2aED2. Experience teaching acoustics at the senior-undergraduate and first-year graduate levels

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Perhaps without appreciating it, college students are more fully equipped to understand and study acoustics than any other field of science. This assertion stems from the fact that most college students have two exquisite broadband receivers with impressive dynamic range (ears), and a matched multi-functional sound projector (voice). Given that nearly all college students have used their ears and voice for many years before arriving in an acoustics classroom, the advanced-acoustics instructor's task is primarily to link theoretical results with the acoustic intuition that students already possess. Thus, a worthy pedagogical goal is to activate this submerged knowledge and connect it to mathematical results through practical examples, classroom demonstrations, and relevant homework. At the senior-level, useful demonstrations include: acoustic resonances of a cardboard tube, the dipole characteristics of small raw loudspeaker, directional reflection with a metal salad bowl, and sound volume changes as a loud speaker is lifted out of a cabinet. At the graduate level, useful homework assignments include boundary-element and finite-element calculations with commercial software that can be checked with established theory. In addition, worthwhile homework problems that attempt to provide sufficient reward for students who master the mathematical content have been developed for both classes.
Introduction

Acoustics is a broad discipline that impacts many fields: engineering, physics, music, architecture, psychology, and medicine, to name a few. Thus, it has no obvious home department within the usual academic divisions of research and teaching topics. Perhaps more importantly, it is part of everyday human experience and has been studied for centuries. Therefore, acoustics instruction at the advanced undergraduate and graduate levels presents a variety of challenges and opportunities.

The challenges result from the inherent diversity of acoustics and from differences between students and instructors. First, even in a mechanical engineering classroom, stated student interests include music, general noise control, speech and hearing, sound quality, sound-structure interaction, transduction, architectural acoustics, and automotive noise, vibration, and harshness (NVH). The challenge here is to direct a confined portion of the instructional effort in these varied directions while also indicating the technical depth of each topic. My finding is that delivering topical breadth via quick handbook and how-to approaches, although effective in specific circumstances, seldom encourages the students to develop the intellectual depth that marks true understanding. Instead, placing lecture emphasis on the underlying physical principles with applications appearing in examples and homework problems has proven to be a satisfactory means for keeping students’ attention while satisfying their desire for diverse acoustics training.

A second challenge is the difference between students and instructors in their willingness to use analytical mathematics. Many of the classical mathematical techniques developed for solving partial differential equations were developed for solving the wave equation. Instructors commonly embrace these techniques because they form the foundation of the instructor’s own training that likely occurred (many) years ago. Modern college students educated in the computer age are comparatively reluctant to use analytical techniques and are likely to be unimpressed by the elegance of mathematical results. However, this reluctance and skepticism can be overcome if students perceive that analytical techniques are worthwhile for generating understanding and producing answers, and that elegant results are often useful too. Asking students to plot and interpret results obtained analytically is a possible means to show students the value of such results. In addition, it is possible to maintain students’ willingness to pursue and use analytical techniques by connecting results from analytical formulae to readily observed acoustic phenomena, and to acoustic design features of products and devices.

Fortunately, the opportunities available in acoustics instruction can overwhelm the challenges. First, the near universality of human experience with acoustics means that it will never go out of style, and the mechanical engineering students who fill my classes can be motivated by this fact; a good noise control engineer can easily find work. Second, nearly all college students already have a wealth of acoustic experience developed from using their ears and voice. For example, they all know that sound can diffract around objects that block lines of sight, that outdoor sounds typically become quieter with increasing distance from their source, that common plate glass provides a satisfactory acoustic barrier in many circumstances, that larger speaker elements (woofers) are needed to produce lower frequency sounds, and that hard surfaces reflect sound while 10 cm of freshly fallen snow absorbs it; this list can go on and on. Successful acoustics instruction activates this prior knowledge so that initially reluctant students are motivated to persist through the potentially difficult portions of an acoustics course at the senior-undergraduate or graduate student level. Difficulties most commonly arise because the quantitative aspects of acoustics are new to students. Thus, successful acoustic instruction links the acoustic experience that students already possess with quantitative (analytical) results derived from the wave equation. When formed, this link motivates students and enhances learning.

