ICA 2013 Montreal
Montreal, Canada
2 - 7 June 2013
Physical Acoustics
Session 3pPAa: Borehole Acoustics Logging for Hydrocarbon Reservoir Characterization II

3pPAa5. Underrelaxed drained bulk modulus for fluid-saturated rocks in full frequency range
Yongjia Song*, Hengshan Hu, Huanhong Du and Tao Lu

*Corresponding author’s address: Astronautic Science and Mechanics, Harbin Institute of Technology, Postbox 344, 92 West Dazhi Street, Harbin, 150001, Heilongjiang, China, songyongjia06122011@126.com

Local flowing between cracks and pores is called squirt-flow that usually induce elastic moduli dispersion and waves attenuation. Expression of underrelaxed drained bulk modulus in full frequency range is derivated in this paper. Underrelaxed drained bulk modulus’s real part increases with frequency, and underrelaxed drained bulk modulus’s imaginary part is nonzero. This study also shows that liquid pressure in cracks equals to zero in the low-frequency limit, that is to say liquid pressure in cracks have sufficient time to relax and in this case the drained bulk modulus corresponds to Biot’s static dry bulk modulus. In the high-frequency limit, the underrelaxed drained bulk modulus approximate to Mavko-Jizba’s expression. The underrelaxed drained bulk modulus can be used to calculate fast P-wave and slow P-wave velocities and attenuation instead of static drained or dry bulk modulus in Biot’s theory.

Published by the Acoustical Society of America through the American Institute of Physics
INTRODUCTION

Sedimentary rocks are typical porous media which are generally saturated or partly saturated with fluids. The pore space in a rock usually contains principle pores and microcracks or contacts between solid grains. The filtration through the principle pores brings about elastic wave dispersion and energy dissipation. This type of flow is called ‘Biot flow’ because Biot (1956) set up a theory to predict the wave propagation behavior associated with such a macroscopic flow in a porous medium. However his theory greatly underestimates the observed intrinsic dispersion and attenuation (such as Sams et al, 1997). Many studies (Winkler et al, 1979; Johnston et al, 1979; Pride et al, 2004) showed that local flow and heterogeneous microscopic flow through microcracks or contacts are the dominant energy loss mechanisms in elastic waves. Liquid will be squeezed out of thin (or soft) cracks and then flow into stiff (or spherical) pores because a greater fluid-pressure is generated in fluid-filled microcracks compared to the pressure in the principle pores when the sample is under pressure. Dovrkin et al (1995) called this kind of local flow “squirt-flow”, and they studied the squirt-flow in fully saturated porous rocks. According to their analysis, the squirt flow induces much greater dispersion and attenuation than those caused by the Biot flow. Before Dovrkin et al (1995), Mavko and Nur (1979) studied another kind of “squirt-flow” in partially saturated cracks, and in their model the squirt flow also induces greater energy loss than the Biot flow does, for both P-wave and S-waves. But Mavko-Nur (1979) described only the loss due to squirt flow, neither the moduli nor velocity was predicted. Mavko and Jizba (1991) proposed a quantitative model for estimating the moduli at the low and high frequency limits. They argued that the fluid pressures in cracks is higher than that in the pores when the sample is under pressure, but the difference in pressure does not cause a flow into the pores at the high frequency limit because of a too short time duration, making the pressure in the crack unrelaxed. At the low frequency limit, the pressure in the cracks are fully relaxed, and moduli approached to the Biot moduli. But Mavko-Jizba (1991) did not consider the behavior of the rock at a frequency between the two extremes, at which the pressure in cracks is underrelaxed. Thus the moduli are also underrelaxed. In order to explain the dispersion and attenuation in full frequency range, an analytical expression is derived in the present paper for the underrelaxed drained bulk modulus (by dividing the total pore space into stiff pores and microcracks).

1 DERIVATION OF THE UNDERRELAXED DRAINED BULK MODULUS

1.1 Porespace Deformation

Consider the rock to be a linear, elastic isotropic, homogeneous (i.e., composed of only one solid material), fluid-saturated porous medium, in which the porespaces are interconnected. Berryman (1992) gave a principle of effective pressure for total pore volume in such a rock, which is expressed in the form of

\[
\frac{1}{K_d} \left( \frac{K_s}{K_s} - \frac{1}{K_s} \right) \left( P_f - P \right) - \frac{\phi}{K_s} p_f = \frac{\Delta V_p}{V_p},
\]

where \( K_d \) is the drained (or dry) bulk modulus of the porous medium; \( K_s \) is the solid grains bulk modulus; \( \phi \) is the porosity; \( \Delta V_p \) is the volume chang of total porespaces \( V_p \); \( P \) and \( p_f \) are the confining pressure and fluid pressure, respectively. In many cases physical rocks contain microcracks and solid contacts as well as pores. These microcracks or solid contacts are usually soft and thin, and the principle pores are much more stiffer than the cracks. By dividing total pore space into pores and cracks, we have that the total pore space change is a sum of crack volume change and pore volume change

\[
\Delta V_p = \Delta V_c + \Delta V_p.
\]

