4aUWb4. Three-dimensional acoustic propagation under a rough sea surface

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A three-dimensional propagation model using stepwise coupled modes is applied to calculate the acoustic field under a rough sea surface. The model is formulated in a cylindrical coordinate system and the solution for the three-dimensional acoustic field is approximated by accounting for mode coupling in the radial direction and including horizontal refraction in the azimuthal direction. The atmosphere above the sea surface is modeled as an acoustic half space having the properties of air and sea surface height is allowed to vary arbitrarily as a function of range and azimuth. For the sea surfaces presented in this work, the amplitude spectrum of the surface waves is modeled according to the JONSWAP spectrum and the directionality is included by assuming cosine-squared spreading. The acoustic field is calculated for sea surfaces determined for varying levels of wind intensity and fetch. A modal decomposition of the acoustic field is used to provide insight into the effects of the rough sea surface on the predicted transmission loss. The importance of using three-dimensional versus two-dimensional models for acoustic propagation under rough sea surfaces is investigated. [Work supported by ONR]

Published by the Acoustical Society of America through the American Institute of Physics
INTRODUCTION

Scattering from rough sea surfaces has been studied extensively for the two-dimensional (2D) propagation problem and acoustic models suitable to axisymmetric environments with variable sea surface height have been developed [1, 2]. Recently, three-dimensional (3D) propagation has been investigated for surfaces composed of straight sinusoidal waveforms [3, 4]. In this work, a 3D propagation model using stepwise coupled modes is applied to calculate the acoustic field under realistic sea surfaces modeled according to the Joint North Sea Wave Project (JONSWAP) spectrum [5]. Realizations of the sea surface are considered by varying wind intensity and fetch length, and the effects of the resulting sea surfaces on the acoustic field are examined. The importance of using 3D versus 2D models for acoustic propagation under rough sea surfaces is also investigated.

OCEAN SURFACE SIMULATION

The JONSWAP spectra is an empirical relationship that defines the frequency distribution of surface wave action [5]. The underlying spectral equation is

\[ S(\omega) = \frac{\alpha g^2}{\omega^5} \exp \left( -\frac{\beta \omega^4}{\omega_p^4} \right)^\gamma, \]

where \( \omega \) is the wave frequency, and the other terms are defined empirically as

\[ \sigma = \begin{cases} 0.07 & \text{if } \omega \leq \omega_p \\ 0.09 & \text{if } \omega > \omega_p \end{cases} \]

\[ a = \exp \left( -\frac{(\omega - \omega_p)^2}{2\sigma^2 \omega_p^2} \right) \]

\[ \omega_p = 2.84g^{0.7}L_F^{-0.3}U_W^{-0.4} \]

\[ a = 0.033 \left( \frac{\omega_p U_W}{g} \right)^{2/3} \]

where \( U_W \) is the wind speed 10 m above the sea surface in m/s, and \( L_F \) is the fetch length in m. The remaining terms in Eqs. (1) – (5) have constant values: shape factor \( \beta = \frac{5}{4} \), peak enhancement factor \( \gamma = 3.3 \), and gravity \( g = 9.81 \text{ m/s}^2 \).

The description of the JONSWAP spectra in frequency is used to obtain the distribution in wavenumber using the first order dispersion relation

\[ S(k) = S(\omega) \frac{\partial \omega}{\partial k}. \]

For shallow water, the dispersion relation is given by

\[ \frac{\partial \omega}{\partial k} = \frac{g}{2\omega} [k \text{sech}^2(kd) + \tanh(kd)] \]

where \( d \) is water depth in m [6].

