4pUW5. Sensitivity of co-prime arrays to shape perturbation

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Co-prime arrays offer savings in both implementation and computation by reducing the number of array elements. For passive beamforming, a pair of specially-spaced sparse arrays organized as a co-prime array provides unambiguous source bearings through the cancellation of the grating lobes inherent in the pattern response of each array if processed by itself. In the ocean environment, however, towed line arrays are difficult to keep aligned and take on a time-varying shape. Hodgkiss (IEEE JOE, 1983) showed that array shape perturbation can lead to beam broadening, an effect which may interfere with the grating lobe cancellation of co-prime arrays. In the present paper, performance degradation of co-prime sparse arrays are examined under the condition of perturbed array shape. Simulations are used to compare a co-prime array of known element spacing and position to an array with small uncertainties in both element location and interelement spacing along the array. Possible correction methods are examined. Work sponsored by ONR Undersea Signal Processing.

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Line arrays are often used in the underwater environment for detecting and determining the angle of arrival of acoustic signals. To better resolve arrival angle, large aperture arrays are desirable. However, as arrays grow larger, so do the number of elements required to fully populate them. As a way to reduce both cost and computational intensity, strategies for sparsely populating large arrays have been examined. Typically, these methods depend upon precise knowledge of the element locations. However, as the array bends and responds to turbulent forces, uncompensated errors in element position may occur, degrading performance.

Co-prime arrays have been shown capable of reducing the number of elements in an array without introducing ambiguity caused by grating lobes [1]. For two arrays with \( N \) and \( M \) elements, respectively, where \( N \) and \( M \) are co-prime integers which share no common factors aside from unity, direction of arrival measurements can be made with \( NM \) distinct beams. To achieve this, the beam patterns from two undersampled uniform arrays are combined such that the grating lobes, resulting from their undersampled nature, are cancelled. Each array has an element spacing of \( N\lambda/2 \) and \( M\lambda/2 \), and consists of \( N \) and \( M \) ideal pass beams with beamwidths proportional to \( 1/M \) and \( 1/N \) respectively. As a result of the co-primality of \( N \) and \( M \), the evenly spaced lobes of the two arrays are coincident only at a single angle. However, it has been shown that because real filters are not ideal, there is some overlap between the grating lobes. This effect can be reduced by increasing the numbers of elements in the arrays while maintaining the spacing, corresponding to an increase in array length.

Array distortion is a chief concern for the practical application of arrays. While a common assumption is that the elements are co-linear with a known inter-element spacing, this configuration is not always realized in practice. Towed array shape can be deformed during turns, as well as at high speed, when turbulence can generate random forces along the length of the array. For uniformly spaced arrays, the effect of such array deformation has been examined by Hodgkiss [2]. For a bowed line array, it was shown that the main lobe in the beam pattern is broadened. This effect can be reduced, however, if the element locations are known. Real-time element localization is not always feasible, however, so the effect of element perturbation on array performance remains important.

The effect of perturbation on co-prime arrays is examined in this paper. Consideration of known effects on single line arrays does give some insight. Each array of a co-prime array is uniformly spaced, and will thus experience the same bow-induced beam broadening seen in previous research. While the effect on grating lobes was not specifically examined by Hodgkiss, two conceptual models can give guidance. Treating the grating lobes as repetitions of the main lobe, one might expect beam broadening when the array is bowed. Co-prime array performance is largely determined by grating lobe overlap. Thus broadening of grating lobes might be expected to increase overlap and increase the magnitude of side lobes. Conversely, aperiodic arrays have been shown to provide reduced peak grating lobe levels [3], which may also be anticipated for co-prime arrays. While this effect may lead to an overall reduction in sideband level in the case of overlap, it will be shown that band overlap is not completely offset by the grating lobe reduction of a perturbed array.

The effect of uncompensated element location errors on co-prime array performance will be investigated through simulation. Using a planewave signal at a known angle, and time-delay beamforming, co-prime array response is calculated. Element location errors are simulated by adjusting the sensor locations and calculating a new response for the same plane wave. The co-prime array response will be calculated using both the intended element locations and actual element locations after perturbation. Both results may be of interest, because real-time element localization is not always feasible.
**SIMULATION**

**Setup**

In this computer simulation, planewaves are generated at an angle of 45° with the array. Sound speed is assumed to be uniform in space and time \((c = 1500\text{m/s})\) and the source is single frequency \((f = 100\text{Hz})\), such that the wavelength is \(\lambda = c/f = 15\text{m}\). In the case of an unperturbed (co-linear) array, elements are evenly spaced in a line. The co-prime array will be designed such that \(M = 6\) and \(N = 5\). The distance between elements is \(M\lambda/2\) in the first array, and \(N\lambda/2\) in the second. The total length of the array, \(L\), is bounded by \(L \geq NM\lambda/2\), and is chosen to be \(L = NM\lambda/2 = 225\text{m}\). To find the beam response of each component line array in the co-prime array, conventional time-delay beamforming is used.