The Mechanical Engineering Department at the University of Michigan offers two one-semester courses in acoustics. The intended instructional outcome for the students in both classes is to have the ability to apply the fundamentals of acoustics to new situations. Thus, exam questions emphasize basic concepts applied to new situations. The senior level course, ME 424 Engineering Acoustics, uses the textbook by Kinsler et al.¹, and largely avoids Fourier transforms and vector calculus. The graduate level course, ME 524 Advanced Engineering Acoustics, uses the textbook by Pierce², and Fourier transforms and vector calculus are prominent. As a side benefit from these textbook choices, students who take both courses learn first hand about the $e^{j\omega t}$ and $e^{-j\omega t}$ conventions for complex sinusoids. In both courses textbook and instructor-generated homework problems are used in approximately equal amounts. Both courses are electives, and both commonly endure a 10 to 20% enrollment decline during the first week of each semester. While this enrollment decline is lamentable, an effective means to prevent it has not been found.
The remainder of this paper discusses specific instructional details of the two courses, and a half-dozen example problems.

**Senior-Level Acoustics Instruction**

This one semester course is taught annually with the goal of providing broad exposure to the quantitative aspects of acoustics in engineering. The course proceeds from a quick review of the simple harmonic oscillator to one-dimensional waves and then to three-dimensional waves. The primary topics in the course are: 

1. Derivation and fundamental solutions of the acoustic wave equation, 
2. Reflection and transmission at flat interfaces, 
3. Waves in pipes and pipe systems, 
4. Acoustic radiation from simple and extended sources, and 
5. Cavities and waveguides. A variety of other student- and instructor-chosen topics are covered with less depth throughout the term.

The course is punctuated throughout the semester, with demonstrations such as the following.

(i) *Dipole radiation from simple loudspeaker.* Start with a small moving-diaphragm loud speaker, an ordinary function generator, and a long BNC cable with clips. Drive the little speaker with the function generator at a sound frequency that is low enough to make the speaker acoustically compact, but high enough so it can still be heard. When the sound frequency is correct, a 10 dB or so difference in sound level may be heard when the speaker is viewed edge-on compared to when it is viewed from the front or back. Allow the students to pass the small operating speaker around the class so that everyone can investigate dipole radiation with their own hands and ears. Experience has found that students readily assist with the cable management issues that arise as the little speaker travels to all the occupied seats.

(ii) *Resonances of a cardboard tube.* Start with the tube from the center of a roll of paper towels, and a small moving-diaphragm loud speaker hooked to an ordinary function generator. For a tube that is 20.5 cm long with a 3.8 cm diameter, its first open-open resonant frequency (with end corrections) is about 750 Hz. Its first open-closed resonance (with end corrections) is about 390 Hz. Place the little speaker on a flat surface and set it to produce a tone at the first open-open resonant frequency of the tube. Moving an open end of the tube to within a centimeter or two of the speaker causes the radiated sound level to jump up by 10 to 20 dB, depending on the location of the observer. Reset the driving frequency to the first open-closed resonant frequency. Moving an open end of the tube to within a centimeter or two of the speaker causes little change in radiated sound level. However, if the other tube opening is blocked (with the palm of your spare hand) and the open end of the tube remains near the speaker, the sound level should jump by 10 to 15 dB. This demonstrates the surprising fact that closing an opening may cause an acoustic system to radiate more sound.

(iii) *Creating an acoustic beam with a salad bowl.* Start with a large metal salad bowl and a small moving-diaphragm loud speaker hooked to an ordinary function generator. Set the speaker to operate at the highest audible frequency that is not too annoying; 3 kHz is suggested. The magnet on the back of the speaker should allow it to stick to the inside of the salad bowl. With the bowl acting as an acoustic reflector, the speaker-bowl combination can make an acoustic beam with a 5 to 10 dB peak that can be swept over the student audience.

(iv) *Demonstrate the effect of a speaker cabinet.* Start with a loudspeaker mounted in a cabinet, an audio amplifier, an ordinary function generator, and appropriate cabling. Hook up the components and set the sound frequency to a low frequency cabinet resonance. A ported cabinet that works like a Helmholtz resonator is suggested. Set the cabinet so that the speaker faces upward and alternately lift the speaker in and out of the cabinet. The radiated sound level should fall (rise) by 10 or more dB when the speaker is lifted out of (placed into) the cabinet.

And, to further illustrate the character of the course, the following exam and homework problems are provided.

