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5aBAa5. Determination of bone properties from Lamb type of waves
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Our domain of interest is cortical bone characterization, i.e. determination of structural and material properties by means of ultrasound waves. The frequency spectrum of Lamb type of waves are the input data of the method of parameters identification, based on a least mean square algorithm. Specificities of clinical measurements of long bones require first to address the issue of measurements with limited access [Minonzio & al., J. Acoust. Soc. Am., 2010] and second, to develop methods for a combined determination of structural and elastic properties. The first physical model was the free plate under plane strain assumption for transverse isotropic materials with 4 elastic parameters and one structural (thickness). Experiments on flat plates and tubes of circular cross section are used as test case and the method was validated by comparison to independent techniques dedicated to material properties [Bernard S. & al. 2012]. The same method was applied on in vitro experiments on cortical bone samples. Here, the bone thickness was measured independently by X ray technique. This study opens the perspective of updating the physical model to determine additional relevant parameters, the tube diameter and/or the soft tissue properties in case of in vivo measurements.

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INTRODUCTION

Our domain of interest is the ultrasound-based characterization of long cortical bone, \textit{i.e.} determination of structural and material properties by means of ultrasound waves. Targeted bone sites under concern are mainly the radius (forearm) and the tibia. They are approximately of tubular shape and due to a limited access along their circumference, they are naturally explored by ultrasound devices along their axis with a linear arrangement of ultrasound emitter/receiver used in pitch-catch technique called thereafter axial transmission.

More precisely, we intend to exploit frequency dependent properties of Lamb type of waves as ultrasounds guiding by cortical walls was evidenced in several \textit{in vitro} measurements on bovine and/or human long bones \cite{1-4}. Few attempts of bone characterization based on guided waves spectrum, expressed as frequency-wavelength or frequency-velocity relationship were reported. Moilanen \textit{et al.} have exploited a unique branch of the spectrum, namely the fundamental flexural branch known to be predominantly sensitive to thickness of the waveguide \cite{1}. Cortical thickness of 40 ex vivo human radius specimens was determined and compared well with independent measurements. The model for bone was an isotropic hollow tube model with prescribed elastic properties and diameter. F11 dispersion curve was obtained after selection of its transient response inside the multi-component measurements. The model for bone was an isotropic hollow tube model with prescribed elastic properties and diameter. F11 dispersion curve was obtained after selection of its transient response inside the multi-component measurements.

In our group, the approach was first to design an experimental set-up dedicated to clinical measurements and second to adapt the signal processing to it. Our choice of using a compact linear array for axial transmission on bones monitored by a multichannel electronics led to a quasi-real time processing in our previous clinical study based on time analysis of the signal. To reduce the impact of heterogeneity and absorption of bone and soft tissue properties, the length of the array is reduced to a few cm. To resolve multiple branches of guided waves in the Fourier plane despite a reduced receiving length, a signal processing based on multichannel response has been introduced, \cite{6} which takes benefit of the singular value decomposition of the bone response \cite{6}. One of the advantage of the SVD-based method of signal processing is to allow upgrading, as reported in \cite{7}. Currently, the SVD based method allows to experimentally determine the spectrum in the Fourier plane (frequency $f$, wavenumber $k$) of guided waves from the measurements conducted with an about 5 cm compact array with 5 emitters and 24 receivers. The method retrieves ($f, k$) values which would have been absent of the spectrum otherwise due to attenuation of guided waves along the receiving length related for example to material absorption. Thanks to these efforts, a procedure of characterization is currently explored.

