4pPAa2. General nonlinear acoustical equation of relaxing media and its stationary solutions

Nonna Molevich*, Rinat Galimov, Vladimir Makaryan, Dmitriy Zavershinskii and Igor Zavershinskii

*Corresponding author's address: P.N. Lebedev Physical Institute of RAS, Samara, 443011, -, Russia, molevich@fian.smr.ru

During previous years, the conditions for the negative second (bulk) viscosity existence were found in a large number of nonequilibrium media. The media with negative viscosity possess a number of new properties including acoustical activity. In the present paper we investigate the nonlinear stage of acoustical perturbation evolution in acoustically active nonequilibrium media using three models: the vibrationally excited gas with the exponential model of relaxation, the chemical active two component mixture with a nonequilibrium reaction and media with the general heat-loss function. The general nonlinear acoustical equation describing stationary density profiles behind the shock wave front in these media is obtained and solved. Its low- and high- frequency limits correspond to the Kuramoto-Sivashinsky equation and the Burgers equation with a source, respectively. Stationary structures of general equation, the conditions of their establishment and all their parameters are found analytically and numerically. In acoustically active media it is predicted the existence of the stationary solitary pulse. Unstable weak shock waves disintegrate into the sequence of solitary pulses. Their amplitude, form and speed are rigidly defined by the nonequilibrium degree and do not depend on the initial weak perturbation amplitude. For weak nonequilibrium degree, this solitary pulse is described analytically.

Published by the Acoustical Society of America through the American Institute of Physics
INTRODUCTION

In nonequilibrium media the acoustical perturbations can be amplified due to the positive feedback between the nonequilibrium heating and acoustical compressions (Rayleigh’s mechanism of acoustical instability). For a wide class of nonequilibrium homogeneous gas media, Rayleigh’s criterion of acoustical instability exclusive of thermal conduction and shear viscosity is satisfied under the condition $\frac{g_{31}}{g_{91}} < 0$, where $g_{54}$ is the bulk (second) viscosity coefficient. In earlier works, the conditions for the negative bulk viscosity existence were found in a large number of nonequilibrium media such as a vibrationally excited gas, atomic or molecular nonisothermal plasma, chemical active mixtures, media with nonequilibrium phases (e.g., volcanic magma, atmosphere regions with nonequilibrium condensation etc), interstellar media and stretched horizons of black holes [see references in 1,4]. The coefficient $\xi_0(\omega)$ depends on the type of nonequilibrium and a relaxation model. In such media it is possible the existence of stationary nonlinear acoustical structures that are different from the shock wave monotonic structures. We show that the generalized nonlinear acoustical equation (GNAE) of nonequilibrium media with one relaxation processes describes some of these structures.

THE GENERALIZED NONLINEAR ACOUSTICAL EQUATION

The generalized nonlinear acoustical equation (GNAE) of nonequilibrium media has a form [2-4]

$$
\tau_0 C_{V0} \left[ \frac{\partial^2 p'}{\partial t^2} - c_s^2 \frac{\partial^2 p'}{\partial x^2} - \frac{\Psi_{c_0} c_{\infty}^2}{\rho_0} \frac{\partial^2 p'}{\partial x^2} \right] + C_{V0} \left[ \frac{\partial^2 p'}{\partial t^2} - c_s^2 \frac{\partial^2 p'}{\partial x^2} - \frac{\Psi_0 c_{0}^2}{\rho_0} \frac{\partial^2 p'}{\partial x^2} \right] = 0. \tag{1}
$$

Here, quantities $\Psi_{c_0}, \Psi_0$ are the high - and low – frequency nonlinear hydrodynamic coefficients, $c_{s_0}, c_0$ are the high - and low – frequency sound speeds, $C_{V0}, C_{V0}$ are the high - and low – frequency specific heats, respectively. In next subsections, we give coefficients obtained for the vibrationally excited gas with simple relaxation law, for media with generalized heat –loss function, and for the stationary non-equilibrium chemically active mixture with a reversible chemical reaction and an external mass reagent source.

