4pPAb5. Early onset of sound in Rijke tube with abrupt contraction

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Rijke tube is a system convenient for studying thermoacoustic instabilities both experimentally and theoretically. Common Rijke setups involve tubes with constant-area cross-sections. With the goal to reduce supplied heat necessary to excite acoustic modes, a segmented Rijke tube comprising two pipes of different diameters is modeled and constructed. A simplified energy-based model developed in this paper predicts the onset of instability to occur at much lower heat addition rate in a tube with abrupt contraction in comparison with a constant-area tube. Experiments with regulated mean flow and heat supply demonstrate similar tendency. This finding suggests that simpler experimental means can be used for studying thermoacoustic instabilities due to significant reduction of required heat addition and highest temperatures in the system. Some practical devices with complicated resonators may appear to be more prone to instabilities of this sort. Stronger coupling between acoustic modes due to their enhanced non-orthogonality in segmented resonators can result in richer nonlinear dynamics effects.
INTRODUCTION

In many practical systems with heat release and mean flow, such as industrial burners and aerospace motors, acoustic instabilities may develop due to thermoacoustic transformation of supplied heat. Sound amplitudes can reach very high levels affecting system performance or even leading to the system failure. Rayleigh (1945) formulated a criterion for this phenomenon suggesting that acoustic oscillations are encouraged if heat fluctuates in phase with acoustic pressure. Culick (1987) developed a mathematical formalism for calculating thermoacoustic power conversion.

One of the most common experimental systems used for demonstrating and studying thermoacoustic instabilities is a Rijke tube (Rijke, 1859; Raun, 1993). Its classical version represents a vertical pipe with open ends and metallic gauze heated by flame (Fig. 1). At the appropriate position of the gauze (usually near the quarter-tube length from the bottom) and sufficient heat supply, a loud sound is generated corresponding to the excited fundamental acoustic mode of the resonator. In order to decouple supplied heat from mean flow through the tube and to obtain more accurate experimental data, highly-controllable horizontal Rijke setups can be constructed with mean flow produced by air blowers and heat addition provided by electric heaters (Matveev and Culick, 2003).

In most previous studies with Rijke tubes, constant-area pipe sections were employed. Although such shapes have the simplest geometry, many practical devices have more complicated forms. Moreover, other resonator shapes may be more prone to thermoacoustic instabilities. It was recently discovered that a simple segmented tube consisting of two pipe sections with different diameters, such as shown in Fig. 1, leads to about twice lower heating threshold for the sound excitation (Hernandez and Matveev, 2010). The present study addresses modeling of such a system using an energy-based approach.

MATHEMATICAL MODEL

The transition to instability (or sound onset) in a Rijke tube can be found by balancing cycle-averaged thermoacoustic power generation and losses including thermoviscous dissipation the resonator walls and sound radiation from the open tube ends,

\[
\dot{E}_{Td} = \dot{E}_{res} + \dot{E}_{rad}
\]  

(1)

Thermoacoustic power production is estimated using a quantitative form of the Rayleigh criterion derived by Culick (1987), which involves integration over the resonator volume \( V \) and cycle period \( T \),

\[
\dot{E}_{Td} = \frac{2\pi \gamma - 1}{\omega \gamma P_0} \int \int q' \rho' dv dt
\]  

(2)
where $\omega$ is the angular sound frequency, $\gamma$ is the gas specific heat ratio, $p_0$ is the mean pressure, $q'$ is the unsteady component of the heat addition rate per unit volume, and $p'$ is the fluctuating acoustic pressure.

Assuming a compact heater and predominantly convective type of heat transfer from the heater, a linearized form of the unsteady heat addition rate can be expressed via a non-dimensional transfer function $Tr$ relating $q'$ to the acoustic velocity fluctuation $u'$ (Merk, 1957),

$$\frac{\dot{Q}}{\dot{Q}_0} = |Tr| \frac{u'(x, t - \tau) \delta(x - s)}{u_0} \delta \frac{1}{S}$$

where $\dot{Q}_0$ is the steady heat addition rate, $u_0$ is the mean flow velocity, $\tau$ is the time lag of heat transfer, $\delta$ is the delta function, $s$ is the heater position (Fig. 2), and $S$ is the resonator cross-sectional area at the heater location. The magnitude of the transfer function $|Tr|$ and phase delay $\theta = \omega \tau$ depend on the heater geometry. In the present study, these values were determined empirically from experiments by fitting modeling results to test data. The following values are adopted for both constant-area and segmented resonators: $|Tr| = 0.2$ and $\theta = \omega \tau = 6^\circ$.

The fluctuations of pressure and velocity in the fundamental acoustic mode can be presented as follows (Culick, 1976),

$$p'(x, t) = p_0 \eta(t) \psi(x)$$

$$u'(x, t) = \frac{\dot{\eta}(t)}{\gamma k^2} \psi_x(x)$$

where $\eta(t) = A \sin(\omega t)$ is the time-dependent component, $A$ is the non-dimensional amplitude, $\psi$ is the mode waveform, and $k$ is the wavenumber. The waveforms and oscillation frequency are determined by satisfying boundary conditions: pressure release at the open tube ends and pressure and volumetric velocity continuity at the junction between tube segments.