The two-dimension wavenumber spectrum \( S(k_x, k_y) \) is created by assuming azimuthal symmetry about \( (k_x, k_y) = (0, 0) \). Cosine-squared spreading is included by

\[ S_{\cos^2}(k_x, k_y) = S(k_x, k_y) \cdot \cos^2(\theta) \]

where \( \theta = \text{atan}(\frac{k_y}{k_x}) \). A realization of the rough sea surface is obtained from \( S_{\cos^2}(k_x, k_y) \) by assuming random uniformly distributed phase and applying a 2D Fourier transform.
Acoustic Propagation Model

A 3D acoustic propagation model based on the stepwise coupled-mode approach [7] implemented as a single-scatter solution is applied. This technique is based on a hybrid modeling approach for which normal modes are applied in the vertical dimension and a PE solution is applied in the horizontal dimension.

The inhomogeneous Helmholtz equation for pressure \( P(r, \theta, z) \) at range \( r \), azimuth \( \theta \), and depth \( z \) from a point continuous wave source of amplitude \( S(\omega) \) located at range \( r = 0 \) and depth \( z = z_0 \), is given by

\[
\rho(r, \theta, z) \nabla \cdot \left( \frac{1}{\rho(r, \theta, z)} \nabla P(r, \theta, z) \right) + k^2(r, \theta, z) P(r, \theta, z) = -4\pi S(\omega) \frac{\delta(r)}{r} \delta(z - z_0),
\]

where \( k(r, \theta, z) = \omega/c(r, \theta, z) \) is the acoustic wavenumber, \( \omega = 2\pi f \), \( f \) is the acoustic frequency, \( c(r, \theta, z) \) is sound speed, and \( \rho(r, \theta, z) \) is density.

The solution for pressure is found by a separation of variables

\[
P(r, \theta, z) = \sum_{m=1}^{M} A_m(r, \theta) \phi_m(z; r, \theta),
\]

where \( A_m(r, \theta) \) are the modal amplitudes, and \( \phi_m(z; r, \theta) \) are the modal eigenfunctions.

The modal eigenfunctions \( \phi_m(z; r, \theta) \) satisfy

\[
\rho(r, \theta, z) \frac{\partial}{\partial z} \left( \frac{1}{\rho(r, \theta, z)} \frac{\partial \phi_m(z; r, \theta)}{\partial z} \right) + \left( k^2(r, \theta, z) - k_m^2(r, \theta) \right) \phi_m(z; r, \theta) = 0,
\]

where \( k_m(r, \theta) \) is the horizontal wavenumber of the \( m \)th mode. Boundary conditions are defined by the plane wave reflection coefficient below the upper halfspace and above the lower halfspace. In this work, the Pekeris branch cut is chosen such that the total pressure is calculated from a Pekeris branch line integral, plus a finite sum of trapped modes, plus an infinite sum of leaky modes. A small gradient is introduced in the lower halfspace which effectively removes the branch point and associated branch cut from the problem [8]. As a result, the leaky modes eventually decay as a function of depth in the lower half space. The eigenfunctions are normalized so that

\[
\int_{-\infty}^{\infty} \frac{1}{\rho(r, \theta, z)} \phi_m(z; r, \theta) \phi_n(z; r, \theta) dz = \delta_{mn}.
\]

In practice, a finite sum of trapped and leaky modes are included in the solution and the integration in Eq. (12) is calculated over a finite range of depths: the maximum depth is determined from the depth highest order mode included in the solution has decayed to an acceptable level, and the minimum depth is chosen according to the maximum height of the surface waves.

In the absence of mode coupling, the adiabatic solution for the mode amplitudes \( \tilde{A}_m(x, y) \) satisfies

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \tilde{A}_m}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \tilde{A}_m}{\partial \theta^2} + k_m^2(r, \theta) \tilde{A}_m = -4\pi S(\omega) \frac{\delta(r)}{r} \frac{\delta(z_0)}{\rho(r, \theta, z_0)},
\]

This equation must be solved for each mode with the horizontal refraction determined by the modal phase speed \( c_{ph_m}(x, y) = \omega/\mathcal{R}e[k_m(x, y)] \) and modal attenuation \( a_m(x, y) = \mathcal{I}m[k_m(x, y)] \). Such a 2D Helmholtz equation can be solved by standard techniques. In this work, the solution to the horizontal refraction equation is obtained from a PE model [9] which has been modified for cylindrical coordinates [10].

