The use of graphical processing unit processing in rough surface scattering

Ahmad T. Abawi* and Paul Hursky

*Corresponding author's address: HLS Research, La Jolla, CA 92037, abawi@hlsresearch.com

The use of Graphical Processing Units (GPU's) in scientific computation has drawn significant interest in recent years. In this paper we use GPU processing to evaluate the performance of a number of approximate techniques in computing scattering from two-dimensional rough surfaces by comparing their results with those obtained using the boundary element technique, which produces a numerically exact solution of the problem. To compute scattering from a two-dimensional surface, we use a technique that we developed for computing scattering from compact objects, which uses an analytical expression for scattering from a single, flat triangle. In this technique the surface is meshed using triangular patches and the scattering is computed as a coherent sum of scattering from individual triangles. This technique not only provides accurate evaluation of the surface integrals that appear in scattering theory, but it also lends itself easily to the benefits of GPU processing. We apply this technique to the Kirchhoff approximation, the small slope approximation and to a rather less familiar technique based on the work of Dashen et. al. [Dashen, R. and Wurmser, D., J. Math. Phys., 32, 986-996, 1991].

Published by the Acoustical Society of America through the American Institute of Physics
INTRODUCTION

The use of Graphics Processing Units (GPU) in computing large numerical problems has attracted a lot of attention in recent years. While a typical CPU can have access to a handful of cores, a modern GPU can have access to hundreds of cores. The ability to harness such computational power has made it possible to tackle numerical problems large enough that just a few years ago they were considered as utterly intractable. One of these problems is the computation of scattering from a two-dimensional rough surface.

In this paper we use GPU processing to evaluate the performance of a number of approximate techniques in computing scattering from two-dimensional rough surfaces by comparing their results with those obtained using the boundary element technique, which produces a numerically exact solution and we thus refer to it as the reference solution.

To compute scattering from a two-dimensional surface, in Section 2 we describe a technique, which computes scattering from a surface as a coherent sum of scattering from individual triangular elements that make up the surface. This method uses an analytic formula for scattering from a single triangle. In Section 3 we apply this technique to compute scattering from a pressure-release (acoustically soft), doubly sinusoidal surface and compare its results with those obtained using the reference solution.

THE TRIKIRCH SCATTERING MODEL

The Helmholtz-Kirchhoff integral for the scattering amplitude can be approximated by

\[ T = \frac{-i}{4\pi} \int_S G(\vec{k}, \vec{q}) e^{i\vec{Q} \cdot \vec{X}'} d^2 \vec{X}', \tag{1} \]

where in the above \( \vec{k} \) is the incident wavevector, \( \vec{q} \) is the outgoing wavevectors, \( \vec{Q} = \vec{k} - \vec{q} \), \( \vec{X} \) is the coordinates of a point on the surface, \( \hat{n} \) is the unit normal to the surface and the kernel \( G(\vec{k}, \vec{q}) \) is specified by the surface scattering model. In the TriKirch scattering model, (1) is evaluated on a single triangle by assuming that \( G(\vec{k}, \vec{q}) \) is constant over the size of the triangle. The result is

\[ T_{\text{triangle}} = \begin{cases} 
-\frac{AG(\vec{k}, \vec{q})}{2\pi} e^{i\vec{Q} \cdot \vec{R}_0} e^{i\vec{Q} \cdot \vec{T}_{12}/2} \Gamma \left( \vec{Q}, \vec{T}_{12}, \vec{T}_{23}, \vec{T}_{31} \right), \\
\frac{iAG(\vec{k}, \vec{q})}{2\pi} e^{i\vec{Q} \cdot \vec{R}_0} \left[ -1 + i\vec{Q} \cdot \vec{T}_{31} + e^{-i\vec{Q} \cdot \vec{T}_{31}} \right] \left( \vec{Q} \cdot \vec{T}_{31} \right)^2 \tag{2} \\
-\frac{iAG(\vec{k}, \vec{q})}{4\pi} e^{i\vec{Q} \cdot \vec{R}_0}, \text{ if } \vec{Q} \cdot \vec{T}_{12} \to 0 \text{ and } \vec{Q} \cdot \vec{T}_{31} \to 0,
\end{cases} \]

where

\[ \Gamma \left( \vec{Q}, \vec{T}_{12}, \vec{T}_{23}, \vec{T}_{31} \right) = \left[ e^{-i\vec{Q} \cdot \vec{T}_{31}/2} \text{Sinc} \left( \vec{Q} \cdot \vec{T}_{23}/2 \right) - e^{i\vec{Q} \cdot \vec{T}_{23}/2} \text{Sinc} \left( \vec{Q} \cdot \vec{T}_{31}/2 \right) \right]. \]

