ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013

Physical Acoustics
Session 1aPAa: Acoustics in Microfluidics and for Particle Separation I: Standing Waves, Streaming, and Radiation Forces

1aPAa6. Direct numerical simulations of acoustic streaming in standing wave tubes using the Lattice Boltzmann Method

Yasser Rafat*, Kaveh Habibi and Luc Mongeau

*Corresponding author’s address: Mechanical Engineering, McGill University, 845 Sherbrooke Street West, Montreal, H3A 0G4, Quebec, Canada, yasser.rafat@mail.mcgill.ca

One important factor in the efficiency of thermoacoustic engines is acoustic streaming, which causes convective heat transfer between high and low temperature reservoirs. Most experimental and numerical studies performed so far have focussed on Rayleigh streaming. Less work has been done on acoustic streaming due to stack. Most numerical studies of Rayleigh streaming were performed using Navier-Stokes based numerical methods. In this study, direct numerical simulations were performed using schemes based on the lattice Boltzmann method (LBM). Acoustic streaming in a simplified thermoacoustic refrigerator was modelled as a rectangular standing wave resonator with a flat plate. Low-amplitude results obtained for Rayleigh streaming velocity magnitudes were compared with linear acoustic theory for verification. High amplitude recirculated streaming flow structures around the edges of the flat plate spoiler were identified. These are likely to contribute to heat transfer much more than Rayleigh streaming. The results confirm that vertical edge streaming flows play a significant role in thermoacoustic heat transport.

Published by the Acoustical Society of America through the American Institute of Physics
INTRODUCTION

Acoustic streaming is described as a second order steady flow induced by a first order oscillatory flow in acoustic standing waves. In thermoacoustic systems, acoustic streaming is assumed to affect the heat transfer inside the thermoacoustic core. Several experimental studies [1-3] were performed on Raleigh streaming inside a standing wave resonator. Lord Rayleigh[4] provided an analytical explanation of the acoustic streaming phenomenon observed in Kondt’s tube. Some analytical models were developed for streaming in standing wave thermoacoustic devices [5, 6]. These analytical models were for slow streaming in simple geometries and idealized conditions. This limitation can be overcome by numerical methods. Direct numerical simulation (DNS) of fully coupled Navier-Stokes (N-S) equation is the most accurate approach. Early numerical simulations for acoustic streaming were performed by Kawahashi et al.[7] and Yano[8]. Recently there have been many numerical studies on the topic by Aktas et al.[9, 10]. Due to high computational cost DNS is often not suited to real thermoacoustic systems.

The Lattice Boltzmann Method (LBM) is a numerical method which is simple to implement and computationally less expensive than Navier-Stokes based DNS for low Mach numbers. The incorporation of complex geometries inside the computational domain is straight forward in LBM. The LBM code can be easily parallelized, and straight forward to implement [11]. A study of acoustic streaming produced by a standing wave between two parallel plates was reported by Stansell et al.[12]. They used lattice gas approach (considered to be a predecessor of LBM) to simulate full N-S equation and streaming appeared as a small correction to the oscillatory flow field. However, lattice gas simulations (LGA) are inherently noisy, and streaming could only be seen for high amplitudes and viscosities. LBM was developed to overcome noise problems. Like LGA, LBM also simulates the full N-S equation. There are very few studies available on the study of acoustic streaming using LBM. The first study was reported by Haydock et al. They studied acoustic streaming in both standing [13] and travelling [14] waves. They also simulated acoustic streaming around various obstacles. They concluded that the acoustic drive ratio \( P_v/P_m \) determines that after how many acoustic cycles streaming would be established. Habibi et al. [15] simulated standing wave acoustic flows over a flat plate inside a resonator. They compared the instantaneous acoustic velocity profiles and the vorticity results from LBM with experimental data.

Almost all the DNS studies reported were performed for acoustic streaming inside an empty resonator. In an actual thermoacoustic engine there would be stacks/regenerators with pores of different shapes and heat exchangers with complex geometries. In the present study, as a first step, acoustic standing wave inside a resonator was numerically simulated. Validations were performed with results obtained from linear acoustic theory for both first order acoustic velocities and the Raleigh streaming velocity. Secondly, a flat plate was introduced in the computational domain and the resulting streaming environment was determined.

