1aPAa7. Three-dimensional analysis of the acoustic radiation pressure: Application to single-beam acoustical tweezers

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Recent studies on the acoustic radiation forces exerted by sound impinging spherical objects suggest the use of structured wavefronts for particle entrapment and controlled manipulation. In the scope of understanding why it is made possible to trap and manipulate small particles with sound, we present a general model for the acoustic radiation forces in three dimensions. A first generalization comes from the extension of well known results for the radiation pressure of plane waves to incident wavefields having arbitrary wavefronts. Secondly, the elastic spherical target of any dimension is allowed to be arbitrarily located within the wavefield. Introducing a new class of "single-beam" acoustical tweezers, we discuss the capabilities of different acoustical beams to achieve particle trapping and manipulation tasks. In addition, using an efficient experimental setup, we report the propagation of a peculiar beam carrying orbital angular momentum, namely an acoustical vortex, which is our selected candidate to achieve the first three-dimensional acoustic trap for elastic particles.

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INTRODUCTION

More than two decades and a half ago, Ashkin’s seminal work on the trapping of dielectric particles with a tightly focused laser beam enhanced an exciting investigation field: optical trapping. The underpinning physical concept called radiation pressure was exploited both as a rich example of physical phenomena and an efficient tool to design sensitive measurement experiments [1]. Various reviews bear witness to the wide and efficient application of optical tweezers across the physical and life sciences [2, 3, 4]. Now more than ever, the idea of making use of acoustic radiation forces as acoustical tweezers is appealing. In fact, acoustic devices may present several advantages to their optical analogous going from the magnitude of the forces obtained, the larger size and nature of the particles that could be manipulated, the extended volume of operation to the better penetration of acoustic waves in soft tissues.

Trapping of particles has been reported in standing wavefield experiments [5, 6] and is the basis of many applications in lab-on-a-chip, microfluidic-type systems [7]. Considering the more complex picture of using a propagative single-beam to trap particles, only two experiments are available. Wu reported the trapping of latex particles and clusters of frog eggs. Lee et al. trapped lipid droplets near the focus of a high frequency transducer. It is to be noticed that neither of these experiments circumvented an inherent problem to single-beam acoustical tweezers: the axial trapping failure. In fact to overcome this axial pushing force, the first counter-propagated two opposite beams and the latter constrained the axial direction with a mylar film.

Recent research on particle manipulation with acoustic or optical tweezers has illustrated important features of radiation forces of helicoidal vortex beams on a sphere. The central dark core of this vortex structure provides an entrapment region where the optic or acoustic intensity is minimum. In the case of a helicoidal Bessel beam, predictions that the sphere may be attracted towards the source of the beam is a result of a negative axial force that arises when appropriate parameters of the particle and the beam are selected [8, 9]. Furthermore, these beams carry orbital angular momentum (OAM) [10] that can be transferred to disks [11, 12, 13] or induce a rotation of the particle around the propagation axis [14]. In addition, if the particle absorbs energy, it may spin about its own axis [15, 16].

Thorough theoretical investigations of the acoustic radiation force of progressive or standing plane waves acting on a sphere established the basic principles of acoustic traps [17, 18, 19]. The radiation forces of acoustic vortices have deserved very little attention perhaps because of various limitations either on the size of the particle or the nature of the incident wavefield of the seminal work previously mentioned. However, a case can be made for recent work dealing with helicoidal Bessel beams [9, 20] were the axial force exerted on a sphere lying on the axis of propagation was derived.

In this work we will focus on the forces arising as an incident Bessel vortex beam impinges an elastic sphere in an inviscid fluid. By presenting in the first section some new experimental results obtained, we will introduce the concept of a propagating vortex beam. Then, since these peculiar acoustic beams are intrinsically three dimensional, in the second section we will present results obtained with a recent theoretical framework that will enable us to get a clear three-dimensional picture of the behavior of a particle within the wavefield.