Vibrationally Excited Gas

The initial system of gas dynamics equations is as follows

$$
P = \frac{\rho T}{M}, \quad \frac{dp}{dt} + \rho \frac{dv}{dx} = 0, \quad \rho \frac{dv}{dt} = -\frac{\partial p}{\partial x}, \quad C_{V0} \frac{dT}{dt} + \frac{dE_v}{dt} = -\frac{T}{\rho} \frac{d\rho}{dt} = Q - I, \quad \frac{dE_v}{dt} = \frac{E_v - E_v}{\tau_v(T, \rho)} + Q \tag{2}
$$

Here, $E_v$ is the energy of the vibrational degrees of freedom of the molecules, $E_v$ is its equilibrium value, $\tau_v$ is the vibrational relaxation time, and $Q$ is the power of an external heat source (in particular, electric pumping in the discharge, chemical or optical pumping) sustaining the nonequilibrium degree $S = (E_v - E_v)/T = Q/T$. In [2,3], Equation (1) is obtained from the system (2) using the perturbation theory of the second order (i.e. neglecting by $\varepsilon^3$) for weak perturbations ($\rho'/\rho_0 \sim \varepsilon$) and weak dispersion

$$
|p| = \left| \frac{c_{0}^2 - c_{\infty}^2}{c_{\infty}^2} \right| \sim \varepsilon \ll 1 \tag{3}
$$

Here, $C_{V0} = C_{V\infty} + C_v + S_0 \tau_T$, $C_{P0} = C_{P\infty} + C_v + S_0 (\tau_T + 1)$ are the low-frequency heat capacities at constant volume and constant pressure in the vibrationally excited gas, $C_v = dE_v/dT_0$.
\[ \tau_T = \frac{\partial \ln \tau_{v0}}{\partial \ln T_0}, \quad c_0 = \sqrt{C_{p0} T_0 / M C_{V0} \approx \sqrt{\gamma_0 T_0 / M}}, \quad c_{\infty} = \sqrt{C_{p\infty} T_0 / M C_{V\infty} \approx \sqrt{\gamma_\infty T_0 / M}}. \]

\[ \Psi_0 = \left[ \frac{S_0 \tau_T (1 + S_0)}{C_{p0} C_{V0}} + \frac{1 + 2C_{V0}}{2C_{V0}} \frac{S_0 (1 + S_0)^2}{2C_{p0} C_{V0}} \tau_{TT} \right], \quad \tau_{TT} = \frac{T_0^2}{\tau_{v0}} \frac{\partial^2 \tau_{v0}}{\partial T_0^2}. \]

The gas with the stationary nonequilibrium and Landau-Teller dependence of the relaxation time has the five fields of the nonequilibrium degree \( S \) with qualitatively different properties [2] as follows.

**Field I:** \( S < S_{th} = C_V / (C_{V\infty} - \tau_T) \). Here, we have the positive bulk viscosity \( \xi > 0 \), the positive dispersion \( c_0 < c_\infty \), and the positive nonlinearity coefficient \( \Psi_0 \approx (\gamma_0 + 1)/2 \) similar to equilibrium media.

**Field II:** \( S_{th} < S < S_n \). The dispersion and the second viscosity are negative (\( \xi < 0 \); \( c_0 > c_\infty \)). The low frequency nonlinear coefficient \( \Psi_0 > 0 \). Here, \( S_n \) is defined from the equation \( \Psi_0 (S_n) = 0 \).

**Field III:** \( S_n < S < S_V = \frac{C_{V\infty} + C_V}{\tau_T} \). Here, \( \xi < 0 \), \( c_0 > c_\infty \), \( \Psi_0 = \gamma_0 \Psi_0 < 0 \).

**Field IV:** \( S_V < S < S_P \); \( S_P = \frac{C_{p\infty} + C_V}{(\tau_T + 1)} \). Here, \( \xi < 0 \), \( \gamma_0 < 0 \), \( C_{V0} < 0 \), \( C_{P0} > 0 \).

**Field V:** \( S > S_P \). Here, \( \xi < 0 \), \( c_0 < c_\infty \), \( \Psi_0 > 0 \), \( C_{V0} < 0 \), \( C_{P0} > 0 \).

