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1pPAa1. Acoustic bubble behavior in a standing wave field
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This paper focuses on the experimental and numerical studies of acoustic cavitation induced micro bubbles in a standing waveguide filled with water. It is shown that the cylindrical geometry of the system used in this study allows the micro bubbles to self organize into particular patterns. At high pressure amplitudes, the cavitation bubbles tend to aggregate into well know cluster patterns and at relatively low pressure amplitudes, the cavitation micro bubbles aggregate into ring patterns. This study highlights that the shape of these ring patterns is directly related to the Bjerknes force distribution in the resonator. It is also shown both experimentally and numerically that cavitation bubbles may exhibit spiraling behavior around this ring pattern. This spiraling phenomena is numerically studied and the conditions for which a single cavitation bubble follows an orbital trajectory in the cylindrical waveguide are established, and the influences of the acoustic pressure amplitude and the initial bubble radius are investigated.

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INTRODUCTION

Depending on forces it experiences, a gas bubble in a liquid exhibits different kind of motions: zigzagging or spiraling motions due to hydrodynamic forces [1], spiraling motions due to the competition between hydrodynamic and acoustic forces [2], motions towards a pressure node or antinode due to the primary Bjerknes force [3]. When a bubble is only subjected to acoustic forces, most of the authors focuses attention on the study of radial oscillations of bubbles. For instance, Marmottant et al [4] showed that, by isolating a single microbubble and studying its dynamics, microstreamlines generated in the surrounding fluid are responsible of controlled deformation, motion and rupture of vesicles, opening applications in a diverse range of engineering and biomedical applications such as particle trapping and transport [5, 6], mixing [7] or cell sonoporation [8]. Even if the space location of bubbles is of great practical importance, bubble motions are much less studied than radial bubble dynamics. An interesting study by Doinikov et al [9, 10] showed that the coupling between radial oscillations and translational motions of bubbles in a strong acoustic field can lead to translational instabilities of bubble motions around the nodal plane. Even when extended to the three-dimensional case [11], no particular attention was paid to the motion that bubbles could exhibit when taking into account more complex and realistic acoustic field.

The aim of this study is to investigate the acoustic bubble motions in a cylindrical acoustic field. Supported by experimental observations, we investigate theoretically how bubbles concentrate into specific patterns whose shape can be linked to the geometry of the system. The radial and translational motions of the bubbles located on these patterns are also studied both experimentally and numerically.

EXPERIMENTAL OBSERVATIONS

The system used for the experiments consists of a cylindrical waveguide of inner radius $a = 4.0 \times 10^{-2}$ m. The resonator is filled up to a fixed height $z = h$ with water at standard conditions of temperature $T_0$ and pressure $P_0$. A 30 kHz continuous monochromatic acoustic wave is generated in the medium using an ultrasonic transducer (ASMT28D50) located at $z = 0$. The sinusoidal signal is supplied to the transducer by a function generator (Agilent 33220A) subsequently amplified by a power amplifier (TREK 50/750) enabling the adjustment of the acoustic pressure amplitude from 0 to 8 bars. The displacements of the cavitation induced micro bubbles are measured with a high speed camera (Vision Research Phantom v12.1). When an ultrasonic wave field is established in the cylindrical waveguide at a resonance of the water column ($h = 4.8 \times 10^{-2}$ m), several phenomena appear. At relatively high pressure amplitudes ($P_a > 2$ bar), the cavitation bubbles self-organize into well-known cluster patterns, located at the pressure antinodes. A picture of this kind of structure is presented in Fig. 1 a. The formation is due to the attracting and repelling forces arising from interactions with other bubbles and interactions with the acoustic field. At relatively low pressure amplitudes ($0.5 < P_a < 2$ bar), the cavitation induced bubbles aggregate into ring patterns (bubble radii $R_0$ varying from 70 to 120 μm) located at pressure nodes. A picture of this kind of structure is presented in Fig. 1 b. The radius of this bubble ring is of about $2.5 \times 10^{-2}$ m. The radiation force set up by the acoustic pressure gradient causes bubbles to accumulate to pressure nodes if their radii are larger than the resonant radius or to pressure antinodes if their radii are smaller [3]. Bubbles located on the ring patterns may exhibit spiraling behaviors in the ($r$, $z$).

![Figure 1](image1.png)

**Figure 1**: (a) Picture of a bubble cluster ($P_a \approx 5$ bar), and (b) picture of a bubble ring ($P_a \approx 1.5$ bar).
z) plane, around the pressure nodal line. Fig. 2 presents the $r$ and $z$ displacements of a spiraling bubble as a function of time (given in terms of acoustic period number). The two components of the displacement are sinusoidal and the phase between these components is of about $\pi/2$. The bubble describes a quasi circular trajectory of radius $R_p \simeq 1.0 \cdot 10^{-3} \text{ m}$ and of center located at $r_c \simeq 2.5 \cdot 10^{-2} \text{ m}$ (on the pressure nodal line, i.e. on the bubble ring).

![Figure 2](image)

**FIGURE 2:** $r$ and $z$ displacements of a single bubble with respect to time. Insert: Zoom of Fig. 1 b) showing a spiraling bubble ($P_a \simeq 1.5$ bar and $R_0 \simeq 110 \mu\text{m}$).

