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1aSP5. Evaluation of system configuration to check the suitability for the sound field rendering using the inverse approach

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Sound field control by the inverse approach based on the acoustical holography is useful to render an arbitrary target sound field within a selected control zone if the target condition is given in a detailed format. This method needs information on the various factors constituting the total system: source array configuration, relative position of source and control region, assigning method of target field condition, conditioning method, etc. Because these factors heavily affect the accuracy of the generated sound field, a proper definition of the problem including all factors related to the system configuration is important. In this work, the method to evaluate the suitability of system to achieve the target sound field was investigated in the viewpoint of efficiency and precision. Because the difference between target and generated sound field strongly depends on the noise and the information error existing in the actual situation, the expected accuracy should be calculated in relation to the characteristics of system transfer matrix. To this end, variances of uncorrelated noise, condition number, and linear independency of the transfer matrix are evaluated to check the suitability of transfer matrix for accurately rendering the sound field with different types of array system.

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INTRODUCTION

Sound field control by the inverse approach employing the theory of acoustical holography is appropriate for the purpose of rendering an arbitrary shape of sound field, if the target condition is given in a form of detailed sound pressure response \(^1\)\(^-\)\(^3\). The method is potentially attractive if a wide target zone is aimed at, and the implemented domain is a small irregular enclosure which is affected from the boundary condition severely and the near field effect is not negligible. Such design approach of inverse sound field rendering is involved with many design parameters by definition. One needs the information on various factors like source array configuration, relative position of source and control region, assigning method of target field condition, conditioning method, boundary conditions, etc. Because these factors heavily affect the accuracy of the generated sound field at the target zone, a proper definition of the problem including all design factors related to the system configuration is very important.

In this work, the evaluation parameters for the suitability of the overall system condition to achieve the desired sound field are investigated in the viewpoint of efficiency and precision. The entire system characteristics are described by a transfer function (TF) between source array and target control region. Treatment method of design factors constituting the TF are studied to obtain the appropriate values fitted to the arbitrary target sound field.

HOLOGRAPHIC INVERSE DESIGN OF SOUND FIELD

If the target sound field response is given by \(H_{\text{target}}\), the source condition \(A_F\) to archive it can be obtained by the least-square solution as\(^1\)

\[
H_{\text{target}} = G A_F, \quad \tilde{A}_F = \left(G^T G\right)^{-1} G^T H_{\text{target}},
\]

where \(G\) is the transfer matrix between sources and control region rearranged as\(^3\)

\[
G = \begin{bmatrix} G_{s,1} & G_{s,2} & \cdots & G_{s,N} \end{bmatrix} \begin{bmatrix} T_1 Z_1 & T_2 Z_2 & \cdots & T_N Z_N \end{bmatrix},
\]

\(G_{s,n}\) is the column vector of TF related to the \(n\)-th source, \(T_n\) is the relative source strength distribution constructing the \(n\)-th source, and \(Z_n\) is the relation between electrical input and resulting mechanical force.

As a first step of the inverse design, the initial configuration of source and target control region should be selected. This is usually decided by practical constraints such as number of control sources and available space in a room. With this configuration, the TF between sources and control region can be estimated. To construct a transfer matrix, either measurement or numerical estimation can be employed. After conditioning for TF\(^6\), the target field condition should be assigned based on the analytical formulation of sound propagation and the solution can be obtained by taking the pseudo-inverse.

The conditioning process to obtain the practically plausible solution can be accomplished by optimizing the related parameters\(^5\),\(^6\). The number of sources is most important constraints among all parameters. The contribution of each source is determined by its relative position against the control region. To evaluate the amount of contribution of each source among candidate positions, linear independency of source elements in the transfer matrix provides useful information. To quantify such linear independency, the effective independence (EfI) method can be applied\(^7\). With the EfI method, the linear independency of each matrix element is given by EfI value which is estimated as\(^7\)

\[
G = U \Lambda W^H, \quad E_w = \text{diag} \left( W_u W_u^H \right),
\]

where \(W_u\) is the 1st to \(a\)-th columns of the \(W\) vector after the singular value decomposition of \(G\). The EfI value ranges 0-1, of which a high value means a highly linear independent condition than the other rows or columns. Therefore, the source located at the position of highest EfI value can be chosen as the most proper position of source.