In (2), \( \vec{T}_{i,j} \) are the triangle edge vectors, pointing from node \( i \) to node \( j \), \( A \) is the area of the triangle and \( \vec{R}_0 \) is the position vector to node 1. The kernel \( G(\vec{k}, \vec{q}) \) for the Kirchhoff approximation is given by

\[ G_K(\vec{k}, \vec{q}) = \hat{n} \cdot \vec{Q}, \tag{3} \]
in the Arctan model for an acoustically soft surface it is given by [3]

\[ G_D(\vec{q}, \vec{k}) = Q_n + W_n - Q_n^2 \tan^{-1}\left(\frac{\sqrt{Q_n^2 - Q_n^2}}{Q_n}\right), \tag{4} \]

where in the above \( \vec{W} = \vec{k} + \vec{q}, \) \( Q_n = \vec{Q} \cdot \hat{n}, \) \( W_n = \vec{W} \cdot \hat{n}; \) and in the first-order small slope approximation it is given by [4]

\[ G_{SS}(\vec{k}, \vec{q}) = -4k_z q_z Q_z, \tag{5} \]

where \( k_z = \vec{k} \cdot \hat{z}, \) \( q_z = \vec{q} \cdot \hat{z}, \) \( Q_z = \vec{Q} \cdot \hat{z}, \) and \( \hat{z} \) is a unit normal to the reference surface. It is well known that the accuracy of the Kirchhoff approximation (3) is \( O(\lambda/R) \), where \( \lambda \) is the acoustic wavelength and \( R \) is the radius of curvature of the surface. The accuracy of the first-order small slope approximation (5) is first order in surface height. The accuracy of the Arctan model, on the other hand, is the same as the composite surface model, i.e. it is accurate to first order in both surface height and surface curvature. It therefore should be more accurate than both the Kirchhoff approximation and the first-order small slope approximation in computing scattering from a rough surface. In the next section we apply these models to compute the scattering amplitude from a doubly sinusoidal surface and compare the results with those obtained from the reference solution.

**APPLICATION TO SCATTERING FROM A 2D ROUGH SURFACE**

The surface is described by

\[ \xi(x, y) = H \sin(20\pi x) \sin(20\pi y), \quad -0.5 \leq x \leq 0.5, \quad -0.5 \leq y \leq 0.5, \]

and shown in Fig. (1), where it is meshed using triangular elements. These triangular elements are used in (2) to compute the scattering amplitude for the surface using (3), (4) and (5), collectively referred to as the approximate solutions. In this preliminary study, we have not made any attempt to suppress edge effects. In these simulations, the incident field is a 6 kHz plane wave incident in the direction \( (\phi_i = \pi/3, \theta_i = \pi/3) \) toward the surface, where \( \phi_i \) is the azimuthal angle and \( \theta_i \) is the polar angle measured from the plane of the surface. The scattering amplitude is computed around the semicircle \( \phi = \pi/3 \) and \( 0 \leq \theta \leq \pi, \) where again \( \theta \) is measured from the plane of the surface. The results are shown in Fig. (2), where the square of the scattering amplitude, \( T \), is plotted as function of \( \theta. \) The top panel in Fig. (2) shows the results for the case when the surface is flat and
Figure 2: Comparison of the scattering amplitude from a doubly sinusoidal surface computed using three approximate rough surface scattering models with the boundary element solution. The boundary element solution in black, the Arctan solution is in red, the small slope approximation is in green and the Kirchhoff approximation is in blue. The surface height varies from 0 in the top panel to 6 cm in the bottom panel. Notice that the Arctan model has the best performance followed by the small slope and the Kirchhoff approximations.

all the approximate solutions agree pretty well with the boundary element solution, particularly near specular ($\theta = 120^\circ$). In the middle panel, the surface height is 3 cm, where we see that the approximate solutions begin to deviate from the reference solution, and in the bottom panel the surface height is 6 cm, where the approximate solutions begin
to break down, as the severe surface height is beyond their range of validity. Based on these results, we rank the Arctan model as having the best performance followed by the small slope and the Kirchhoff approximations. The reason for the poor performance of the Kirchhoff approximation is that it is only accurate to first-order in surface curvature and it breaks down for non-zero surface heights. This can be seen in Fig. (2) as the surface height increases. The reason for the superior performance of the Arctan model is that it is accurate to first order in both surface height and surface curvature. The first-order small slope approximation is accurate to first order only in surface height and thus its performance lies in between the Arctan model and the Kirchhoff approximation.

REFERENCES