MODEL

The general framework of an inversion process starts with the choice of \textit{a} model in which the quantities intended to be experimentally measured vary with some set of parameters (model parameters). By minimizing the deviation between experiments and model predictions, parameters of the experiments are identified to values taken by the parameters in the best model. A cost function is build to evaluate the minimal shift reached through the application of an optimization algorithm. In this study, the conventional non linear least square algorithm is used and the cost function is based on the shift between experimental and calculated frequency for values of wavenumber experimentally measured. The model used is the Rayleigh-Lamb waves on isotropic or transverse isotropic homogeneous plate. The model requires several assumptions to be verified. First, hypothesis of elastic homogeneity is supported by previous studies of bone characterization and also by several independent studies \cite{8-10}. All indicate that effective homogenized material bone properties are investigated by ultrasound waves with wavelengths of millimetric order on small pieces of bones. Impact of radial variations of elastic properties related to a profile of porosity is disregarded in this model. Second, assumption of constant thickness might appear stronger, as studies indicate that in the region of measurement in the forearm, variation of thickness is systematic (decrease from proximal to distal end) but small compared to wavelength and is then disregarded by the present study. Third, the relevance of assumption of flatness to describe bone geometry must be questioned. According to experimental measurements on homogenous tubes of standard material with circular cross section, the probe does not induce...
displacements in the tangential direction. However, axisymmetrical waves depart only slightly from Rayleigh-Lamb waves. One additional issue to be mentioned is that we assume that bone is perfectly transverse isotropic with a well defined symmetry axis and aligned on an hypothetic geometrical axis along which the probe is aligned. The following conditions are strictly assumed: the material constituting the plate is transversely isotropic; the motion induced by the excitation lie strictly in the \((O_{x_1x_3})\) plane; the sagittal plane is supposed to coincide with the plane of symmetry \((O_{x_1x_3})\) in Fig. 1) where \((O_{x_1})\) is the direction of the symmetry axis of the material.

**FIGURE 1.** Plate geometry.

It can be envisioned that effects other than these induced by the limited thickness of bone could be taken into account in upgraded models. The major purpose of this study is to evaluate the current state-of-art of the whole procedure of measurements regarding the goal, namely reconstructing both thickness and elastic properties of cortical bone. The current state-of-art of the measurement procedure provides experimental \((f,k)\) measured with the current prototype dedicated to clinical measurements processed with the SVD-based method upgraded for attenuated waves. The set-up operates at a central frequency of 1 Mhz, cortical bone at the forearm is about 2 mm thick, according to X ray measurements on previous database in our lab, velocity of compression wave is likely around 3-4 m/s according to in vitro measurements. It follows that the experimental database is expected to include branches of higher order than the two fundamental ones (A0 and S0). In addition, the current prototype of the probe do not render systematically these fundamental branches. Special care is then given to potential of higher order modes to reconstruct both elastic properties and thickness.

**MODEL PARAMETERS**

For a free monolayer, roots of the dispersion equation depend on material properties through the slowness \(V^{-1}_J\) of bulk waves \(J\) involved in the problem. \(V_J\) is the phase velocity of bulk wave \(J\), where \(J=L\) or \(T\) for an isotropic material or \(J=QP\) or \(QT\) for an anisotropic plate \([11,12]\). It follows that elastic stiffness and mass density cannot be determined unambiguously from the characteristic equation of the free monolayer. This is not a universal rule for all free waveguides, namely for a free bilayer, the ratio of mass density of each layer shows up explicitly in the dispersion equation. Subsequently, the free monolayer model offers a characterization of the plate in terms of thickness and bulk velocities or a function of them, unless an a priori value of mass density or individual elastic stiffness is postulated.

It can be argued that in an anisotropic medium, the mass density and individual stiffnesses show up explicitly in the Christoffel equation which define the slowness in the material. We have introduced an adimensionalized form of the Christoffel equation which writes with quantities reduced to \(V_T\), the bulk wave velocity of the pure shear wave in the direction parallel to the symmetry axis of the material. Moreover, plane strain is assumed, and after some manipulations, the dispersion equation of the transverse isotropic plate of symmetric (subscript S) or antisymmetric motion (subscript A) may be expressed as a function of 4 elastic parameters:

\[
F_{AS} \left(\epsilon_k, \epsilon_e ; P_{el}\right) = 0 \quad \text{with} \quad P_{el} = \begin{bmatrix} c_{13} & c_{33} & c_{11} & c_{55} \end{bmatrix} V_T^{-1} \tag{1}
\]

The stiffness \(c_{13}\) and \(c_{55}\) respectively are related to compression and shear, along the symmetry axis of the material. The stiffness \(c_{11}\) and \(c_{33}\) respectively are related to compression and shear in the direction normal to the symmetry axis of the material respectively. The stiffness \(c_{13}\) is the non-diagonal term which participates to the
deviation from the isotropic case. Note that these stiffness ratios are related to bulk wave phase velocities ratio and to the plane strain Poisson’s ratio \( \nu_p \) as follows:

\[
\frac{c_{13}}{c_{11}} = \left( \frac{V_{L3}}{V_{L1}} \right)^2; \quad \frac{c_{11}}{c_{55}} = \left( \frac{V_{L1}}{V_T} \right)^2; \quad \nu_p = \frac{c_{13}}{c_{11}}
\]  