NUMERICAL SETUP

In order to study acoustic streaming in a rectangular channel (Figure 1), a two dimensional planar flow was considered within the X-Y plane. The channel was excited at resonance at an acoustic frequency, \( f_{ac} \) of 824.25 Hz. The channel length, which corresponds to one-half wavelength, was 0.21 m. The width, H of channel was 0.01 m. The initial pressure and temperature inside the channel was 1 atm and 298 K respectively. The fluid domain was considered isothermal. In order to resolve flow in the acoustic boundary layer (\( \delta_v=2.16e-4 \) m) yet taking into consideration the computation cost, variable grid (voxel) resolution was used. Three levels of grid resolution were used as shown in Figure 3. Coarser grids were used away from the rigid walls. The size of each voxel, \( L_v \) in a coarser level is four times the size of each voxel in the previous finer level. In the acoustic boundary layer, 12 grid points were used with a resolution corresponding to level 1. The non-dimensional size, \( L_v/\delta_v \) of the finest voxel (spatial resolution) which were located in the boundary layer was 0.083. The non-dimensional simulation timestep (temporal resolution: \( ts/T \)) was 2.48e-5, where T corresponds to time period of one acoustic cycle. A no-slip boundary condition was imposed on the walls of the resonator and the flat plate. A sinusoidal time varying pressure boundary condition was imposed at the inlet, \( x=0 \) as shown in Figure 1. The amplitude of the dynamic pressure was imposed and the velocity of the flow was allowed to float to any value as the fluid responded. A pressure antinode occurred at the two ends of the channel, while a pressure node occurred in the center of the channel. The acoustic particle velocity distribution is similar to that of the pressure with the location of nodes and antinodes interchanged. In order to observe the transient nature of the simulation results, virtual probes were located at the node and antinode locations of the dynamic pressure and acoustic velocity. Four probes were also located on either side of the flat plate edges. The distance between each probe was equal to the maximum acoustic displacement, \( x_{ac} \).
The fluid data were acquired at a sampling rate of 1000 data points per acoustic cycle. Each simulation was evolved over a time period equivalent to 300 acoustic cycles. This was found to be sufficient for the flow to reach a steady state within two orders of the acoustic velocity magnitude. This was necessary because the Rayleigh streaming velocity is two orders of magnitude smaller than the peak acoustic velocity. The total non-dimensional simulation time, Ts/T was 300. The streaming velocity was calculated as the time average of acoustic velocity over 30 acoustic cycles. The simulation domain is symmetric about the horizontal axis; therefore, simulation was carried out only for half the domain. The total number of voxels for the cases without flat plate was 0.6 million and with the introduction of flat plate the total number of voxels was 1.1 million. Each simulation was evolved on 64 cores for 40 hours.

**VERIFICATION**

In order to verify the validity of the numerical results obtained from LBM, comparisons were made with analytical results available from linear acoustic theory. The axial dimensions were non-dimensionalized by one quarter wavelength, and the transverse dimensions were non-dimensionalized by the channel height. One quarter wavelength was chosen as the characteristic axial dimension due to the fact that a Raleigh streaming cell is one quarter wavelength long. The numerical results presented are for the cases tabulated in TABLE 1. The analytical results are from a linear acoustic model developed by Nyborg [16]. The analytical linear acoustic model considers no slip at the wall.

**Acoustic velocity**

Comparisons between analytical and numerical results for case 2 (TABLE 1) are shown in axial and transverse directions in Figure 4. The axial component of the instantaneous acoustic velocity is compared. The axial direction acoustic velocity is plotted at two time instants in Figure 4.a. Very good agreement between analytical and numerical results was obtained. Comparison for the two time instants corresponding to $\theta = \pi/2$ and $\theta = \pi/3$ is shown in Figure 4.a.