INCIDENT VORTEX BEAM

Phase Singularities: The pioneering work on phase singularities that can appear in propagating trains of acoustic waves is owed to Nye and Berry [21]. They demonstrated the existence of a peculiar wavefront dislocation that they called screw-type phase singularity. The phase of the beam is indeterminate on the propagation axis and as a consequence the wavefronts are twisting around this same axis (see Figure 1). This enhanced a new topic in the optics community now re-
ferred as singular optics going from the fundamental understanding of the intrinsic and extrinsic nature of the angular momentum carried by light [22, 23] to its application in dexterous particle manipulation tasks with optical tweezers [24, 25]. In acoustics, vortices were successfully synthesized and analyzed in both linear and non-linear regimes [10, 26, 27].

Along with Gauss-Laguerre beams and the r-vortex, a helicoidal Bessel beam is an example of vortex beams [28]. In cylindrical coordinates, the spatial part of the complex acoustic velocity potential may be written for a Bessel beam as follows:

\[ \phi(\rho, \phi, z) = \phi_0 J_m(\kappa \rho)e^{i(m \phi + k_z z)} \]  

\( \phi_0 \) is a real amplitude constant and the \( (e^{-i\omega t}) \) time convention is adopted. \( \omega = 2\pi/T \) is the pulsation and \( T \) the acoustic period. As shown in Fig.1, \( (\rho, \phi, z) \) are the coordinates of a cylindrical basis centered on the sphere. \( J_m \) is the Bessel function of radial order \( m \). Sometimes referred as the topological charge [10], \( m \) is an integer whose sign defines the handedness of the wavefront rotation and its magnitude determines the pitch of the helix. \( \kappa \) and \( k_z \) are the radial and axial wave numbers respectively, related by the usual dispersion relation \[ \kappa^2 + k_z^2 = (\omega/c_0)^2. \]

Bessel beams propagate without diffracting [29]. This property is generic of separated variable solutions of Helmholtz equation in a Cartesian or cylindrical coordinate system. Consequently, their transverse dependence is invariant along the propagation axis and so is the radiation force.

**Figure 1:** Geometry of the radiation force problem. The sphere is illuminated by an acoustic helicoidal Bessel beam. The reference frame \((x, y, z)\) and the spherical coordinate system \((r, \theta, \phi)\) are centered on the elastic sphere. The incidence direction of the Bessel beam is described by the half-cone angle \( \beta \).

**Experimental Propagation:** Experimentally obtaining a propagating Bessel beam remains tricky and has only been reported for a non-vortex Bessel beam obtained in a restricted area [30]. Furthermore, in the aim of trapping particles with a single-beam, they may not be as pertinent as other propagating acoustic beams. Here we show new results on the propagation of tightly focused vortex beams.

The experimental setup is similar to the one used in various experiments on the synthesis of linear and non-linear vortices in acoustics [10, 27]. The experiments take place in a water tank in which a spherically focused array of 127 ultrasonic transducers (Vermon, France) is immersed (see Fig.2). Each transducer is driven at its central frequency of 1 MHz corresponding to a wavelength \( \lambda = 1.5\text{mm} \). The matrix of transducers forms a compact hexagon 10 cm wide. A fast response membrane hydrophone can measure the instantaneous pressure field propagating in the tank.

The generation of these waves relies on the inverse filter technique [31] suitable to synthesize complex ultrasound scenes. Defining the sources and a set of prob points (here a control plane), the method relies on the knowledge of the medium separating each transducer-probe couple. The green function (impulse response) of each couple is recorded and gathered together to form the final propagation operator. Then, an appropriate numerical treatment is applied to inverse that
operator and compute the signals to be emitted by the array suitable to obtain the desired “target” wave field.

The geometrical focal length of the array is initially 450mm. To obtain sharply focused wavefields, a spherical acoustic lens carved in PMMA and directly mounted on the array was used in addition. It focuses at a length of $r_0 = 80$ mm. The control plane scanned by the hydrophone and the focal plane of the acoustic lens fit together.