### Media with Generalized Heat-Loss Function

The initial system of hydrodynamical equations is

\[ \frac{d \rho}{dt} + \rho \frac{\partial u}{\partial x} = 0, \quad \rho \frac{du}{dt} = -\frac{\partial P}{\partial x}, \quad c_{\text{v}0} \frac{dT}{dt} - \frac{P}{\rho^2}, \quad \frac{d \rho}{dt} = -\mathcal{S}, \quad \mathcal{P} = k_b \rho T. \tag{4} \]

In Equations (4), \( u, T, \rho, P \) are, respectively, the velocity, temperature, density, and pressure, \( \mathcal{S} \) is the generalized heat-loss function, \( c_{\text{v}0} \) is the specific heat at constant volume, \( k_b \) is the Boltzmann constant, \( m \) is molecular mass, and \( d / dt = \partial / \partial t + u \partial / \partial x \). For simplicity, we do not take into account the viscosity and heat conductivity coefficients.

Setting \( \rho = \rho_0 + \overline{\rho} \exp(-i \omega t + ikx) \), \( u = \overline{u} \exp(-i \omega t + ikx) \) etc. and linearizing on the small quantities \( \overline{\rho}, \overline{u}, \overline{T}, \overline{P}, \overline{u} \) gives the dispersion equation [3,4]

\[ \frac{\omega^2}{k^2} = \frac{c_{p0} - i \omega \tau_{0} c_{p0}}{c_{v0} - i \omega \tau_{0} c_{v0}} = \frac{k_b T_0}{m}. \tag{5} \]

In Equation (5), \( \tau_0 = k_b T_0 / \Gamma_0 m \) is the characteristic time of heating, \( \Gamma_0 = \Gamma(\rho_0, T_0) \) is the heating rate in the stationary medium, \( c_{\text{v}0} = c_{\text{v}0} + k_b / m \), \( c_{\text{v}0} = k_b T_0 \mathcal{S}_{0T} / \Gamma_0 \), \( c_{p0} = k_b T_0 \mathcal{S}_{0p} / \Gamma_0 \), \( \mathcal{S}_{0T} = (\partial \mathcal{S} / \partial T)_{\rho=\rho_0,T=\rho_0} \), \( \mathcal{S}_{0p} = (\partial \mathcal{S} / \partial \rho)_{\rho=\rho_0,T=\rho_0} \). The quantities \( c_{\text{v}0}, c_{p0} \) are the low-frequency specific heats at constant volume and constant pressure in a gas with energy gains.

If Equation (6) is solved for \( k = k_R + ik_I \), then \( k_I < 0 \) implies acoustic amplification, \( k_I > 0 \) implies acoustic damping, \( k_R = \omega / c_s \). The quantity \( c_s \) is a sound speed. For a case

\[ k_I \ll k_R. \tag{6} \]
we obtain from Equation (5) the acoustic gain and the sound speed

$$\alpha = k_I = \frac{\omega^2 \xi}{2 c_s^3 \rho_0^2}, \hspace{1cm} c_s = \frac{\omega}{k_I} = \sqrt{\frac{c_{v0}^2 + \omega^2 v_{0c}^2 c_{\infty}^2}{c_{v0}^2 + \omega^2 v_{0c}^2 c_{\infty}^2}},$$  \hspace{1cm} (7)$$

where

$$c_{\infty} = \sqrt{c_{p0} k_B T_0 \gamma_\infty v_{0c}}, \hspace{1cm} \xi = \frac{c_{p0} k_B T_0 \gamma_\infty v_{0c}}{c_{v0}^2 + \omega^2 v_{0c}^2 c_{\infty}^2},$$

Accordingly to Equations (7), (8), the sound amplitude increases if $\xi_0 < 0$, that is

$$\frac{c_{v0} m \rho_0 \beta_0}{k_B T_0} + \mathcal{J}_{0T} = \frac{\rho_0 \beta_0}{(\gamma_\infty - 1) T_0} + \mathcal{J}_{0T} < 0.$$  