Array bowing is characterized using a radius of curvature, \(R\), relevant for a towed array being pulled by a ship which is changing course. The radius of curvature is directly related to the array bowing parameter used by Hodgkiss \(\text{bow} = R(1 - \cos(L/2R))\). Fig. 1 shows the three sets of element locations (corresponding to three radii of curvature) which are used in this simulation. An infinite radius of curvature corresponds to a straight line, which are the desired element locations.

![Diagram of array element locations](image)

**Figure 1:** Array element locations calculated for co-prime arrays with \(M=6\) and \(N=5\), and element positions for two different radii of curvature, given \(L = 225\text{m}\).

For visual comparison, all graphs will be normalized by the peak value, and plotted against sin\(\theta\). Use of this independent variable causes the grating lobes of the component arrays to be evenly spaced. In addition to visual comparison of the beam patterns, two values will be measured and compared: the peak level of the first sidelobe of the main beam, and the grating lobe suppression, determined from the maximum peak level outside the main lobe and its adjacent side lobes.

**Results**

When used in a passive configuration, the response of a co-prime array is found by multiplying the responses of the two component arrays [1]. As shown in Fig. 2, the component arrays each have a lobe in the direction of the source, as well as a number of grating lobes determined by the values of \(M\) and \(N\). However, the grating lobes of the component arrays are not at the same locations as a result of the co-primeness of \(M\) and \(N\). As a result, through multiplication of the two patterns, the grating lobes are significantly reduced. In Fig. 2 it can be seen, however, that the remaining grating lobe is slightly greater than the sideband level of either of the component arrays, and is determined by the amount of overlap of the grating lobes.
**FIGURE 2:** Beam response of a co-prime array with $M=6$ and $N=5$, shown with the component arrays for comparison. The actual source location is $\sin(-45^\circ) = -0.707$. The side lobe level is -24.6 dB and the grating lobe suppression is 11.0 dB.

Fig. 3 shows the response of a single component array to bowing when the true element locations are not used for beamforming. Instead, the planned element locations, corresponding to a straight array or an infinite radius of curvature, are used for processing. It can be seen that as the radius of curvature decreases, both the main beam and the grating lobes become wider. The effects of this beam widening can be seen in the co-prime arrays in Fig. 4. The increased overlap of the component array grating lobes has directly lead to reduced grating lobe suppression, in addition to an increase in sidelobe level.

Co-prime array performance degradation caused by bowing can be mitigated somewhat if the true element locations are known. Element localization could be achieved using heading and depth sensors built into the array or perhaps using coherent ambient noise [4]. Hodgkiss showed that using the true element locations restores the performance of a uniform line array [2]. Fig. 5 shows the response of a co-prime array with and without correct element locations. When true element locations are used, the performance is almost entirely restored, indicating that this is a solution when element localization is feasible.

**CONCLUSION**

In order to reduce both the material and computational costs of large aperture arrays, alternatives to uniform element spacing have been explored. Co-prime arrays offer one approach, eliminating grating lobes by multiplying the beam responses of two parallel undersampled arrays. However, the performance of such arrays with array bowing has not heretofore been considered. The present work shows that the beam broadening that occurs when uniform line arrays are bowed reduces the grating lobe suppression of a co-prime array. However, performance can be restored if the true element locations are known and used in beamforming.
FIGURE 3: Beam response of a single component array with M=6 and two different unmodeled radii of curvature for $L = 225m$. The side lobe levels for the three component arrays are -12.5 dB, -10.4 dB, and -5.8 dB, respectively.

FIGURE 4: Beam response of a co-prime array with M=6, N=5, and two different unmodeled radii of curvature for $L = 225m$. The side lobe levels are -24.7 dB, -20.3 dB, and -11.0 dB, respectively. The grating lobe suppression is 11.1 dB, 10.0 dB, and 5.2 dB, respectively. The $R = \infty$ response is the same as in Fig 2.

The use of true element locations is only one possible solution to the problem of reduced grating lobe suppression due to array bowing. Moreover, array element localization may not
Figure 5: Beam response of a co-prime array with M=6, N=5, and a radius of curvature of $R = 5 L$ for $L = 225$ m. The bowed response shows the beam pattern derived when the planned element locations are used in beamforming. The side lobe level is -11.1 dB and grating lobe suppression is 5.2 dB. The recovered response shows the beam pattern achieved when the actual element locations are used in beamforming. The side lobe level is -24.4 dB and grating lobe suppression is 11.2 dB.

always be feasible. Other means of improving performance of a curved array should be investigated. Possible solutions include, but are not limited to, increasing the length of the array to reduce grating lobe width and applying array shading to reduce sideband levels. Additionally, this paper has focused on time-delay beamforming. Alternative beamforming schemes applied to the component arrays may have different responses to perturbation, such as a complete failure to determine angle of arrival, which was not demonstrated here.

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REFERENCES