Mode-coupling is incorporated into the solution for the modal amplitudes by a stepwise coupled-mode technique [7]. This approach, originally derived for the 2D Helmholtz equation, is...
applied by discretizing a range-dependent environment into a series of range-independent segments for which continuity of pressure and particle velocity must be satisfied at the vertical boundaries between segments. Application of the boundary conditions results in a set of linear equations from which the coupling matrix \( R_{m,n}(r,\theta) \) is obtained,

\[
R_{m,n}(r,\theta) = C_{m,n}^{RL}(r,\theta) + C_{m,n}^{LR}(r,\theta) \frac{k_n(r_L,\theta)}{k_n(r_R,\theta)},
\]

(14)

where

\[
C_{m,n}^{LR}(r,\theta) = \int_{-\infty}^{\infty} \frac{1}{\rho(r_L,\theta,z)} \phi_m(z;r_L,\theta)\phi_n(z;r_R,\theta)\,dz
\]

(15a)

\[
C_{m,n}^{RL}(r,\theta) = \int_{-\infty}^{\infty} \frac{1}{\rho(r_R,\theta,z)} \phi_m(z;r_L,\theta)\phi_n(z;r_R,\theta)\,dz,
\]

(15b)

where the suffixes \( L \) and \( R \) denote the properties to the left and right of a vertical interface. This form of the mode-coupling matrix describes the single-scatter approximation, such that each pair of segments is treated as an individual problem, thus neglecting higher-order terms resulting from multiple scattering at other interfaces.

In the 3D coupled-mode model, the range-independent segments are defined in the radial direction, forming angular sectors in the range-bearing plane. The solution for the coupled-mode amplitudes is obtained in a two-step process. First, the coupling of energy between the modes is calculated using the mode-coupling matrix,

\[
A_m = \sum_{n=1}^{N} R_{m,n}(r,\theta)A_n.
\]

(16)

Then the coupled-mode amplitudes are propagated in range and azimuth according to

\[
\frac{\partial A_m}{\partial r} = ik_0 \sqrt{1 + k_0^{-2} \left( \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + k^2 - k_0^2 \right)} A_m.
\]

(17)

where \( k_0 = \frac{\omega}{c_0} \) is the reference wavenumber, and \( c_0 \) is the reference sound speed. According to this solution technique, at each range step, mode-coupling is calculated by Eq. (16) and horizontal refraction is calculated by Eq. (17). Although mode-coupling occurs only in the radial direction, the coupled energy is refracted out of the range-depth plane.

**RESULTS**

Propagation in a shallow water environment was investigated for various wind intensities and fetch lengths. The propagation environment consists of an isovelocity water column \((c = 1500 \text{ m/s}, \rho = 1.0 \text{ g/cm}^3)\) over an acoustic half space having the properties of sand \((c = 1767.3 \text{ m/s}, \rho = 1.845 \text{ g/cm}^3, \alpha = 0.75225 \text{ dB/} \lambda)\). The mean water depth is 50 m. In all cases, the direction of wind forcing is along the x-axis. For the acoustic modeling, a frequency of 100 Hz is considered and the solution is obtained from a sum over 10 modes. For the environments considered, these parameters provided for convergence of the coupled-mode solution as well as good agreement with a rough surface PE solution [2].

Sea surface height calculated for a wind speed of 32 m/s and fetch length of 100 km is shown in Fig. 1. The cosine squared spreading is evident from the longer wavelengths observed in the direction perpendicular to the wind forcing. The spreading is further illustrated by Fig. 2 which shows sea surface height along the positive x-axis in the top panel and sea surface height along the positive y-axis in the bottom panel.