In figure 3, the pressure amplitude of two synthesized beams is represented. These fluctuations were measured in a plane $(x, y)$ transverse to the propagation axis $z$ and to appraise the motion of the wavefronts, four successive time frames at $t = [0, T/4, T/2, 3T/4]$ are selected. The top figures correspond to a propagating vortex of positive topological charge $m = 1$ whereas on the bottom, the sound beam with twist has a charge $m = -2$. Hence, from left to right, the rotation of the wavefronts are counterclockwise and clockwise respectively. It can also be seen that while the $m = 1$ vortex will achieve a complete rotation during a period $T$, the higher order vortex will only achieve half a twist (see the positive lob flagged by a black arrow). Indeed as mentioned before, the topological charge determines the pitch of the helix. More information is made available by analyzing the phase and the intensity of the wavefields in the control plane. In fig.??, at first glance one retrieves the helocoidal motion of the wavefronts in the variations of the phase. It achieves $|m|$ jumps of $2\pi$ around the propagation axis (center of the plane). In addition, nearby the beam center the phase turns indeterminate. One can notice that two symmetric points about the center of the beam are dephased of $\pi$ and as a consequence, in the intensity pattern, destructive interferences are responsible of the central dark core of null amplitude. The silent central core is surrounded by an intense ring and as a whole, the typical “doughnut” intensity diagram is retrieved [22]. In addition, tightly focusing properties are observed. The beam is focused into a ring of radius $R \approx 0.6\lambda$ flirting with the diffraction limit. As the topological charge $m$ increases so does the radius $R$ of the ring.

**THREE DIMENSIONAL ANALYSIS OF THE RADIATION FORCE**

From the flux of momentum that is altered by the particle through scattering arises a radiation force acting on particles [32]. Recently, an efficient model to study the three-dimensional radiation forces acting on an elastic sphere in an arbitrary wavefield has been established [14]. The analysis found there encompasses several difficulties that appear when considering the scattering problem and the calculation of the radiation forces for a sphere off-axis [9, 20]. The procedure is as follows. The incident field defined in Eq.(1) is decomposed in the spherical basis centered on the sphere (Fig.1):

$$
\phi_i = \phi_0 \sum_{n=0}^{\infty} \sum_{m=-n}^{n} A_n^m j_n(k_0 r) Y_n^m(\theta, \phi) \quad \text{with} \quad A_n^m = i^{n+m}(2n+1) \frac{(n-m)!}{(n+m)!} P_n^m(\cos \beta)
$$

(Fig.1)
FIGURE 3: Instantaneous pressure field (Pa) measured in a plane transverse to the propagation axis. Snapshots are taken at four different instants: $t = 0$, $T/4$, $T/2$ and $3T/4$. (On top) Vortex of charge $m = 1$ rotating counterclockwise around the propagation axis. (Bellow) Vortex of charge $m = -2$, the arrow flags the clockwise rotation of one of the positive pressure lobes.

FIGURE 4: Analysis of focused AV beams of charge $m = 1$ (top) and $m = 2$ (bottom) in the focal plane (dimensions in units of $\lambda$). a) The phase variations that are obtained by scanning the focal plane with the hydrophone. b) Intensity of the field in arbitrary units, the typical “doughnut” intensity distribution of vortex beams can be observed. There is respectively 1 and 2 jumps of $2\pi$ of the phase around the propagation axis for the vortices of charge $m = 1$ and $m = 2$.

The beam shape coefficients $A_n^m$ are necessary for radiation force calculations along with the scattering coefficients $R_n = \alpha_n + i\beta_n$. For every location of the particle, the coefficients $A_n^m$ are translated and/or rotated using the addition theorem for spherical harmonics and their rotation properties as explained in [14]. Finally the three components of the radiation force can be calculated from Eqs.(14)-(16) in [14]. It will be useful further on to calculate the acoustic radiation forces in a cylindrical basis ($\rho, \varphi, z$) as shown in Fig.5.

**Numerical results:** The example that follows is carried out for a sphere of radius $a$ immersed in water of density $\rho_0 = 1000$ kg/m$^3$ and the speed of sound is $c_0 = 1500$ m/s. The sphere will
be made of polystyrene having a density $\rho' = 1080 \text{ kg/m}^3$ and compressional and shear wave velocities of respectively 2350 m/s and 1120 m/s. The incident pressure amplitude is $p_0 = 1 \text{ MPa}$ and the angle of incidence of the Bessel beams will be $\beta = 60^\circ$ (see Fig. 1).