Along the transverse direction, Figure 4.b, the axial component of the acoustic velocity is shown over the top half of the channel. Three time instants corresponding to $\theta = \pi/2$, $\theta = \pi/3$ and $\theta = \pi/180$ were selected for comparison. As for the axial direction, the numerical results are in excellent agreement with the analytical results. The velocity overshoot, most pronounced for the time instant corresponding to $\theta = \pi/180$, is well captured in the simulation. The numerical results appear to be accurate in the acoustic boundary layer and in regions away from the wall.

**Streaming velocity**

Rayleigh streaming results using LBM were compared with the analytical results. The axial (Figure 5.a) and transverse (figure 5.b) variation of Rayleigh streaming velocity magnitude is shown in Figure 5. The 1D velocity profiles are extracted from the 2D numerical simulation results along the centerline of the channel corresponding to $Y=0$ in the axial direction. The velocity profile in the transverse direction is at the location of streaming velocity maxima, which corresponds to $X=\lambda/8$. The numerical results presented here correspond to cases 1, 2 and 3 of TABLE 1. The numerical results for Rayleigh streaming variation in axial direction are in satisfactory agreement with the analytical results. The axial variation of the streaming velocities corresponding to Mach number, $M=0.014$ and $M=0.031$ are in very good agreement with analytical results. As $M$ is increased, the deviation from linear acoustic theory increases. This can be attributed to the non-linearity occurring in the flow which is not taken into account by the streaming results obtained from linear acoustic theory. The drive ratio corresponding to case 3 is approximately 5.9%. The acoustic flow tends to become nonlinear near this value of pressure ratio. Another deviation of the numerical results from the analytical results with increasing $M\theta$ is the axial dimension of a complete Rayleigh streaming cell. For Mach number corresponding to $M=0.043$ the axial dimension of the streaming cell is smaller than the analytically predicted value of $\lambda/4$.

A comparison between numerical and analytical results for transverse variation of Raleigh streaming velocity magnitude shows very good agreement at $M=0.014$ and $M=0.031$. Both inner and outer streaming cells are well resolved by the numerical scheme using LBM. As with the axial variation, the numerical results tend to deviate from the analytical results with increasing amplitude. The transverse dimension of the inner streaming cell tends to decrease compared to predicted dimensions for higher Mach numbers. The analytical results slightly under predict the streaming velocity magnitudes in the inner streaming cell while they over predict in outer streaming cell when compared with the numerical results obtained at high drive ratios as in case 3.
The verification results for acoustic and streaming velocities suggest that LBM can be utilised as a numerical method for study of acoustic streaming in a standing wave resonator.

STREAMING FROM FLAT PLATE

In the presence of a flat plate inside the standing wave resonator, two types of acoustic streaming were observed. The first type of streaming was Rayleigh streaming. The key features observed for Rayleigh streaming were that the axial dimension of a complete streaming cell was \( \lambda/4 \). Both inner and outer streaming cells were observed. The maximum streaming velocity amplitude for each case was two orders of magnitude smaller than the maximum acoustic velocity amplitude, as shown in TABLE 1.

The second type of streaming observed was near the edges of flat plate spoiler. For convenience we now refer to it as edge streaming. In order to study this type of acoustic streaming we introduced a flat plate in the simulation domain corresponding to case 2 of TABLE 1. The length (\( L_p \)) of the flat plate was taken to be very small, 2\% of the acoustic wavelength and the plate thickness (\( t \)) was 10\% of the resonator width (\( H \)). As the blockage ratio (\( t/H \)) is \( \leq 10\% \) we can assume safely that the wall effects due to resonator can be neglected. The center of the plate was located at \( X=3 \lambda/8 \) and \( Y=0 \) according to the coordinate system shown in Figure 1.