Substituting Eqs. (3-5) into Eq. (2), one can obtain more detailed expression for the acoustic power generation,

$$\dot{E}_{ac} = A^2 |Tr| \frac{\dot{Q}_0}{u_0} \sin(\omega t) \gamma \frac{\psi^2}{k^2} s(s)$$

where $\omega$ is the speed of sound. The loss terms in Eq. (1) can be also expressed via the acoustic mode parameters using standard expressions for thermoviscous dissipation on the resonator walls and sound radiation at the open ends (Swift, 1988),

$$\dot{E}_{ac} = A^2 \pi \frac{\rho a}{2} \left[ \frac{(p_0 \psi')^2}{\rho a^2} \delta_k (\gamma - 1) + \frac{\rho a \psi^2}{\gamma} \delta_v \right] R \text{d}x$$

$$\dot{E}_{rad} = A^2 \pi \frac{\rho a^2}{8} \gamma \left[ \psi_x^2(0) R^4(0) + \psi_x^2(L) R^4(L) \right]$$

where $\rho$ is the gas density, $R$ is the tube radius, and $\delta_k$ and $\delta_v$ are the thermal and viscous penetration depths, respectively (Swift, 1988). When Eqs. (6-8) are substituted into the energy balance (Eq. 1), the non-dimensional
amplitude cancels out, and the critical heat addition rate $\dot{Q}_c$ is obtained from other given parameters that include flow rate, system geometry, and fluid properties. If supplied heat exceeds this value, then the system becomes self-excited and sound is generated. Theoretical predictions for the transition to instability in a Rijke tube are compared with experimental data in the next section.

**EXPERIMENTAL RESULTS**

An experimental investigation of instabilities in Rijke tubes was conducted in a modular system with controllable conditions. A system photograph is shown in Fig. 2. The detailed description of this setup is given by Matveev and Hernandez (2012). Rijke tubes in this system are made of aluminum or steel pipes. They are placed horizontally to minimize the effect of natural convection on mean flow. The heater is a nichrome wire wound on a ceramic ring, which is inserted into the tube from the upstream end. A heating power is controlled via a regulated power supply. Mean flow is produced by and controlled using an air blower. A damping chamber is installed between the Rijke tube and blower to ensure open-end boundary conditions at the tube ends.

![Figure 2](image)

**FIGURE 2.** Experimental Rijke tube system with controllable conditions.

Upon increasing heating power at a given mean flow rate, the system becomes acoustically unstable and loud sound is generated at the fundamental frequency of the resonating Rijke tube. A map showing stability boundaries is given in Fig. 3 for a constant-area tube and a segmented tube with abrupt contraction. The diameter and length of the constant-area tube and in a wider section of the segmented tube are 5.2 cm and 60 cm, respectively. The diameter and length of the narrow portion in the segmented tube are 2.4 cm and 30 cm, respectively. The heater positions were selected to minimize the input heat required for excitation; the distances from the upstream tube end were 12.5 cm and 35 cm for the segmented and constant-area pipes, respectively. The rate of heat transferred to air flowing through the heater is estimated by excluding thermal radiation and heat conduction losses from the supplied electrical power. As one can see in Fig. 3, sound is excited at much lower heat input in the tube with a sudden contraction.

Additional tests with a 90-cm-long constant-area tube indicated that heating power required for sound excitation is even greater than in the short constant-area tube. In the excited regimes, Rijke tubes may also exhibit strongly nonlinear behavior (Matveev, 2003). In the present system, a well-pronounced hysteresis in the sound level at a fixed mean flow rate and a heater position was observed. This effect can be caused by thermal inertia of the system, nonlinear heat transfer from the heater in unsteady flow, non-orthogonality of acoustic modes in the resonator with complex geometry and non-uniform conditions, and possibly other reasons.

Results of the above mathematical theory for the stability boundaries are shown in Fig. 3 by lines. The agreement between experiments and simplified theory is reasonably good. Similar to experimental findings, the mathematical model also predicts significantly lower heat values needed for the sound onset. As follows from calculations, the main reason for decreased threshold of instability is reduced acoustic losses in the segmented setup, which are associated with lower sound frequency. In the tube with an abrupt contraction, the frequency was about 165 Hz, while in the constant-area Rijke tubes the frequencies were about 310 Hz and 200 Hz for the setups with lengths 60 and 90 cm, respectively. The constant-area tubes represent a half-wavelength resonator, while the segmented setup has a resemblance with a quarter-wave resonator.
CONCLUDING REMARKS

Both experimental and modeling studies have demonstrated that instability threshold in a Rijke tube with abrupt contraction is much lower than that in commonly used constant-area Rijke tubes. This may have significant implications for industrial devices where thermoacoustic instabilities are a concern, since practical chambers with heat release often have more complicated geometries than simple constant-diameter resonators. Another possible application of the observed phenomenon is for experimental studies of thermoacoustic instabilities. Lower heating requirements make it easier for researchers to construct controllable Rijke tubes. Segmented resonators may also lead to more interesting nonlinear effects at the onset of instability and in the excited regimes, because of more pronounced non-orthogonality of acoustic modes and complications of mean flow patterns, such as the appearance of recirculation zones at sudden pipe contractions.

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