The second order perturbation theory gives GNAE (2), where [4]

$$\Psi_0 = \frac{\gamma_\infty + 1}{2},$$

$$\Psi_0 = \frac{2 \gamma_0 - 1}{\gamma_0} - \frac{1}{2} B_0 \mathcal{J}_{0T} \left[ \frac{\mathcal{J}_{2T} (\gamma_0 - 1)^2}{T_0 \mathcal{J}_{0T}} \right],$$

The Non-equilibrium Gas with the Reversible Chemical Reactions and the External Injection of Reagents

The initial system of gas dynamic equations which describes the behaviour of the investigated model of the non-equilibrium medium with the reversible chemical reaction and the external reagent pumping has the form:

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = m_x + m_{\text{mix}},$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \nabla) \mathbf{u} = -\nabla p + f_\mathbf{x} (\mathbf{u}^\mathbf{4} - \mathbf{u})$$  \hspace{1cm} (9)

$$\frac{dU}{dt} = -\frac{p}{\rho} \frac{dp}{dt} = \frac{Q_{\text{at}}}{\rho},$$

$$\frac{dx}{dt} = \frac{(M_x \chi + M_y (1-x))^2}{M_y \rho} \left( \frac{1-x}{M_x \chi + M_y (1-x)} m_D - \frac{v_g \Omega(x, \rho, T)}{M_y \chi + M_y (1-x)} \right)$$
Here, $\rho$ is the density of the mixture, $\vec{u}$ is the velocity of the medium, $m_A$ is the intensity of the reagent A inflow, $m_{mix} < 0$ is the intensity of the mixture outflow, $p$ is the pressure, $\vec{u}^A$ is the velocity of the pure reagent A which is delivered by external reagent source, $U$ is the internal energy per mass unit, $Q_{ext}$ is the external heat source per volume unit which does not associated with heating during chemical reaction, $x$ is the mole fraction of the reagent A, $M_A$ and $M_B$ are molar masses of the reagents A and B, $\Omega(x, \rho, T)$ is the chemical reaction rate.

The system of equations (1) was derived on basis of conservation laws. Two equations should be added to the initial system of the equations (1). It is the equation of the state, in which the pressure depends on the mole fraction of the reagent A:

$$ p = \frac{\rho RT}{M_A x + M_B (1-x)}, $$

where $R$ is the universal gas constant, and the equation for the internal energy per mass unit:

$$ U = \frac{x U_A}{M_A x + M_B (1-x)} + \frac{(1-x) U_B}{M_A x + M_B (1-x)}. $$

Here $U_A = RT/(\gamma_A - 1) + E_A$ is the internal energy per mole unit of the pure reagent A, $U_B = RT/(\gamma_B - 1) + E_B$ is the internal energy per mole unit of the pure reagent B, $\gamma_A$, $\gamma_B$ are adiabatic exponents of the pure reagent A and B, $E_A$ and $E_B$ are chemical energies per mole unit of the pure reagents A and B, respectively.

The external heat source per volume unit $Q_{ext}$ in the system of equations (9) depends on the external inflow and outflow of the reagent. It has the following structure:

$$ Q_{ext} = Q_m + Q_k + Q_A + Q_V $$

Heat sources $Q_m$, $Q_K$, $Q_A$ are related to volume inflow of the reagent A and volume outflow of the reacting mixture. Heat sources $Q_m = m_A(U_A^0 - U)$, $Q_k = m_A(\vec{u}^A - \vec{u})^2/2$ appear due to the different thermodynamical and mechanical states of the inflow and outflow substances, $U_A^0$ is the internal energy per mass unit of the inflow substance A. The heat source $Q_A = -(m_A + m_{mix})p/\rho$ appears due to the mass exchange leading to the additional thermodynamic activity. The heat source $Q_V$ provides the stationary state of the mixture. This source is cooling for exothermic chemical reaction and heating for the endothermic chemical reaction.

The first equation of system (9) is the mass balance equation. The right hand member of the first equation includes the volume mass pumping of the pure reagent A $m_A$ and the volume outflow of the reacting mixture $m_{mix}$. The second equation of system (1) is the Euler equation. Its right hand part includes the ordinary force which related with gradient of pressure and the additional force which related with inflow and outflow of the substance. The inflow and outflow substances have the different mass velocity. It leads to the additional volume force appearance. The additional volume force has the dissipation character. Third equation of system (1) is the energy balance equation.