Input power is another important design parameter and it is usually optimized by\(^7\)

\[
J = \left[H_{\text{target}} - G \tilde{A}_F\right]^H e + \beta \tilde{A}_F^H \tilde{A}_F, \quad \tilde{A}_F = \left[(G)^H (G) + \beta I\right]^{-1} (G)^H H_{\text{target}},
\]

where \(\beta\) is the weighting constant for input power. Actually, these descriptions are identical to the typical form of Tikhonov regularization to obtain stable inverse solution of ill-conditioned matrix\(^8\). This means that the minimization of power consumption and stabilization of solution can be achieved by same processes.
SUITABILITY OF SYSTEM CONDITION FOR FIELD RENDERING

Accuracy of the constructed sound field in the ideal situation without any noise and stability of the generated sound field in a practical situation are considered. The RMS error is the simplest measure to evaluate the accuracy of the generated sound field; however, the low RMS error does not always mean better rendering quality. This is due to the fact that RMS error is to evaluate scalar difference, but the actual sound field is composed of propagating vectors. There are some cases requiring a measure directly related to the target sound field condition such as sound pressure ratio between designated areas when the sound is to be focused at a specific area. In spite of many limitations in using the RMS error, one can use it as a reference measure for evaluating the control result in general.

It is not easy to quantify the level of difficulty in rendering a sound field for a given condition. If the effort to generate a specific sound field condition is concerned, one may quantify the difficulty of rendering process as a function of the essential control parameters to achieve a specific level of accuracy. Typical two parameters are: number of acoustic actuators as active sources and the required input power consumption. If the target sound field characteristic is already specified, the RMS error can be obtained through a simulation under the ideal condition, which does not contain any noise and discrepancy with the initial setting. By using this RMS error under ideal condition, one can decide that the most suitable system for the rendering is the case that requires the smallest source parameters. However, such an evaluation with single or small number of the given target sound fields is not thought to work similarly in practical situations that are involved in various types of sound field. One can therefore choose an evaluation method using the information included in the transfer matrix as suitable for the general purpose.

Condition number of TF is one of the important parameters to investigate the characteristics of matrix. It can be obtained by the ratio of the minimum to maximum of singular value. In an actual enclosed field for the control, there are a number of uncertainties in comparison with its design stage. Uncertainties related to the source array system can be considered as noise input and it can be written as

\[ p_{\text{tot}} = p_f + \delta p = H_{\text{target}}(x_i + \delta x) = H_{\text{target}}x_i + GA_{F,\text{target}}\delta x, \]

where \( \delta x \) means the noise to the input signal. Denoting the condition number as \( \text{CN} \), the range of resultant noise radiated into the sound field can be estimated by

\[ \frac{\delta H}{H_{\text{target}}} < \text{CN}(G) \frac{A_f}{A_F} \frac{(x_i/\delta x)}{x_i} = \text{CN}(G) \frac{\delta x}{x_i}. \]

Not only the range of noise amplification, but also the required input power is comparable with condition number. Because the regularization process such as Eq. (4) omits the low singular value, the condition number is also increased. Therefore, the condition number of TF can be employed as an observation parameter to compare the required input power obtained by the regularization and the input power minimization. Although the condition number shows the bound of noise amplification, it sometimes suggests too large value for practical usage. From this reason, the use of expectation of variance would be more appropriate in many cases:

\[ E\left[ \left( \bar{H} - H_{\text{target}} \right) \right] = E\left[ tr\left( \left( \bar{H} - H_{\text{target}} \right) \left( \bar{H} - H_{\text{target}} \right)^H \right) \right] = \sigma^2 tr\left( G^H G \right)^{-1}. \]