1. Although actual vibration isolation problems may be more complicated, we can use acoustic barrier formulae to assess potential vibration isolation designs for two slabs of concrete separated by gap of width $L$ (see Fig. 1). For the following transmission loss ($TL$) calculations, assume normal-incidence plane waves at a frequency of 1.5 kHz. Estimate the $TL$ between slabs when: $L = 5.0$ cm and the gap is filled with steel, and when $L = 0.5$ cm and the gap is filled with air. Which design do you recommend for vibration isolation in building foundations?
2. This problem illustrates that the unexpected may occur in acoustic systems when a sound-control element is added to block a propagating sound wave. In particular it involves a constant-area duct, an ideal sound source, and a sound control barrier intended to block transmission of sound along the duct (see Fig. 2).

Consider a hard ideal piston located at \( x = 0 \) that oscillates with speed amplitude \( U_o e^{\omega t} \) at the end of a tube with cross-sectional area \( S \) that contains air having speed of sound \( c \) and density \( \rho \). At a distance \( L \) from the piston, a thin sound barrier having a mass per unit area of \( \rho_s \) spans the inside cross-section of the tube. The complex acoustic pressure and velocity fields inside the tube are \( p(x,t) \) and \( u_x(x,t) \), respectively. The mass of the barrier causes a pressure difference to occur at \( x = L \): \( p(L,t) - p(L^+,t) = \rho_s \frac{dU(L,t)}{dt} \). Consider only plane waves in this problem.

\[ C = \rho c U_o \left[ 1 - \left( \frac{\omega \rho_s}{\rho c} \right) e^{j\kappa L} \sin(kL) \right]^{-1} \]

3. Linear transducer arrays are commonly used in directional microphones, biomedical ultrasound, and underwater acoustics. Here we will study how sound radiation from a continuous harmonic line source of length \( L \) and radius \( a \) can be altered when the radial surface velocity is not uniform along the length of the line source.

Assume \( ka << 1 \) throughout this problem but do not assume anything about the size of \( kL \), and only worry about radiated sound characteristics in the \( x-z \) plane.
FIGURE 3. Schematic drawing of a line-source lying along the x-axis that radiates sound into the x-z plane.

a) If the radial surface velocity on the surface of the line source is described by a weighting function $\Gamma(x)$ so that:

$$u_{surf}(x) = U_s e^{j\omega t} \Gamma(x),$$

show that the radiated sound in the Fraunhofer far-field is:

$$p(r, \theta, t) = \frac{j}{2r} \rho_c c U_s \exp\{j(\omega t - kr)\} \int_{-\frac{L}{2}}^{\frac{L}{2}} \Gamma(x) \exp\{j k x \sin \theta\} dx.$$

b) The main radiation beam may be steered away from the broadside direction, $\theta = 0$, when the weighting function provides a spatially dependent phasing along the line source: $\Gamma(x) = \exp\{j y x\}$. For this case, use the results of part a) to show:

$$p(r, \theta, t) = \frac{ja}{2r} \rho_c c U_s k L \exp\{j(\omega t - kr)\} \sin\left[\frac{L(\gamma + k \sin \theta)}{2}\right] \frac{L(\gamma + k \sin \theta)}{L(\gamma + k \sin \theta)}.$$

c) What is the main beam angle, $\theta_{max}$, where the radiated sound amplitude is maximum?

d) When $\gamma = 0$ what is $\theta_{max}$? In terms of the product $kL$, find the full half-angle of the main beam $\theta = \frac{\theta_{max} - \theta_1}{2}$, where $\theta_1$ is the nodal direction closest to the main beam. Evaluate $\theta_{1/2}$ for $kL = 24$ and 48.

e) Repeat part d) for $\gamma = -k$ (commonly called endfire).

f) Are longer or shorter line sources better at radiating sound in narrow beams? For line sources that are more than a wavelength long ($kL > 2\pi$), there are nodal directions in the radiated sound in the Fraunhofer far field. A different amplitude distribution of the line source’s surface velocity can eliminate these nodal directions.

g) Consider $\Gamma(x) = \exp\{-x^2/l^2\}$; plot this function for $0 \leq x/l \leq 3$.

h) Using $\Gamma(x) = \exp\{-x^2/l^2\}$ show that the far-field acoustic pressure produced by the continuous line source is given by:

$$p(r, \theta, t) = \left(\frac{ja}{2r}\right) \rho_c c U_s k \sqrt{\frac{l}{\pi}} \exp\{j(\omega t - kr)\} \exp\{-k^2 \frac{\beta^2}{4}\} \sin^2 \theta$$

where $\beta$ is a real constant.