The fourth equation of system (1) is the equation for mole fraction of the reagent A $x$ which dynamics is defined by the rate of the external mass exchange and the rate of the chemical reaction in fluid. This equation does not include the intensity of the outflow of the mixture $m_{mix}$ because the outflow of the mixture leads to the outflow of the both reagents. In the present work all the analytical results were obtained under general form of the chemical reaction rate model, but during computing, we use the Arrhenius law of the chemical reaction rate.
In Equation (10), $k_1, k_2$ are the rate constants of the direct and reverse reactions respectively, $E_f, E_b$ are energies of activation of the direct and reverse reactions, respectively. However, all the analytical results have been obtained for the any chemical reaction rate model.

Homogeneous stationary state of the investigated mixture, which is described by parameters $\rho_0, T_0, p_0, \tilde{u}_0 = 0, x_0$, is defined by following conditions:

\[
\begin{align*}
\frac{m_A + m_{mix}}{M_A x_0 + M_B (1-x_0)} = & \quad \frac{1-x_0}{M_A x_0 + M_B (1-x_0)} m_A - \nu_b \Omega(x_0, \rho_0, T_0) = 0
\end{align*}
\]

System (11) allows defining the dependence of the stationary mole fraction $x_0$ of the reagent A on the density $\rho$ and temperature $T$: $x = x_{st}(\rho, T), x_0 = x_{st}(\rho_0, T_0)$.

Let us consider the small perturbations propagating in direction of the axis $Oz$. After linearization of the initial system of the equations (9), we have obtained the following equation which describes the perturbations of the density $\rho_1$ in first infinitesimal order:

\[
C_{V0}\tau_{eff} \frac{\partial}{\partial t} (c_v x^2 \frac{\partial^2 \rho_1}{\partial z^2} - \frac{\partial^2 \rho_1}{\partial t^2}) + C_{V0} (c_v x^2 \frac{\partial^2 \rho_1}{\partial z^2} - \frac{\partial^2 \rho_1}{\partial t^2}) = 0
\]

In Equation (12), the specific heat capacities are defined by the following expressions:

\[
\begin{align*}
C_{V0} &= (\partial U / \partial T)_{\rho, x}, \\
C_{p0} &= C_{V0} - ((\partial U / \partial \rho)_{T,x} - p_0 / \rho_0^2) (\partial p / \partial T)_{\rho, x}, \\
C_{V0} &= C_{V0} + (\partial U / \partial x)_{\rho, T} (\partial x_{st} / \partial T)_{\rho}, \\
C_{p0} &= C_{V0} \left((\partial p / \partial T)_{\rho, x} + (\partial p / \partial x)_{\rho, T} (\partial x_{st} / \partial T)_{\rho}\right), \\
x(T(X / \partial p)_{T,x} - p_0 / \rho_0^2 + (\partial U / \partial x)_{\rho, T} (\partial x_{st} / \partial \rho)_{T} )
\end{align*}
\]

where $C_{V0}, C_{p0}$ are the frozen specific heat capacities at constant volume and pressure, respectively, $C_{V0}, C_{p0}$ are the equilibrium specific heat capacities at constant volume and pressure, respectively, all derivatives are evaluated under the stationary parameters $\rho_0$ and $T_0$. Relaxation time $\tau_{eff}$ is defined by following expression:

\[
\frac{1}{\tau_{eff}} = \frac{1}{\tau_r} + \frac{1}{\tau_m},
\]
where \( \tau_r \) is the relaxation time of the chemical reaction, \( \tau_m = \rho_0 / m_A \) is the character time of the external reagent source. The sound velocities are defined by following expressions:

\[
\begin{align*}
&c_{\infty}^2 = \frac{C_{\rho_0}}{C_{\rho}} \left( \frac{\partial \rho}{\partial \rho} \right)_{T,x}, \quad c_0^2 = \frac{C_{\rho_0}}{C'_V} \left( \frac{\partial \rho}{\partial \rho} \right)_{T,x} + \frac{\partial \rho}{\partial x} \left( \frac{\partial x}{\partial \rho} \right)_{T}, \\
&\text{where } c_{\infty} \text{ and } c_0 \text{ are high-frequency and low-frequency sound speeds, respectively.}
\end{align*}
\]