Notwithstanding the fact that the foregoing two parameters are appropriate in the evaluation of the effect of noise and the relative level of required power, these two parameters do not give any information on the expected error in an ideal condition, i.e., called an inherent error, which is most important in evaluating the system performance. One possibility to evaluate the inherent error is to adopt a representative sound field, e.g., plane wave propagation, as a test example. However, it is hard to say that such a simple case can represent general practical sound field. From this reason, a mathematical definition of error employing a resolution matrix can be a better choice. The resolution matrix \( R \) is defined as

\[ R = GG^* = G \left( G^T G \right)^{-1} G^T. \]

By using Eq. (8), the inherent error is given by

\[ \bar{H} - H_{\text{target}} = G\hat{A}_F - H_{\text{target}} = G \left( G^T G \right)^{-1} G^T H_{\text{target}} - H_{\text{target}} = \left[ R - I \right] H_{\text{target}} = G_{\text{res}} H_{\text{target}}, \]

where \( N \) means the number of field points to be estimated. The performance evaluation parameter in achieving a desired sound field is chosen as

\[ E_{\text{inherent}} = \sum_m \sum_n G_{\text{res},m,n}. \]
This parameter can be a reference value to compare the maximum accuracy between the generated sound fields with respect to fulfilling the desired sound field. However, the tendency of variation of $E_{inherent}$ would not be very similar to the RMS error for all types of target sound fields because the difference between the realized field and the desired field depends on the target field condition vector $H_{target}$ as indicated in Eq. (9).

**TEST EXAMPLES**

As discussed in the previous section, the condition number, expected variance, and singular value of TF can be used to evaluate the suitability of system condition for the precise sound field rendering in a selected region. The RMS error and the required input power to achieve the same RMS error are used to support the prediction results. For the validation, two examples of line and circular array system are adopted for the simulation.

**Field Generation by a Line Array**

Figure 1 shows the configuration of source array and the target control zones. The source system consists of an equally spaced line array with 16 spherical shaped loudspeakers. Two target control zones are assigned to realize different shapes of wave fronts. In Case 1, the whole control zone is placed within a distance of a wavelength $\lambda$ from the source array, of which the field condition in the domain can be regarded as the near-field. In Case 2, the distance of a half of control region from the source is less than $\lambda$, while the whole control zone is in the farfield in Case 3.

To construct the transfer matrix, the boundary element method is employed and the model for each loudspeaker is composed of 284 linear triangular elements with the high frequency limit of $6v/\lambda$, in which $v$ is the speed of sound. With this model, the transfer matrix between the source array and the control region is estimated as listed in Table 1. The condition number and the expected standard deviation of error increase drastically according to the distance between the source array and the control region. If the desired field for each target zone is set as traveling plane wave, in the left zone, and spherical wave, in the right zone, the generated sound field can be simulated as depicted in Fig. 2. Without noise, the desired sound field condition is successfully achieved in the target control region. However, one can observe a relatively high sound level at the outside of control region due to the consumption of high input power as can be seen in Table 1. Figure 2(b) suggests that only Case 1 can obtain similar wave fronts to the desired condition with input noise, which can be foreseen by observing the expected value of $\sigma$ for each case. The RMS error without noise in Case 1 is largest among the tested cases and $E_{inherent}$ is also larger than the other cases. In case of near field, a lot of non-propagating components persist in actual situation although the desired field condition does not include any non-propagating components, that is one reason of relative high RMS error in Case 1.