Transmission loss (TL) in the range-bearing plane at a depth of 25 m is shown in Fig. 3. The effects of the range-dependent sea surface are evident from the asymmetric acoustic field.
Generally, there is higher loss in the direction parallel to the wind forcing. This result is also observed in Fig. 4 which shows TL in the range-depth plane along bearings of 0° and 90°. The higher loss along the 0° bearing results from surface waves which have shorter wavelengths and induce more mode coupling. The scattering of energy from trapped to leaky modes, which are more rapidly attenuated, is responsible for the higher loss.

### Effect of Wind Intensity and Fetch Length

In this section, the effects of varying the wind intensity and fetch length on the acoustic field are investigated. Over longer fetch lengths, the surface waves evolve into longer crested waves. The effect of fetch length on acoustic propagation is examined by considering fetch lengths of 1 km, 10 km, and 100 km. Greater wind intensities increase the height of the surface waves. Wind speeds of 17 m/s, 24 m/s, and 32 m/s are considered.

Depth integrated TL for the three different fetch lengths and wind intensities calculated using the 3D coupled-mode model are show in Fig. 5. The top row of plots show depth integrated TL in the direction parallel to the wind forcing and the bottom row of plots show depth integrated TL in the direction perpendicular to wind forcing. For reference, the red line in the plots on the far right were calculated from the acoustic fields shown in Fig. 4. Comparing the top and bottom rows of Fig. 5, the general trend of greater loss in direction parallel to wind forcing is observed. From comparison of the solutions orientated in a single direction, it is found that shorter fetch lengths and higher wind intensities are associated with the greatest loss.
Examination of 3D Effects

As shown in Fig. 5, the falloff in the depth integrated TL curves calculated using the 3D coupled-mode model are not smooth. Furthermore, the curves corresponding to the direction perpendicular to wind forcing are characterized by the greatest fluctuations. To understand this, consider depth-integrated TL calculated for identical conditions using an N×2D model. These solutions are shown in Fig. 6. The same trends discussed for the 3D modeling are viewed here as well, i.e., higher loss in the direction parallel to wind forcing and higher loss associated with shorter fetch lengths and higher wind intensities. However, the N×2D solutions are characterized by smoother curves. The fluctuations in the depth integrated TL of the 3D solutions are caused by refraction of sound into and out of the range-depth plane. A comparison of Figs. 5 and 6 shows that horizontal refraction effects can cause a difference in depth integrated TL of more than 10 dB. The most significant horizontal refraction effects are observed in the direction perpendicular to wind forcing for which the longer wave crests more efficiently channel sound. Additionally, the greatest 3D effects are observed for short fetch lengths and high wind intensities because these conditions result in the largest gradients in modal phase speed which control the horizontal refraction according to Eq. (17).

CONCLUSIONS

A 3D propagation model using stepwise coupled modes was applied to calculate the acoustic field under realistic sea surfaces modeled according to the JONSWAP spectrum with cosine-squared spreading. A number of realizations of the sea surface were considered by varying wind intensity and fetch length. The acoustic simulations showed the acoustic field depended on azimuthal angle with the greatest loss occurring in the direction parallel to the
FIGURE 6: Depth integrated TL calculated using an N×2D coupled-mode model for various wind speeds and fetch lengths in directions parallel and perpendicular to wind forcing.

wind forcing. The greatest loss was associated with the shortest fetch lengths and highest wind speeds.

The importance of using 3D versus 2D models for acoustic propagation under rough sea surfaces was investigated. It was found that horizontal refraction effects can cause a difference in depth integrated TL of more than 10 dB. The acoustic simulations showed that 3D effects were most important in the direction perpendicular to wind forcing. The most significant horizontal refraction effects were associated with the shortest fetch lengths and highest wind speeds.

ACKNOWLEDGMENTS

This work was sponsored by the Office of Naval Research under Grant N00014-12-1-0074. The Texas Advanced Computing Center (TACC) at The University of Texas at Austin provided High Performance Computing (HPC) resources that have contributed to the research results reported within this paper.

REFERENCES