i) How many beams are produced in this case?

j) From (1) of part h), sketch polar-plot beam patterns for $kl = 0.5, 1.0, 2.0,$ and $5.0$ with $l$ fixed.

k) When $\Gamma(x) = 1$, the far-field radiation from a line source with $L = k \sqrt{\pi}$ is:

$$p(r, \theta, t) = \frac{ja}{2r} \rho_c c U_s k \sqrt{\frac{l}{\pi}} \exp\{j(\omega t - kr)\} \frac{\sin\left[(kl \sqrt{\pi}/2) \sin \theta\right]}{(kl \sqrt{\pi}/2) \sin \theta}.$$  

The axial pressure and complex phase are the same for (1) and (2). Plot and compare the angular distribution of acoustic radiation verses $kl \sin \theta$ from (1) and (2). Which line source is longer? Which main beam is narrower?

l) If acoustic radiation in the x-z plane from a continuous line source must be directed in a narrow beam along $\theta = \theta_1$ without any side lobes, what form of $\Gamma(x)$ do you suggest?

**Graduate-Level Acoustics Instruction**

This one semester course is taught approximately once every two years with the goal of equipping students with the knowledge necessary for pursuing research in acoustics. The nominal coverage includes the topics from the
senior-level course, but they are explored with greater mathematical sophistication and depth. For example, the impacts of viscosity, heat conduction, and inhomogeneity in the acoustic medium on the acoustic wave equation and boundary conditions are considered. The graduate-level course also includes a few topics not mentioned in the senior-level course. The Helmholtz-Kirchhoff integral is derived and used to explain the boundary element formulation of computational acoustics, and temporal Fourier transforms are used to establish the duality of the time and frequency domains. In addition, based on instructor preference, scattering, diffraction, ray theory, computational acoustics, fluid-structure interaction, and approximate techniques for interior acoustics may also be covered in the graduate level course.

When present, the computational acoustics component of the course involves calculations with a commercial software package. Step-by-step instructions are provided to the students who learn how to call-up the software, load computational grids, plot computed output, and save results. The software package is used for several homework problems intended to illustrate the strengths and limitations of boundary-element and finite-element methods. The first problem asks students to compare numerical and analytical sound radiation results for a pulsating sphere at frequencies from 4 Hz to 500 Hz. The results match well at low frequencies where there are many computational elements per wavelength, but the numerical results lose accuracy at high frequencies when the necessary grid resolution is not maintained. The remaining problems allow students to investigate how well analytical and numerical mode shapes and frequencies match for harmonic fields in a rectangular enclosure, and how a sound source in such an enclosure may or may not excite particular interior acoustic modes based on its frequency and location.

And, as above, three analytical exam and homework problems are provided to further illustrate the character of the course.

4. A clever acoustician and entomologist decides to estimate wild cricket populations from multiple sound pressure level (SPL) measurements made at different heights above a large grassy field in the evening when the crickets are chirping. Laboratory measurements in an anechoic chamber show that the time-average SPL at a distance of 1 m from a chirping cricket is $SPL_o$.

a) For an ideal flat circular field with radius $R$ (see Fig. 4) having $\beta$ crickets per square meter, show that the sound pressure level measured at height $H$ above the center of the field is approximately:

$$SPL(H) = SPL_o + 10 \log_{10} \left\{ \pi \beta \ln \left[ 1 + \left( \frac{R}{H} \right)^2 \right] \right\}.$$  

[Hints: i) treat the crickets as incoherent sources, ii) assume that the sound waves from each cricket undergo hemispherical spreading, and iii) approximate the sum over crickets by an appropriate area integral.]

b) In real field measurements, the distance $R$ may be ill-defined or hard to measure, but it is very likely to be much larger than $H$. If SPL measurements are made at two different heights, $H_1$ and $H_2$, use the result of part a) to show:

$$\beta \approx \frac{10^{SPL(H_2)/10} - 10^{SPL(H_1)/10}}{10^{SPL_o/10} \pi \ln \left( \frac{H_2^2}{H_1^2} \right)}$$  

for $H/R << 1$, where $\beta$ has units of number/m$^2$.

c) Estimate $\beta$ when $H_1 = 2.0$ m, $H_2 = 3.0$ m $SPL_o = 47$ dB re 20 $\mu$Pa, $SPL(H_1) = 55$ dB re 20 $\mu$Pa, and $SPL(H_2) = 51$ dB re 20 $\mu$Pa.

d) State in words how your approach to part a) would change if the chirping crickets were in-phase coherent sound sources.