The crack volume change \( \Delta V_c \) can be express as

\[
\Delta V_c = -q - \frac{V_c}{K_f} \cdot \Delta V_c,
\]

where \( q \) is the “squirt-flux”, which is the fluid flux out of cracks into pores; \( P_c \) is the averaged fluid pressure in the cracks, which is greater than pore fluid pressure and results in a squirt-flow; \( V_c \) is the volume occupied by cracks. Following Mavko and Jizba (1991), we assume that the principle pores are drained and the fluid pressure in cracks is nonzero (or underrelaxed), which leads to \( p_f = 0 \). due to the cracks occupy little volume although nonzero fluid
pressures exist in cracks. The “underrelaxed” drained bulk modulus can be expressed by substituting (2) and (3) into (1) as

\[ K_d = \left( \frac{\varphi}{K_p} + s + 1 / K_s \right)^{-1}, \]

where

\[ K_p = -\delta P \left( \frac{\Delta V_p}{V_0} \right), \]

\[ s = \frac{\varphi q}{V_0} P + \frac{\varphi V_c}{K_f V_0} \frac{P_s}{P}. \]

\( K_p \) is the bulk modulus of the principle pores (or pore bulk modulus for short). The squirt-flux \( \varphi \) and crack fluid pressure \( P \) vary with frequency and are relevant to the degree of crack fluid pressure relaxation. The quantity given by equation (6) has a dimension of compliance, and is associated with the degree of relaxation of crack fluid pressure. Thus we call \( s \) the “crack relaxation compliance”.

### 1.2 Crack Relaxation Compliance and Underrelaxed Drained Bulk Modulus

The definition (6) of \( s \) shows that \( s \) is composed of two parts

\[ s = s_1 + s_2. \]

The expressions of compliances \( s_1 \) and \( s_2 \) are given by [Song and Hu, 2013]

\[ s_1(\omega) = \frac{\varphi q}{V_0} P = \frac{4 \varepsilon (1 - \nu_d)}{\mu} \frac{f(\omega)}{1 + 2 \frac{K_f (1 - \nu_d) [1 - f(\omega)]}{\pi \mu \gamma}}, \]

\[ s_2(\omega) = \frac{\varphi V_c}{K_f V_0} \frac{P}{P} = \frac{4 \varepsilon (1 - \nu_d)}{\mu} \frac{1 - f(\omega)}{1 + 2 \frac{K_f (1 - \nu_d) [1 - f(\omega)]}{\pi \mu \gamma}}, \]

where \( \varphi \) is the crack density (O’Connell and Budiansky, 1977), \( \nu_d \) is the poisson ratio of the dry rock sample. Here, \( s_1 \) is called the “squirt-flux compliance”, which denotes the compliance associated with the squirt-flux through cracks; while \( s_2 \) is called the “compression compliance”, which is contributed by fluid volume change in cracks. The function \( f(\omega) \) in (8) and (9) is frequency-dependent and can be expressed as

\[ f(\omega) = \frac{2 J_1 \left( \sqrt{i \omega / \omega_r} \right)}{\sqrt{i \omega / \omega_r} J_0 \left( \sqrt{i \omega / \omega_r} \right)}, \]

where \( J_0 \) and \( J_1 \) are Bessel functions and where the relaxation frequency is defined by

\[ \omega_r = \frac{\gamma^2 K_f}{3 \eta}. \]

Here, \( \gamma \) is the crack aspect ratio (thickness to diameter); \( K_f \) and \( \eta \) are the fluid bulk modulus and viscosity in cracks, respectively.

We assume that the pore bulk modulus \( K_p \) is independent of frequency, which allow us to express the underrelaxed drained bulk modulus in a simple form

\[ K_f(\omega) = \left[ s(\omega) - s(0) + K_{dry}^{-1} \right]^{-1}, \]

where (4) is used. \( s(0) = \lim_{\omega \to 0} s(\omega) \) is the crack relaxation compliance in the low frequency limit. Both crack compliance \( s(0) \) and pore bulk compliance \( 1 / K_p \) contribute to the drained bulk modulus. For a homogeneous (i.e., composed of only one solid material) porous medium, Biot (1956) showed that dry bulk modulus is identical to drained bulk modulus

\[ K_{dry} = K_f(0). \]

Equation (12) is the desired expression for underrelaxed drained bulk modulus dispersion in terms of dry rock property.
1.3 Comparison with Mavko-Jizba Model