Substituting the solution of equation (12) in the form \( \rho_i = \bar{\rho} \exp(i\omega t - ikz) \), where \( \omega \) - frequency, \( k \) - wave vector, we obtain the dispersion relation

\[
\omega^2 = \frac{c_0^2 C_{\rho_0} - i \omega \tau_{\text{eff}} c_\infty^2 C_{\rho_0}}{C_{\rho_0} - i \omega \tau_{\text{eff}} C_{\rho_0}}.
\]

For the investigated model, we have obtained the generalized nonlinear acoustical equation (GNAE) (1) describing the second infinitesimal order perturbations under condition of the weak dispersion (3). The coefficients of non linearity have the following complex forms:

\[
\Psi_\infty = 1 + \frac{1}{2c_0^2 C_{\rho_0}} \left( c_{\infty}^2 - 2 \rho_0 \left( \frac{\partial \rho}{\partial T} \right)_{p,x} + \frac{\partial \rho_0}{2c_0^2} \left( \frac{\partial^2 \rho}{\partial T^2} \right)_{p,x} + \frac{\partial \rho}{\partial x} \left( \frac{\partial x}{\partial \rho} \right)_{T,x} \right),
\]

\[
\Psi_0 = \frac{\partial \rho_0}{2c_0^2 C_{\rho_0}} \left( \frac{\partial^2 \rho}{\partial T^2} \right)_{p,x} \left( \frac{c_{\infty}^2 - C_{\rho_0}}{C_{\rho_0}} \right) + \frac{\partial \rho_0}{2c_0^2} \left( \frac{\partial^2 \rho}{\partial T^2} \right)_{p,x} + \frac{\partial \rho}{\partial x} \left( \frac{\partial x}{\partial \rho} \right)_{T,x} + \frac{\partial \rho}{2c_0^2} \left( \frac{\partial^2 \rho}{\partial T^2} \right)_{p,x} + \frac{\partial \rho}{\partial x} \left( \frac{\partial x}{\partial \rho} \right)_{T,x}.
\]

\[
\text{STATIONARY WAVES}
\]

Let consider acoustical waves traveling in the positive direction of \( x \). Taking into account \( \beta << 1 \), \( \rho' / \rho_0 << 1 \), we can use the method of the slowly varying profile and reduce GNAE (1) to the following dimensionless form:

\[
\left( \frac{\partial \tilde{\rho}}{\partial y} + \Psi_\infty \frac{\partial \tilde{\rho}^2}{\partial \zeta} \right) - c_{\infty} c_{\gamma_0} \left( \frac{\partial \tilde{\rho}}{\partial y} + \frac{\beta}{2} \frac{\partial \tilde{\rho}}{\partial x} + \Psi_0 \frac{\partial \tilde{\rho}^2}{\partial \zeta} \right) = 0,
\]

(13)

For the stationary waves traveling with the speed \( D = c_{\infty} + w \), Equation (13) is transformed to the form:

\[
\frac{\partial \tilde{\rho}}{\partial \zeta} \left( w - c_{\gamma_0} \right) - \Psi_\infty \tilde{\rho} + \frac{c_{\gamma_0}}{2} \left( w - \frac{\beta}{2} \tilde{\rho} + \Psi_0 \tilde{\rho}^2 \right) = 0
\]

(14)
The solution of Equation (17) can be written in the implicit form [2]:

\[ z = z_0 + \frac{\sigma \rho \varepsilon}{\rho_i} \ln |\tilde{\rho}| + \frac{\sigma (\rho_i - \rho \varepsilon)}{\rho_i} \ln |\tilde{\rho} - \rho_i|, \]  

(15)

where \( z_0 \) is an arbitrary constant,

\[ \sigma = \frac{2\Psi_\infty c_v}{c_{v0}}, \quad \rho_{cr1} = \frac{w}{\Psi_\infty}, \quad \rho_i = \frac{2w - \beta}{\Psi_0}. \]