![Figure 1. Configuration of the source array and the target control region.](image)

**TABLE 1.** Estimated parameters to evaluate the characteristics of transfer matrix and parameters related to the generated sound field obtained by the estimated source condition.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition number</td>
<td>150</td>
<td>1.24e+05</td>
<td>2.54e+07</td>
</tr>
<tr>
<td>Expected $\sigma$</td>
<td>5</td>
<td>5.05e+03</td>
<td>1.24e+06</td>
</tr>
<tr>
<td>$E_{inherent}$</td>
<td>483</td>
<td>474</td>
<td>475</td>
</tr>
<tr>
<td>RMS error w/o noise (%)</td>
<td>25.2</td>
<td>15.2</td>
<td>11.1</td>
</tr>
<tr>
<td>Required input power (dB re. case 1)</td>
<td>0</td>
<td>52</td>
<td>95</td>
</tr>
</tbody>
</table>
Field Generation by a Circular Array

Next, a circular array with 18 sources, which are the same speaker units employed in the previous example, controls the interior rectangular zones that are designated at several different positions as illustrated in Fig. 3. The transfer matrix is constructed as the same manner with the previous example. The desired sound field is a travelling plane wave emanated from the bottom and heading the upper side.

Figure 4 shows the estimated parameters related to the characteristics of transfer matrix for each case. For the same control area, the case with a target zone closer to the center of circular array appears most stable having small values of both condition number and expected $\sigma$. The simulated sound field generated by the estimated source condition is shown in Fig. 5. The estimated RMS error and required input power for this condition are in Fig. 6. The required input power as a function of frequency is similar to the variation of condition number as well as $E_{\text{inherent}}$, but the latter does not always very similarly matched for each case.

![FIGURE 2](image-url) Simulated sound field with the estimated source condition at 1.0c Hz: (a) without noise, (b) with noise (S/N=20 dB).

![FIGURE 3](image-url) Configuration of the source array and the target control region.

![FIGURE 4](image-url) Estimated parameters to evaluate the characteristics of transfer matrix: (a) condition number, (b) expected standard deviation of error due to noise, (c) $E_{\text{inherent}}$. 
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because the final result largely depends on the characteristics of target sound field condition which is not clearly evaluated. The evaluation parameters do not ensure to find the best configuration for the rendering of the desired sound field quality of generated sound field compared to the prescribed desired field in general. However, the two performance cannot deal with relatively small difference for observing some considerable result.

One can choose a performance evaluation parameter using the information included in the transfer matrix that to some values, such as the number of sources and the required input power to obtain a specific level of accuracy, for the stability of solution, and the inherent error for RMS error of generated sound field without noise. For two examples with different number for required input power, the expected standard deviation of error due to noise for the stability of solution, for observing some considerable result.

Therefore, either the condition number or the expected standard deviation can be used in the prediction of the quality of generated sound field compared to the prescribed desired field in general. However, the two performance evaluation parameters do not ensure to find the best configuration for the rendering of the desired sound field because the final result largely depends on the characteristics of target sound field condition which is not clearly fixed in the design stage.

**FIGURE 5.** Simulated sound field with the estimated source condition: (a) $1.5c$ Hz, (b) $2.0c$ Hz.

**FIGURE 6.** (a) RMS error without noise, (b) relative input power for the estimated source conditions.

**CONCLUSIONS**

In this work, the parameters for the evaluation of the suitability of the overall system condition to achieve the desired sound field are investigated in the viewpoint of efficiency and precision. The required control efforts, limited to some values, such as the number of sources and the required input power to obtain a specific level of accuracy, are the observation parameters; however, those parameters largely depend on the shape and location of target sound field. One can choose a performance evaluation parameter using the information included in the transfer matrix that is suitable for the general purpose. Three performance parameters are adopted as observation indices: the condition number for required input power, the expected standard deviation of error due to noise for the stability of solution, and the inherent error for RMS error of generated sound field without noise. For two examples with different configurations of the array system, it is observed that the condition number and the expected $\sigma$ exhibit same tendency in relative input power and noise amplification due to noisy input. With the inherent error, however, one cannot deal with relatively small difference for observing some considerable result.
ACKNOWLEDGMENTS

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