Mavko and Jizba (1991) proposed a quantitative model for squirt dispersion of elastic wave velocities between low frequency limit and high frequency limit. One of their central results is the expression of the “unrelaxed” frame bulk modulus under the assumption that the stiff pores are drained (or dry) but the soft pores (or cracks) are filled with liquid (namely, there exist unrelaxed fluid pressures in cracks). When the confining pressure is zero, Mavko-Jizba’s expression has the form

\[
\frac{1}{K_{uf}} = \frac{1}{K_{hp}} + \left( \frac{1}{K_f} - \frac{1}{K_s} \right) \phi_c,
\]

where \( K_{uf} \) is the unrelaxed frame bulk modulus, which corresponds to the at high-frequency limit of the unrelaxed drained bulk modulus in this paper; \( K_f \) and \( K_s \) are bulk modulus of the fluid and of the solid grains, respectively; \( \phi_c \) is compliant porosity, which is the crack porosity in this paper; \( K_{hp} \) is the dry bulk modulus under so high a confining pressure that the compliant porosity vanishes, thus \( K_{hp} \) is the \( K_d \) when crack density is zero. We assume that the effects of the magnitude of confining pressure on pore bulk moduli are negligible, which allows us to express \( K_{hp} \) by taking crack density equal to zero in the form of

\[
K_{hp} = K_d \bigg|_{\varphi = 0} = \left( \varphi / K_p + 1 / K_s \right)^{-1},
\]

where (4) is used. Expression (15) is correct or at least reasonable when the cracks are dilute or occupy little space. Substituting (15) into (14) leads to

\[
K_{uf} = \left[ \left( \frac{1}{K_f} - \frac{1}{K_s} \right) \phi_c - s(0) + \frac{1}{K_{dry}} \right]^{-1},
\]

where (4) and (13) are used. Equation (16) is an approximate form of Mavko-Jizba’s unrelaxed frame bulk modulus when the effect of cracks on pore bulk modulus is neglected.

In order to compare the underrelaxed drained bulk modulus with Mavko-Jizba model as well as Biot-Gassmann model, we define a dimensionless parameter

\[
\psi = \frac{K_d(\omega)}{K_{uf}} - K_{dry}.
\]

If \( \psi \) is equal to zero, the underrelaxed drained bulk modulus will be identical to Biot-Gassman’s dry or drain bulk modulus; and if \( \psi \) is equal to one, the underrelaxed drained bulk modulus will be identical to Mavko-Jizba’s unrelaxed frame bulk modulus. There is a modulus operator in (17) because \( K_d(\omega) \) is complex.

2 NUMERICAL EXAMPLES

Some examples of underrelaxed drained bulk modulus \( K_d(\omega) \), crack relaxation compliance \( s \) and its components \( s_1 \) and \( s_2 \) dispersions are now given. Appropriate parameters used for calculations are shown below. In order to model the elastic properties of porous rock with different crack densities we adapt a Biot-consistent theory (Thomsen, 1985) to calculate the dry bulk modulus. We take \( K_p = 37.9 \text{GPa}, \ G_p = 32.6 \text{GPa (quartz)}; \ \varphi = 0.25; \ G_r = 2.25 \text{GPa}, \ \eta = 10^{-7} \text{Pa} \cdot \text{s (water)} \).

FIGURE 1 shows the underrelaxed bulk modulus dispersion that is determined by using Eq.(12). The real part of \( K_d \) increases with frequency, which means that the rock becomes stiffer as frequency increases. And the characteristic frequency \( \omega_c \) of squirt-flow is approximately proportional to \( \gamma^3 \), weights peak shifting. The characteristic frequency is different from the relaxation frequency defined in (11) which is proportional to \( \gamma^2 \). A suitable frequency weighting squirt-flow properties may be

\[
\omega_c \sim \gamma^3.
\]

The location of the lowest point of the imaginary of \( K_d \) is proportional to \( \omega_c \). Complex \( K_d \) reveals the viscoelastic or anelastic feature of the rock. \( \text{Im}(K_d) \) is minus because a factor \( e^{-\omega\tau} \) is used in the derivation (Song and Hu, 2013).

FIGURE 2 shows the dimensionless parameter \( \psi \) that weighs the comparison of the underrelaxed drained bulk modulus with Mavko-Jizba model as well as Biot-Gassmann model. In the low frequency limit the underrelaxed drained bulk modulus is identical to Biot’s drained bulk modulus, and in the high frequency limit the underrelaxed drained bulk modulus tends to Mavko-Jizba’s value. For underrelaxed drained bulk modulus, the aspect ratio
determinates the characteristic frequency between the low frequency limit and the high frequency limit. Notice that although (12) is not identical to (16) exactly in the high frequency limit due to a very small difference between $s(\infty)$ and $\varphi_k(1/K_f−1/K_s)$, the minor difference between $K_s(\infty)$ and Mavko-Jizba’s expression can be neglected.