(2)

where \( V_{L1} = (c_{11}/\rho)^{1/2} \) and \( V_{L3} = (c_{33}/\rho)^{1/2} \) are respectively the bulk compression velocity across and along the symmetry axis of the material. Note that relative locations in \((f, k)\) plane of cut-off frequencies depend entirely on the relative value of \( V_{L1}/V_T \), as they depend entirely on the ratio \( k = V_L/V_T \) for isotropic material.

Lamb spectrum was calculated branch by branch using a root finding algorithm with an initial guess taken from the analytically derived limit of individual branch.

In conclusion, the model parameters are then the set \( P_\delta \) and the thickness \( e \) of the plate. We experienced that alternatively, the set \( P_\delta \) and the parameter \( e/V_T \) gave the most stable results with respect to change of the trial value for the model parameters. Reduced to isotropic material, the model parameters are \( e/V_T \), \( V_T \) and \( k = V_L/V_T \).

**VALIDATION ON STANDARD MATERIAL**

The general procedure consists in identifying trajectories \( \text{Traj}_1, \text{Traj}_2, \ldots \), on the experimental spectrum \((f, k)\) then in assigning a Lamb branch \( \text{A}0 \) or \( \text{S}0 \) or \( \text{A}1 \), \( \ldots \), at each experimental trajectory \( \text{Traj}_i \), then in fixing trial value to model parameters, and optimizing the model parameters using the least square algorithm. Note that trial values are randomized values taken in a large interval around the true values. Care was taken about checking the stability with respect to the change of trial values. The general procedure follows work reported in several studies in the domain of non destructive testing (see for example [13]). The following results are given for 3 repeated measurements with repositioning of the probe. As shown, this lead to a small variability, given the regularity of the geometry.

The procedure of identification of model parameters was validated on plates of isotropic metals, namely copper and aluminium, with samples about 2 mm thick. Results are presented on Table 1 in terms of thickness which can be easily measured with a caliper. There was no systematic independent measurements of bulk wave velocities. Velocities determined by the procedure was \( V_L = 4868 \text{ m/s} \) \( V_T = 2352 \text{ m/s} \) for copper, \( V_L = 6435 \text{ m/s} \) \( V_T = 3125 \text{ m/s} \) for aluminium. Table 1 shows a remarkable agreement for the plate thickness.

**TABLE 1.** Thickness \( (e) \) determined by the procedure of identification of model parameters. \( e_{ref} \) indicates thickness measured with a caliper, repeated thickness measurements in the area under the probe gave a range of values indicated in the table.

<table>
<thead>
<tr>
<th>Material</th>
<th>( e_{ref} ) (mm)</th>
<th>( e ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper plate</td>
<td>2.00 - 2.02</td>
<td>2.07</td>
</tr>
<tr>
<td>Aluminum Plate</td>
<td>1.99 - 2.01</td>
<td>2.02</td>
</tr>
<tr>
<td>PMMA Plate</td>
<td>1.92 - 1.98</td>
<td>1.97 - 1.98</td>
</tr>
<tr>
<td></td>
<td>1.29 - 1.31</td>
<td>1.27 - 1.28</td>
</tr>
<tr>
<td></td>
<td>2.28 - 2.32</td>
<td>2.33 - 2.37</td>
</tr>
<tr>
<td>Glassfibers/epoxy plates</td>
<td>3.33 - 3.37</td>
<td>3.27 - 3.29</td>
</tr>
<tr>
<td></td>
<td>4.13 - 4.17</td>
<td>4.00 - 4.15</td>
</tr>
</tbody>
</table>

Next, the procedure was validated using isotropic absorbing PMMA with a plate of about 2 mm. The SVD-based method upgraded for attenuated waves was applied on experimental data. This material presents a lower impedance mismatch with the silicone which covers the probe compared to the metallic material. It was experimentally observed an interaction between the probe and the guided waves in the PMMA plate. The spectrum was close to the spectrum of a bilayer fluid/solid. Then, experimental data close to Lamb branches were semi-manually selected by the operator. Results of the procedure of identification of model parameters are shown on Table 1. In that case, the procedure determined \( V_L = 2717 \text{ m/s} \) \( V_T = 1385 \text{ m/s} \).