The acoustic streaming due to the interaction between the standing wave and the flat plate is shown in Figure 6. For convenience the origin was shifted along the horizontal axis to coincide with the right side edge of the plate. In order to plot the axial and transverse variation of the edge streaming, velocity magnitude data was extracted along the lines depicted in Figure 2. The interaction between the flat plate and the acoustic flow in a standing wave resonator generated an acoustic streaming pattern which is distinct from the Rayleigh streaming produced due to acoustic standing wave interaction with the wall of a channel without any object in flow path. The most important feature of edge streaming is the maximum velocity magnitude. When comparing edge streaming amplitude with the Rayleigh streaming of Figure 5 for \( M=0.031 \), we could see that plate edge streaming amplitudes are two orders of magnitudes greater than maximum Rayleigh streaming velocity amplitude. The second distinct feature of edge streaming is its localised nature, which affects the streaming amplitude of the first streaming cell which is adjacent to the plate is smaller in size and larger in maximum velocity amplitude than the second streaming cell. The maximum streaming velocity of the second cell is approximately 25\% smaller than the streaming amplitude of the first streaming cell. But it is still an order of magnitude higher than the Rayleigh streaming velocity amplitude. Third distinct feature of plate streaming is that its amplitude is proportional to the magnitude of acoustic velocity at the edge. We could deduce from Figure 6 that the streaming velocity amplitude for both the streaming cells on the left hand side (L.H.S) edge of the plate is higher than the streaming amplitudes of the two cells on right hand side (R.H.S) edge. The magnitude of the acoustic velocity at the edge also supports the observations of the two streaming cells. The axial dimension of each streaming cell on the L.H.S of the flat plate was larger than its respective counterpart on the R.H.S of the plate. An important detail to consider here is the location of plate. The plate center was located at a distance of \( \lambda/8 \) away from the location of maximum acoustic velocity due to which either edges of the plate are not located at acoustic velocity antinode. For the simulation results presented in Figure 6, the acoustic velocity at R.H.S of the plate is 8.24 m/s, while the maximum acoustic velocity is 10.57 m/s. Thus, if either edge of the plate was located at maximum acoustic velocity it would experience higher streaming velocity magnitudes than the present case.

CONCLUSION

The edge streaming is an important type of acoustic streaming. It is distinct from the Rayleigh streaming. Due to its high velocity amplitude and localised nature, edge streaming seems to play a more significant role in heat transfer when compared with Rayleigh streaming. This phenomenon is thus important and has implications for the design of Thermoacoustic engines. In order to further understand edge streaming a parametric study will be performed as a next step. The flow and plate parameters will be varied and their effect on edge streaming would be studied.
TABLE 1. Parameters for the cases simulated in the study of Rayleigh streaming in an empty resonator.

<table>
<thead>
<tr>
<th>Case number</th>
<th>Parameters ↓</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Parameters ↓</td>
</tr>
<tr>
<td></td>
<td>Acoustic velocity amplitude: $u_{ac}$ (m/s)</td>
</tr>
<tr>
<td></td>
<td>Inlet pressure amplitude (kPa)</td>
</tr>
<tr>
<td></td>
<td>Drive ratio (%)</td>
</tr>
<tr>
<td></td>
<td>Mach number ($M$)</td>
</tr>
<tr>
<td></td>
<td>Reynold's number (Re)</td>
</tr>
<tr>
<td></td>
<td>Strouhal number ($St$)</td>
</tr>
<tr>
<td></td>
<td>$u_s$ (m/s)</td>
</tr>
<tr>
<td></td>
<td>$u_s/u_{ac}$ (%)</td>
</tr>
</tbody>
</table>

FIGURE 1. Schematic of the numerical setup.

FIGURE 2. Schematic of the lines along which velocity data is extracted in axial (X-axis) and transverse (Y-axis) directions near plate edge.
FIGURE 3. Schematic of the computational grid.

FIGURE 4. Theoretical and numerical axial component of instantaneous acoustic velocity. (a):- axial direction, (b):- transverse direction. (··· θ =π/2, ↔ θ =π/3, - · · · θ =π/180 and solid line=analytical)
FIGURE 5. Theoretical and numerical streaming velocity magnitude for different Mach numbers (M#) . (a):- axial direction, (b):- transverse direction. (··· M#=0.014, -- M#=0.031, -· M#=0.043 and solid line=analytical).

FIGURE 6. Axial variation of streaming velocity magnitude due to unsymmetric acoustic velocities at the two plate ends.
ACKNOWLEDGMENTS

The authors would like to thank Exa Corporation for providing academic licenses for PowerFLOW and for their continuing technical support. Computations were performed on Colosse and Mammouth supercomputing machines under the auspices of Calcul Québec and Compute Canada.

REFERENCES