**FIGURE 1.** Dispersion of the underrelaxed drained bulk modulus $K_f$ as a function of crack density $\varepsilon$ and aspect ratio $\gamma$. The dispersion corresponding to four groups of parameters with different crack densities and aspect ratios are shown as solid, dashed, dotted, and dashed-dotted lines in the boxes. (a) and (b) are real part and imaginary part of the complex $K_f$, respectively.

**FIGURE 2.** Dispersion of the dimensionless parameter $\psi$ which represents the dispersion of the underrelaxed drained bulk modulus between Biot-Gassmann limit to Mavko-Jizba limit. The four curves correspond to different aspect ratios in the box. The blue horizontal dashed line denotes the Biot-Gassmann limit (namely, low frequency limit) and the red horizontal dashed line denotes the Mavko-Jizba limit (namely, high frequency limit).

**FIGURE 3.** Dispersion of the crack relaxation compliance $s$ as a function of crack density $\varepsilon$ and aspect ratio $\gamma$. The dashed, dotted, and dashed-dotted curves correspond to dispersions in rocks with different crack densities and aspect ratios, as given in the boxes. (a) and (b) are real part and imaginary part of the complex $s$, respectively.
FIGURE 4. Dispersion of the squirt-flux compliance $s_1$ as a function of crack density $\varepsilon$ and aspect ratio $\gamma$. The dispersion corresponding to four groups of parameters with different crack densities and aspect ratios are shown as solid, dashed, dotted, and dashed-dotted lines in the boxes. (a) and (b) are real part and imaginary part of the complex $s_1$, respectively.

FIGURE 5. Dispersion of the fluid compression compliance $s_2$ as a function of crack density $\varepsilon$ and aspect ratio $\gamma$. The dispersion corresponding to four groups of parameters with different crack densities and aspect ratios are shown as solid, dashed, dotted, and dashed-dotted lines in the boxes. (a) and (b) are real part and imaginary part of the complex $s_2$, respectively.

FIGURE 6. Dispersion of the underrelaxed fluid pressure $P_i$ in cracks. The dispersion corresponding to four groups of parameters with different crack densities and aspect ratios are shown as solid, dashed, dotted, and dashed-dotted lines. At the low frequency limit, $P_i$ is relaxed to zero (or pore pressure), and at the high frequency limit, $P_i$ is unrelaxed and equal to confining pressure.
FIGURE 3 shows that the dispersion of \( s \). The real part of \( s \) decreases as frequency increases, which implies that cracks become stiff in the high frequency range. The location of the maximum of the imaginary of \( s \) also corresponds to \( \alpha_c \).

FIGURE 4 shows that the dispersion of squirt-flux compliance \( s_1 \). The real part of \( s_1 \) decreases as frequency increase, which implies that the squirt-flux become fewer, namely, there is not sufficient time for crack fluid squirting, as frequency increases. We can infer that no fluid will squirt from cracks into surrounding pores in the high frequency limit. From FIGURE 1 and FIGURE 3, we have that \( s_1 \) is greatly approximate to \( s \), so \( s_1 \) dominate the crack relaxation compliance. And FIGURE 5 shows that the dispersion of compressive compliance \( s_2 \). It is different from \( s_1 \) that the real part of \( s_2 \) increases as frequency, which means that the compressibility of crack fluid becomes important in the high frequency range. Due to the magnitude of \( s_2 \) is far smaller that \( s_1 \), \( s_2 \) can be neglected.

FIGURE 6 shows that the dispersion of underrelaxed fluid pressure in cracks. Underrelaxed fluid pressure \( P_c \) vary from zero to \( P \). At the low frequency limit, \( P_c \) is relaxed to zero (or pore pressure), and at the high frequency limit, \( P_c \) is unrelaxed and equal to confining pressure. Aspect ratio mainly determinates the pressure characteristic frequency of squirt-flow induced fluid pressure between the low frequency limit and the high frequency limit.

To conclude, we underline that although the crack porosity \( \varphi = 2 \pi \gamma \) is no more than 0.13% in our calculations and models, but the squirt-flow can brings about apparent moduli dispersion and energy loss. The complex underrelaxed drained bulk modulus induced by the squirt-flow can be used to explain the elastic wave dispersion and attenuation in rocks. In the low frequency limit the derived underrelaxed drained bulk modulus approaches the drained bulk modulus predicted by Biot(1956) and Gassmann(1951), and in the high frequency limit it tends to the expression for unrelaxed modulus in Mavko-Jizba(1991).

ACKNOWLEDGMENTS

This study is supported by the National Natural Science Foundations of China (Grant No.41174110).

REFERENCES