We have three types of discontinuous solutions. From [2,3], it follows that the first solution with the smooth growth of the disturbance behind the discontinuity can exist only for

\[ \rho_i > \rho_{cr2} \equiv \frac{\beta}{(\Psi_\infty - \Psi_0)}. \]

For \( \rho_{cr1} < \rho_i < \rho_{cr2} \), there can exist stationary waves of another type: behind the discontinuity the disturbance amplitude decreases smoothly from \( \rho_i \) to \( \rho_i \). Solution (15) for \( \rho_i = \rho_{cr1} \) can exist only in a nonequilibrium medium with negative dispersion and has the form of a shock-wave pulse

\[ \tilde{\rho}(z) = \begin{cases} \rho_p \exp \left[ \frac{(z - z_0) c_v \Psi_0}{2\Psi_\infty c_v} \right] & z \leq z_0, \\ 0 & z > z_0, \end{cases} \]

where \( z_0 \) corresponds to the front position, \( \rho_p \) is the pulse amplitude, \( w_p \) is the speed of this pulse. For \( \rho_i < \rho_{cr1} \), the stationary waves do not exist.

**NONSTATIONARY EVOLUTION**

The infinite small width of the shock wave front is connected with the negligibility by the viscosity coefficient \( \eta \) and the thermal conductivity coefficient \( \chi \). Taking into account these small dissipation coefficients transforms Equations (13), (14) to the forms

\[ \left( \frac{\partial \tilde{\rho}}{\partial \xi} + \frac{c_{v0}}{c_{v\infty}} \frac{\partial^2 \tilde{\rho}}{\partial \xi^2} + \mu \frac{\partial^2 \tilde{\rho}}{\partial \xi^2} \right) - \frac{c_{v0} \Psi_\infty}{c_{v\infty}} \frac{\partial \tilde{\rho}}{\partial \xi} + \beta \frac{\partial \tilde{\rho}}{\partial \xi} + \frac{\Psi_0}{2} \frac{\partial^{2} \tilde{\rho}}{\partial \xi^2} = 0, \]

(16)

\[ \mu \frac{\partial^2 \tilde{\rho}}{\partial \xi^2} + \frac{c_{v0}}{c_{v\infty}} \left[ \frac{w - c_{v0}}{c_{v\infty}} - \Psi_\infty \tilde{\rho} \right] + \frac{c_{v0}}{c_{v\infty}} \left[ \frac{w - \beta}{2} \tilde{\rho} + \frac{\Psi_0}{2} \tilde{\rho}^2 \right] = 0. \]

(17)


The initial step-like disturbance with amplitude \( \tilde{\rho} > \tilde{\rho}_{cr} = -2w \mu / (\Psi_\infty - \Psi_0) \) transforms to the first stationary structure. The second structure is obtained for \( \tilde{\rho}_{cr} > \tilde{\rho} > \tilde{\rho}_{cr1} = -2w \mu / (2\Psi_\infty - \Psi_0) \). The steps with amplitudes \( \tilde{\rho} < \tilde{\rho}_{cr1} \) were unstable and broke down into a periodic sequence of stationary pulses. Each pulse had previous form and amplitude \( \tilde{\rho}_p \). Thus, such pulse is the autowave (self-sustaining wave), whose form and amplitude depend on parameters of the nonequilibrium medium only (Figure 1).
CONCLUSIONS

Using the generalized nonlinear acoustical equation gives the condition of instability of a perturbations with a step-like and bell-like initial forms. They disintegrate into the sequence of the self-sustaining waves. Two types of the travelling self-sustained waves (the solitary pulse and roll wave) are obtained in acoustically active media. Their parameters depend only on nonequilibrium medium properties. The obtained stationary structures have a wide spectrum and can’t be described by the known equations of the low- or high-frequency approximations.

ACKNOWLEDGMENTS

This work was partially supported by the Ministry of education and science of Russia, projects 2.560.2011, 14.740.11.0999, 14.740.11.1140, 14.B37.21.0767, 14.132.21.1423, 14.132.21.1440 and by RFBR grants РФФИ 13-01-97001 r_Volga_region_a, 13-01-97005 r_Volga_region_a, 12-01-31229_mol_a

REFERENCES