the angular coordinate of the center of the sphere. The phase pattern is much like that of the previously studied [2, 3] wavefield of a spreading vortex beam having \(|m| = 1\). The observed property is consistent with the phase of the scattering being dependent on the phase of the incident wave evaluated at the position of the center of the sphere.

Cross-helicity scattering

Now consider the situation with a helicity selective detector centered on the beam’s axis in the cross-helicity signal. In the ideal situation the detected signal vanishes both: (A) in the absence of a sphere, and (B) when the sphere is on the axis. In practice from Fig. 1 a residual background signal is recorded in
situation (A). The weak recorded background is coherently subtracted from the signals detected by the array when a sphere is present. This procedure is sufficient to display the phase evolution of the cross-helicity scattering of a scanned sphere. Figure 2 shows the phase as a function of the position of the center of the sphere. Near the axis the wavefield incident on the sphere is a vortex beam with $|m| = 1$ and along a circular path around the axial position the phase evolves by $2\pi$ radians. Inspection of Figure 2 shows, however, that along a circular path around the axial position the detected phase is found to evolve by $4\pi$ radians instead of $2\pi$ radians. Hence the cross-helicity scattering is found to be analogous to that of a wavefield having a topological charge incremented by unity from that of the incident beam. (For comparison, a phase scan of a beam having $m = 2$ is given in [4].) Using superposition, this observation is also consistent with the phase of the scattering being dependent on the phase of the incident wave evaluated at the position of the center of the sphere. This suggests the phase evolution of the cross-helicity component of the acoustic scattering of symmetric objects is useful for characterizing the topological properties of scattered fields. The amplitude variation of the cross-helicity scattering when the sphere is close to the axis is also found to be approximately similar to that of a wavefield having $|m| = 2$.

**Figure 2.** Instantaneous phase of the receiver array output is shown with the array set in the configuration favoring cross-helicity detection of scattering relative to the incident vortex beam. The horizontal and vertical axes give the horizontal and vertical coordinate of the center of a sphere scanned though the beam in a plane 60 cm from the source. The phase, in radians, is given by the color-bar shown on the right. The displacements of the sphere are measured in mm. Near the center of the pattern, the sphere is very close to the axis of the incident beam; the received amplitude is weak and the phase is indeterminate. When the sphere is displaced from the axis of the beam, while in the proximity of the axis, the phase of the array output evolves similar to a helicoidal wavefield having $|m| = 2$. 
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REFERENCES