In figure 6, first the axial force $F_z$ is plotted as a function of the sphere's radius $a/\lambda$. It can be seen that in this particular regime where the sphere's radius is comparable to the driving wavelength, strong resonance peaks in the force can appear. They are attributed to weakly damped surface waves circulating around the sphere [9]. Furthermore, taking a look into the two graphical insets (red curves), in specific regions for $a$ the axial force turns negative. This counterintuitive results means that the axial force can point towards the source of the propagating beam. Hence the particle may be pulled just like a "tractor beam". To make sure this can be made possible, it is of great important to analyze the radial forces acting on the sphere displaced away from the beam axis. In fact they must be attractive towards the vortex core so that the pulling forces can operate. This result is also shown in Fig. 6. $F_\rho$ is calculated as a function of the radial displacement $\rho$ of the sphere away from the vortex core. A radial equilibrium position requests a force passing by zero with a negative slope. Indeed, only in this case the force is restoring. This is verified in the core, i.e. $\rho = 0$, only for the case $a = 0.35\lambda$. When the sphere is larger $a = 0.84\lambda$, the positive radial force for $\rho > 0$ is repelling and the sphere is urged away from the propagation axis. Thus the negative axial force will not operate. As a general remark, it should be noticed that the predicted forces are much larger than that reported in any optical device.

As the analysis of the forces here is complete, it is possible to get a three dimensional picture of the moving sphere within the wavefield. Having a knowledge of the initial position of the particle, its weight and buoyancy, Stokes' drag forces and the acoustic radiation forces, we computed the trajectory of the sphere inside a Bessel beam of charge $m = 1$. This is achieved by resolving the dynamics problem with a step by step Runge Kutta scheme. We show the predicted trajectory of the polystyrene sphere of radius $a = 0.35\lambda$ in a Bessel beam of charge $m = 1$. Departing from the initial position $(0.25\lambda, 0.25\lambda, 0)$, it can be seen in figure 7 that the sphere is first pushed ($F_z > 0$) by the downward propagating beam while it is attracted towards the propagation axis ($F_\rho < 0$). The spiral motion bears witness to azimuthal forces $F_\phi \neq 0$ and confirms the transfer of orbital angular momentum from the beam to the particle. Once the particle sinks in the vortex core, it is sucked upwards by the negative axial force $F_z < 0$ and against the gravity field $\vec{g}$ just like in a "tractor beam".
FIGURE 6: Axial and radial forces acting on the polystyrene sphere. a) The axial force is plotted as a function of the increasing radius of the sphere $a$ compared to the wavelength $\lambda$. b) The radial force is shown as a function of the sphere’s displacement $\rho$ away from the propagation axis. The plain curve is for $a = 0.35\lambda$ and the dashed curve for $a = 0.84\lambda$ corresponding to values where $F_z$ turns negative.

FIGURE 7: Three dimensional trajectory in a Bessel beam operating as a Tractor beam. The sphere’s radius is $a = 0.35\lambda$ is initially located at $(0.25\lambda, 0.25\lambda, 0)$. The axial axial and transverse intensity of the beam are superposed with transparency.

CONCLUSION

In this study we analyzed the acoustic radiation forces acting on spherical particles within peculiar wavefields. First new results on the focusing properties of acoustical vortices were re-
ported. The experimental setup and its versatility enabled us to synthesize vortices of charges $m = 1$ and $m = -2$. We consider that after dealing with some technical problems (over-heating of transducers, etc...) we will be able to seriously plan the use of the focused vortices to achieve the first three-dimensional single beam trap in acoustics. To shed some light on the complex behavior of particles in these wavefields, we made use of a general model recently proposed [14] to get a three dimensional analysis of the forces acting on the sphere inside a helicoidal Bessel beam. We showed that when the radius of the sphere reaches sizes comparable to the driving wavelength, interesting mechanisms may appear. Beginning with neat resonances in the radiation force. Thus, the sphere’s radius can be tuned to the driving wavelength. Seeking in between these peaks, regions are found for which the axial force can flip towards the source of the beam. We demonstrated that under certain circumstances, Bessel beams may operate against gravity as tractor beams. More generally, the analysis displayed here shall be of practical use to any potential application involving radiation forces and shall be an impetus in the design of original acoustic traps that may have potential applications in physics and life science.

**REFERENCES**