5. A large plate lying a $z = 0$ is struck from the rear by a small high-speed projectile at the origin of the x-y plane (see Fig. 5). The impact causes a near instantaneous surface deflection. The plate response is perfectly plastic and produces a surface velocity in the $z$-direction $v_z$ given by:
\[ v_s(x_s, y_s, t) = \zeta \delta(t) \exp \left\{ -\frac{x_s^2 + y_s^2}{a^2} \right\}, \]

where \( \zeta \) is the peak plate deflection at the point of projectile impact.

**FIGURE 5.** Schematic of the impact source geometry for problem 5.

a) Compute \( \tilde{v}_s(x_s, y_s, \omega) \) the Fourier transform of \( v_s(x_s, y_s, t) \).

b) Use the Rayleigh integral to determine the complex pressure field \( \tilde{p}_s(r, \theta, \omega) \) in the Fraunhofer far-field, with \( r^2 = x^2 + y^2 + z^2 \) and \( \sin^2 \theta = (x^2 + y^2)/r^2 \). The following integral relationship may be useful:

\[
\int_{-\infty}^{\infty} \exp \left\{ -\frac{x^2}{a^2} - ik \frac{xx}{r} \right\} dx = a\sqrt{\pi} \exp \left\{ -\frac{k^2 x^2 a^2}{4r^2} \right\}.
\]

c) Compute the inverse Fourier Transform of \( \tilde{p}_s(r, \theta, \omega) \), to find the time-dependent far-field radiated acoustic pressure:

\[
p(r, \theta, t) = \frac{\zeta \rho \omega c^2 (r - ct)}{a^2 \sqrt{\pi} r \sin^2 \theta} \exp \left\{ -\frac{(r - ct)^2}{a^2 \sin^2 \theta} \right\}.
\]

d) At what angle is the radiated pressure pulse the shortest? Why?

6. A point source located at \( \vec{x}' = (0, 0, -z_0) \) produces sound at radian frequency \( \omega \) in an unbounded uniform environment of fluid having density \( \rho \) and sound speed \( c \). An acoustic receiver is placed at \( \vec{x}_r = (0, 0, +z_0) \). The complex acoustic pressure field from the sound source is given by: \( \hat{\rho}(\vec{x}) = \left( \hat{A} / |\vec{x} - \vec{x}'| \right) \exp \{ i k |\vec{x} - \vec{x}'| \} \) where \( k = \omega/c \).

a) What is \( \hat{\rho}(\vec{x}_r) = \hat{\rho}(0, 0, z_0) \) in terms of \( A \), \( k \), and \( z_0 \).

b) Now enclose the receiver within a closed Helmholtz-Kirchhoff surface composed of the \( x-y \) plane and a hemisphere centered on the origin having a radius \( R \) and lying in the right \( (z > 0) \) half-space as shown in Fig. 6. Using this surface, \( r = \sqrt{x^2 + y^2} \), and the free-space Green’s function \( \hat{G}(\vec{x}_2, 1 \vec{x}_1) = 1/|\vec{x}_2 - \vec{x}_1| \exp \{ i k |\vec{x}_2 - \vec{x}_1| \} \), show that:

\[
\hat{\rho}(0, 0, z_0) = -z_0 A \lim_{R \to \infty} \int_{r=0}^{R} \frac{2ik \sqrt{r^2 + z_0^2}}{(r^2 + z_0^2)^{3/2}} \left( -\frac{1}{\sqrt{r^2 + z_0^2}} + ik \right) r dr.
\]
FIGURE 6. Helmholtz-Kirchhoff integral surface and source-receiver geometry for problem 6. The final derived ratio $R/z_o$ suggests that acoustic rays are impressively broad.

c) Evaluate the integral, and determine the value of the ratio $R/z_o$ necessary to ensure that $\hat{p}(0,0,z_o)$ is within 1% of the correct value determined in part a). What – if anything – does this value of $R/z_o$ imply about the effective cross sectional dimensions of the acoustic ray linking the source and receiver?

REFERENCES