Finally, the procedure was validated on transverse isotropic composite material made of glass fibers embedded in epoxy (Sawbones®). Different plates of different thicknesses were tested. For thicker plates, it was not possible to postulate unambiguously the Lamb counterpart of some experimental trajectories. Indeed, due to change of
excitability and attenuation of a given Lamb branch with frequency. Lamb branch appear experimentally as incomplete trajectories. It was developed a preliminary step aiming at associating experimental trajectories to Lamb branch. This step consists in using a proxy for the material properties and solving a preliminary inversion problem in finding the same thickness which allow to minimize the shift between trajectories and trial branches. At that step, experimental trajectories were reduced to a polynomial fit. In addition, as for PMMA, the impedance of the material was close enough to that of the front cover of the probe to require a pre-selection of experimental data. Tested on different thicknesses, the procedure reconstructs the thickness measured with the caliper within less than 1%. The elastic properties determined by the procedure are represented in the slowness plane on Fig. 2 which shows a small variability from one sample to another. The values obtained for VL1, VL3 and VT are consistent with RUS measurements reported in

FIGURE 2. Slowness curves describing the elastic properties of the composite material constituted of glass fibers embedded in epoxy. Data obtained from the procedure of identification of model parameters.

In conclusion, there was no case of failure. The whole set of tests suggest that the determination of thickness is rather independent of the elastic properties. It was experienced a relative difficulty in relating an experimental branch to a well defined Lamb branch. In the context of the standard samples, this point was solved.

IDENTIFICATION OF MODEL PARAMETERS ON BONE SAMPLES

Identification on standard samples served as training step for identification of model parameters on bones. Experimental data on in vitro samples (here human radius) shows a pronounced greater variability assumed to be related partially to imperfect contact. There are definitely more degrees of freedom let to the relative position of the sample and the probe than for standard samples.

Failure cases were observed and preliminary steps were developed. Both models of material, namely isotropic and anisotropic material, were tested. The isotropic case was expected to be erroneous but to help the anisotropic case. Moreover, the procedure was tested using both the raw data constituting each trajectory and a polynomial fitting of the raw data, to decrease the variability.

Table 2 shows up the resume of the preliminary steps. As for standard samples, we do not have independent measurements of elastic parameters of that sample. However, the HR-pQCT measured thickness of that sample was $\text{CT.Th.ref} = 1.50 \pm 0.20$ mm. As shown by table 2, the procedure based on fitted experimental data allowed a proxy for that thickness within less than 10%.
<table>
<thead>
<tr>
<th>Experimental data</th>
<th>Model</th>
<th>Ct.Th (mm)</th>
<th>$V_T$ (m.s$^{-1}$)</th>
<th>$V_{L1}$ (m.s$^{-1}$)</th>
<th>$V_{L3}$ (m.s$^{-1}$)</th>
<th>$V_{2D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw data isotropic</td>
<td></td>
<td>2.06</td>
<td>2350</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raw data anisotropi</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polynomial fit isotropic</td>
<td></td>
<td>2.07</td>
<td>2360</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polynomial fit anisotropic</td>
<td></td>
<td>1.39</td>
<td>1610</td>
<td>2750</td>
<td>3760</td>
<td>0.31</td>
</tr>
</tbody>
</table>

**CONCLUSION**

Using a least square algorithm, a tool to characterize bulk wave velocities and thickness using a transverse isotropic plate model was developed and applied on standard samples. The experimental data were obtained with a set-up dedicated to clinical measurements on which was applied a SVD based procedure upgraded for attenuated guided waves. The procedure was considered efficient on standard samples and provide a thickness close to measured values. The characterization of bone require preliminary steps but encouraging results were obtained, in which ultrasound based thickness was closed, within 10%, to HR-pQCT determined thickness. Upgrading the signal processing method and improving the detection of trajectories might help for a more robust characterization.

**REFERENCES**