**THEORETICAL BACKGROUND**

Assuming that the acoustic field in the cylindrical resonator is stationary and azimuthally symmetric, the acoustic pressure $p_a$ can classically be expressed in the cylindrical coordinate system as

$$p_a = P_a \sum_{\nu=0}^{\infty} \sum_{m=0}^{\infty} A_{\nu,m} J_\nu \left( k_{w,\nu,m} r \right) \cos \left( k_{z,\nu,m} z \right) e^{i \omega t},$$

(1)

where $\omega$ is the angular driving frequency, $A_{\nu,m}$ is the modal complex amplitude of the mode $(\nu, m)$, $J_\nu$ is the cylindrical Bessel function of $\nu$th order, and $k_{w,\nu,m}$ and $k_{z,\nu,m}$ are the radial and longitudinal wave numbers which satisfy the dispersion equation

$$k_{w,\nu,m}^2 + k_{z,\nu,m}^2 = \omega^2 / c^2,$$

(2)

c being the adiabatic celerity of sound in the medium. Considering a rigid wall condition at the location $r = a$, the radial wavenumber can be expressed as $k_{w,\nu,m} = \chi_{\nu,m} / a$, where $\chi_{\nu,m}$ is the $(m + 1)$ zero of $J'_\nu$. At the experimental driving frequency (i.e. $f = \omega / 2\pi = 30$ kHz), all modes are antisymmetrics or evanescent except the modes $(0,0)$ and $(0,1)$ that propagate into the waveguide.

It is known that a single bubble in a stationary acoustic field is attracted or repulsed at the pressure node or antinode. The force that acts on this single bubble is the primary Bjerknes force which is defined as

$$F_{b1} = -\frac{4}{3} \pi R^3 \nabla p_a,$$

(3)

where $R(t)$ is the time dependent radius of the bubble, and where $\nabla$ is the gradient operator. Figs. 3 (a) and (b) present the normalized spatial distributions of the acoustic pressure in the $(r, \theta)$ plane, and the corresponding primary Bjerknes force, respectively. These results clearly show that zeros of the Bjerknes force in a cylindrical waveguide take the form of a ring. The radius of this ring is of about $2.5 \cdot 10^{-2} \text{ m}$ which corresponds to the value of the experimental bubble ring radius reported in Sec. 2.
The particular spiraling behavior presented in Sec. 2 suggests in turn that there is a coupling between the radial oscillations and the motions of bubbles located on the ring pattern. Indeed, supported by the work of Watanabe and Kukita [12] on the translational motion of a bubble in a standing wave acoustic field, Doinikov showed [10] that the rederivation of the equations of motions of a bubble using an energy approach yields to a missing term from Watanabe and Kukita’s equations. This term enables a feedback between the radial oscillations and the motions as follows:

\[
\dddot{R} + \frac{3}{2} \dddot{r} \dot{R}^2 - \frac{P - \frac{1}{4} \dot{r}^2}{\rho} = 0, \tag{4}
\]

where \( r \) is a position vector in the \((r, z)\) plane, \( P \) is the classical scattering pressure (taking into account the acoustic pressure), \( \rho \) is the density of the fluid, and \( F \) is an external force that takes into account the primary Bjerknes force \( F_{b1} \) and the Levich viscous drag [13] \( F_v = -12 \pi \eta R (\dot{r} - v_a) \), \( v_a \) being the acoustic velocity, and \( \eta \), the viscosity of the fluid.

\[
\dddot{r} + \frac{3}{2} \dddot{R} \frac{\dot{R}^2}{R^3} = \frac{3}{2\pi \rho R^3} F, \tag{5}
\]

\[
\omega_0 = \frac{1}{R_0} \left( \frac{3\gamma P_0}{\rho} + \frac{2\sigma (3\gamma - 1)}{\rho R_0} \right)^{1/2}, \tag{7}
\]

is the resonance frequency of a bubble of initial radius \( R_0 \), \( \gamma \) and \( \sigma \) being the polytropic exponent, and the surface tension of the bubble, respectively. The single bubble describes for both cases (Figs. 4 a and b) a quasi circular trajectory. The center of this circular trajectory is located at the pressure node whatever the ratio \( \omega/\omega_0 \). The radius of the bubble path decreases from \( R_p = 1.0 \times 10^{-2} \) m to \( R_p = 1.0 \times 10^{-3} \) m as the ratio \( \omega/\omega_0 \) increases from 0.8 \( (R_0 = 90 \mu m) \) to 1.0.
CONCLUSIONS

The preliminary results presented in this paper highlight that bubbles in a cylindrical acoustic standing wave exhibit particular dynamics. Experimental observations showed that bubbles can concentrate into ring patterns located at the pressure nodes when the acoustic pressure amplitude is moderate ($P_a < 2$ bar). Supported by this experimental observation, it has been shown theoretically that the shape of these patterns is directly linked to the cylindrical geometry of the system. It has also been observed experimentally, and verified numerically, that bubbles located on this ring may exhibit spiraling behavior around the pressure nodal line (i.e. the ring pattern). A detailed study of this spiraling motion must now be conducted to give a better understanding of the bubble dynamics in cylindrical geometry. The control of this phenomenon could find promising applications in microfluidics for the enhancement of fluid mixing, for instance.

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REFERENCES


